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Feedback Control of Aerodynamic Flows

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Every airplane has an associated external flow field that is extraordinarily rich in terms of its flow physics. It is intuitively obvious that the shape and configuration of the airplane influences the external flow physics, and the nature of the flow field influences the aerodynamic performance. An airplane is designed to perform optimally under certain operating conditions. Its aerodynamic performance is, consequently, sub-optimal at neighbouring operating points. In order to acheive close to optimum performance under such conditions, we need to be able to control the behaviour of the external flow field as required.

The objective of this paper is to explore means for flow control, and derive the necessary control laws. It should be noted that the external flow is nonlinear, and hence finding a feedback control law is a mathematically hard problem.

I. Introduction

An airplane, by its very design, is a flow control device. This becomes clear when one considers a steady air-flow with and without the airplane. Obviously, the very presence of the airplane alters the flow pattern, significantly so. It is not too hard to believe further that the external shape of the airplane determines the exact nature of the surrounding flow-field. It is also fairly obvious that the exact nature of the surrounding flow-field determines aerodynamic performance.

A good measure of aerodynamic performance is usually the drag at cruise conditions. Low drag immediately translates to low fuel consumption, and hence lower operating costs. It has hence been of great interest to the aerodynamics community to find the airplane/wing shapes that have the lowest drag at cruise conditions. During the latter part of the last century, shape optimization techniques for airplanes using CFD and Optimal Control Theory have been studied and perfected by Jameson and his associates ([1], [2], [3]). Shape optimization, however, has the huge disadvantage that the aircraft design is over-optimized for a single operating condition. Any deviation from the design operating condition results in very sub-optimal performance. An alternate method of affecting the flow-field behaviour would prove to be very beneficial for such cases.

Flow control is also talked about in the context of airplane maneuverability. This concept of flow-control has been intimately associated with airplane design ever since the design of the Wright Flyer. The most innovative thing about the Wright Flyer was of course, the concept of wing-warping to achieve control. This has, however, instilled in us the firm belief that any device that can be used to control an airplane should look like an elevator or a rudder.

In this day and age, where CFD has advanced to the stage where aerodynamic design using CFD has become a routine task, and MEMS sensors and actuators are prevalent everywhere, this thought appears fairly crude. We are now in a position to implement localized flow control devices. This has the advantage that we would be spending no more energy than absolutely necessary. We can also control a wider range of flow phenomena using such devices: delay stall, achieve better maneuverability, prevent loss of control authority, reduce drag due to shocks, seperation, and transition to turbulence, etc.

What we need to do to get there, is to have a foolproof way of modeling the flow, the sensors and the actuators, and design the control schemes. The present paper deals with setting up such a computational control framework, to reduce shock based drag. The actuators are jets in the walls through which there is a small mass flow: either by way of blowing or suction. The concept of flow control discussed in this paper

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relies heavily on the Adjoint Methods developed by Jameson and his associates over the last few years. We show how it is possible to reduce drag by blowing and suction and also discuss patterns for feedback flow control.

For the case of fluid flow control, all previous attempts have included the human in the loop. This is feasible if the number of controllers is very small and their behaviour is well documented and reproducible. A very fine degree of control can be achieved only if a large number of miniature controllers are used. In such cases, it becomes absolutely necessary to have a completely automated control mechanism, right from making state measurements down to making control corrections using the actuators.

Considerable stress needs to be placed on the concept of *Feedback Control* ([4]). Feedback Control implies that the current state of the system is always measured, and suitable control input derived such that the system is pushed to a desired optimal state.

II. Virtual Aerodynamic Shaping using Blowing and Suction

A basic review of ideal fluid aerodynamics reveals that including a mass source on the survace of an airfoil has the effect of increasing the curvature and including a mass sink has the opposite effect. Given that, for a pre-determined performance measure, every operating condition has an optimum shape, it follows that a combination of a shape that is optimal for one operating condition, combined with suitably placed sources and sinks will result in a system that meets optimum performance criteria for a wide range of operating conditions.

For the purposes of this study, steady jets are used in order to simplify numerical modeling. Mass flows are prescribed at the wall, and the jets are modeled so as to satisfy the normal velocity/flux conditions at the wall. In addition, the nett. mass flow through the wall is assumed to be zero.

$$\int_{\mathcal{B}} \rho q_n = 0.$$

Here $\mathcal{B}$ is the boundary of the aerodynamic surface, and ρq_n is the prescribed normal mass flux.

III. Optimal Feedback Control of Dynamical Systems

One of the most commonly encountered problem in Control Theory is that of finding the optimum path taken by a system to reach a given state. ([7]). Consider a linear system be described by the following dynamical equation

$$\dot{\mathbf{x}} = A\mathbf{x} + B\mathbf{u} \tag{III.1}$$

where $\mathbf{x}$ is the state of the system at any time and $\mathbf{u}$ is the control input required. An optimal path is one that minimizes a cost function of the form

$$J = \int_0^T \left(\mathbf{x^T}Q\mathbf{x} + \mathbf{u^T}R\mathbf{u}\right) dt + \mathbf{x^T}Q_f\mathbf{x} \tag{III.2}$$

where Q and Q_f are positive semi-definite and R is positive definite.

The optimal control $\mathbf{u(t)}$ can be found by writing down the adjoint equations in the linear case by assuming that the adjoint solution is a linear function of the state. It is found that the optimal control $\mathbf{u(t)}$ can be represented as a feedback control

$$\mathbf{u(t)} = K(t)\mathbf{x(t)}. \tag{III.3}$$

The determination of Optimal Feedback Laws for nonlinear systems is generally intractable.

The theory of Optimal Control for systems governed by PDEs may be found in the book by J. L. Lions. Optimal control theory was applied to Aerodynamic Shape Optimization by Jameson and his associates. In the present paper, we show that feedback control laws based on the adjoint equations can be derived and used for a class of Inverse Problems. The complete development is presented in appendices A and B. Equation (B.9) clearly shows that the Adjoint Gradient depends on the flow variables at the boundaries and is hence clearly Feedback Control.

$$\delta I = \int_{\mathcal{B}_\xi} \left(\frac{\partial \mathcal{M}}{\partial \rho q_n} \delta(\rho q_n) \right) d\mathcal{B}_\xi - \int_{\mathcal{B}_\xi} \left(\psi_1 + \psi_2 u + \psi_3 v + \psi_4 w + \psi_5 \left(E + \frac{P}{\rho} \right) \right) \delta \rho q_n d\mathcal{B}_\xi \ . \quad \text{(III.4)}$$

It should be noted however, that the adjoint equation is solved using the computed state of the flow field and not the measured state. This bypasses the need for global measurements of the flow-field.

IV. Results - Inverse Pressure Design

An adjoint based control law was first derived and implemented for an inverse aerodynamic case. An rae-82 airfoil was optimized for minimum drag at a Mach number of 0.75. The pressure distribution of the optimized section was used as the target distribution for the flow control case, where blowing and suction is used to mimic the shape changes that lead to the desired pressure changes.

The original(solid) and optimized(dotted) airfoil are shown in figure IV.1. The blowing and suction

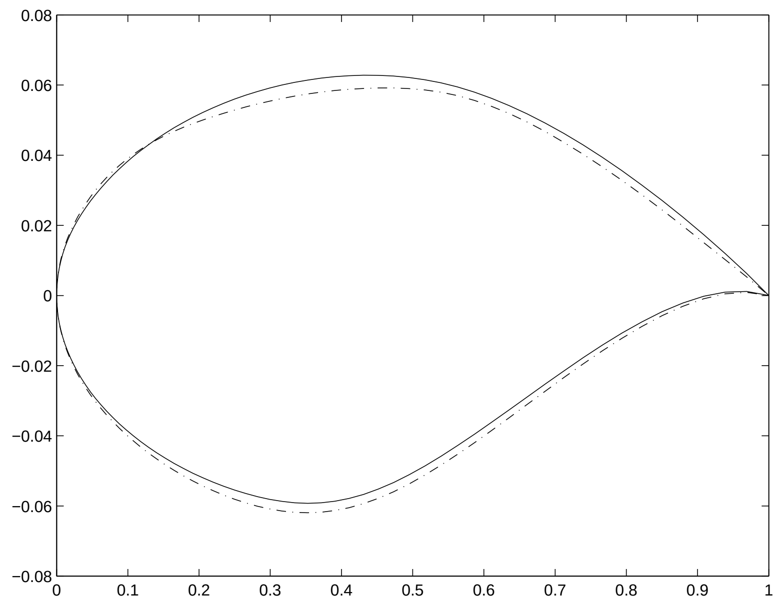

Figure IV.1. rae-82 optimized for minimum drag at Mach 0.75: original - solid line and optimized - dotted line

velocities that produce the same pressure distribution are shown in figures IV.2 and IV.3. The Pressure distributions before and after applying flow control are shown in figures

V. Conclusions

A control law based purely on feedback was designed and tested computationally. This control law works very well in two-dimensional and three-dimensional Inverse design problems. The blowing and suction patterns are shown to correspond to equivalent shape changes. This control law can be used in conjunction with optimum shape design to do multi-point design where we always fly at a configuration that is a minimum drag configuration.

VI. Acknowledgements

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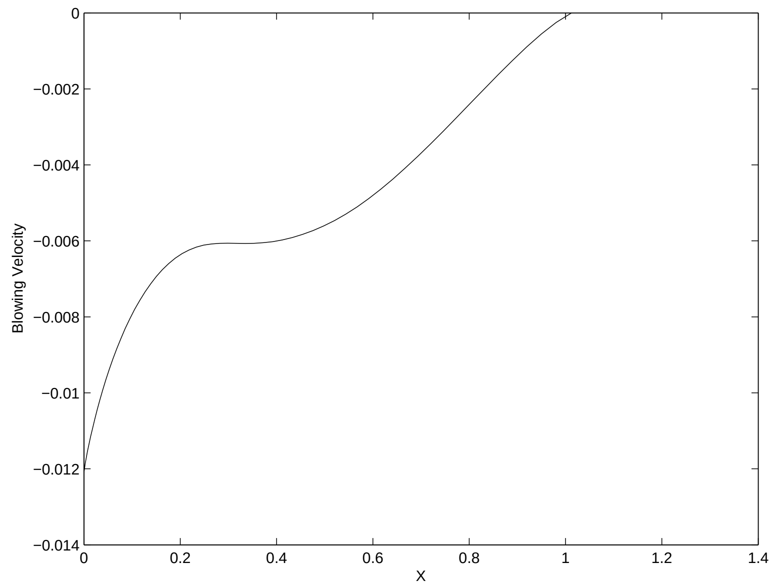

Figure IV.2. rae-82: Flow control Velocities on the lower surface for Inverse Design

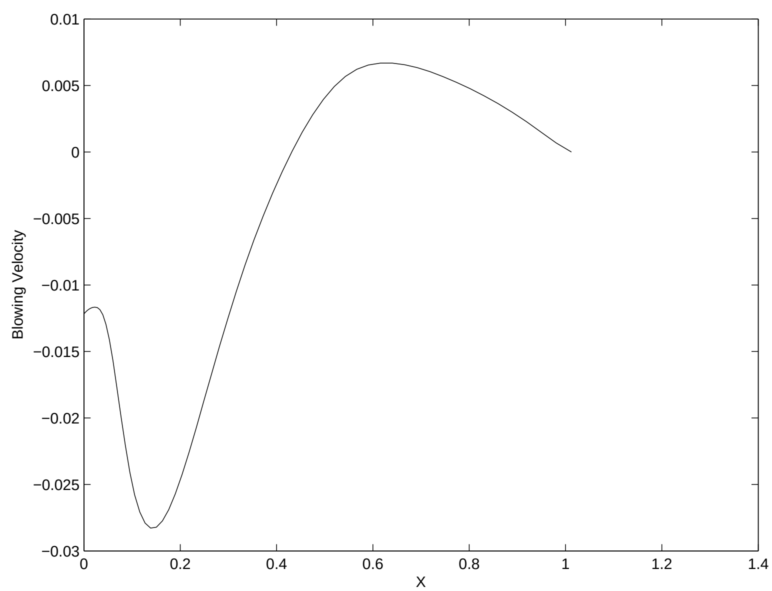

Figure IV.3. rae-82: Flow control Velocities on the upper surface for Inverse Design

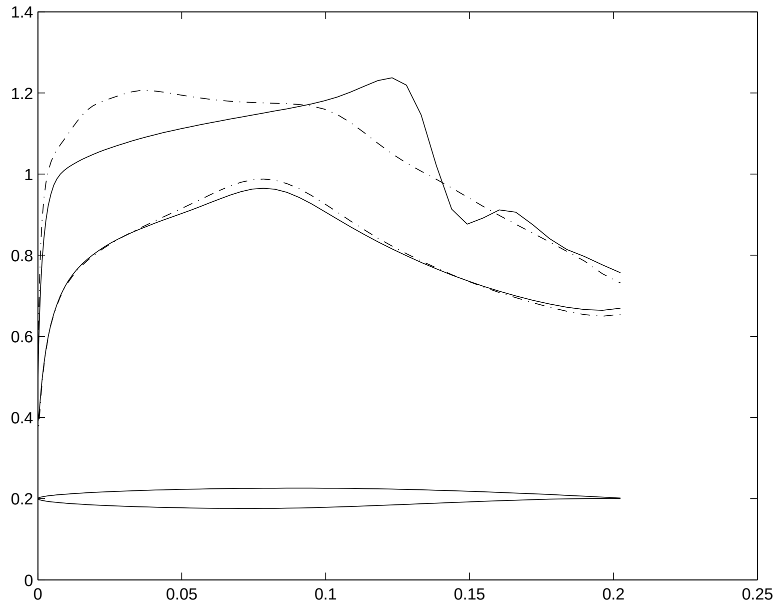

Figure IV.4. Pressure distributions: target(solid) and actual(dotted) before flow control

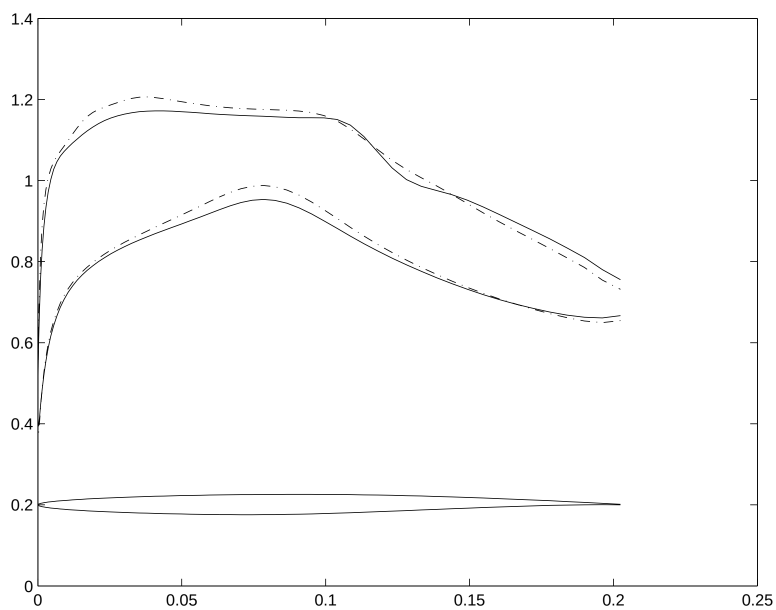

Figure IV.5. Pressure distributions: target(solid) and actual(dotted) after flow control

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A. The Euler Equations for Fluid Flow with Blowing at the Walls

In this paper, the fluid flow is modeled using the Euler Equations. The Euler Equations model the behaviour of invicid, compressible fluids. They are

$$\frac{\partial w}{\partial t} + \frac{\partial f_i}{\partial x_i} = 0 \ . \tag{A.1}$$

Here x_i represent the cartesian co-ordinate directions, w the state variables, and f_i are the corresponding flux vectors, given by

$$w = (\rho, \rho u, \rho v, \rho w, \rho E) \ , \tag{A.2}$$

and,

$$f_i = (\rho u_i, \rho u_i u + \delta_{i1} P, \rho u_i v + \delta_{i2} P, \rho u_i w + \delta_{i3} P, \rho u_i H) \ . \tag{A.3}$$

The Steady State Euler Equations can be written in weak conservation form as follows

$$\int_{\mathcal{B}} n_i \phi^T f_i(w) d\mathcal{B} = \int_{\mathcal{D}} \frac{\partial \phi^T}{\partial x_i} f_i(w) d\mathcal{D} \ , \tag{A.4}$$

where ϕ is any test function. If a transformation is made from physical space to computational space, defined by the mapping functions

$$K_{ij} = \left[\frac{\partial x_i}{\partial \varepsilon_j}\right] \ , \quad J = det(K) \ , \quad K_{ij}^{-1} = \left[\frac{\partial \varepsilon_i}{\partial x_j}\right] \ , \tag{A.5}$$

and

$$S = JK^{-1} \ , \tag{A.6}$$

the Euler Equations (A.4) become

$$\int_{\mathcal{B}_\xi} n_i \phi^T S_{ij} f_j(w) d\mathcal{B}_\xi = \int_{\mathcal{D}_\xi} \frac{\partial \phi^T}{\partial \xi_i} S_{ij} f_j(w) d\mathcal{D}_\xi \ . \tag{A.7}$$

The boundary conditions for the case where we have blowing or suction at the boundary can then be prescribed in terms of the blowing velocity as follows

$$F_2 = (\rho q_n, \rho q_n u + S_{21} P, \rho q_n v + S_{22} P, \rho q_n w + S_{23} P, \rho q_n H) \ , \tag{A.8}$$

where ρq_n is the prescribed mass flow at the boundary, initially set to zero in the design problem.

B. The Adjoint Equations for Boundaries with Blowing and Suction: Existence of a Feedback Control Law for the Euler Equations

Let us assume that we are trying to minimize a cost function of the form

$$I = \int_{\mathcal{B}_\xi} \mathcal{M}(w, \rho q_n) d\mathcal{B}_\xi \ , \tag{B.1}$$

The constraint is given by the Euler Equations (A.7,A.8). Since equation A.7 is true for any test function ϕ, we can choose ϕ to be the adjoint variable ψ. We can then add the constraint given by the Euler Equations to B.1 to form the augmented cost function given by

$$\begin{aligned} I &= \int_{\mathcal{B}_\xi} \mathcal{M}(w, \rho q_n) d\mathcal{B}_\xi \\ &- \int_{\mathcal{B}_\xi} n_i \psi^T S_{ij} f_j(w, \rho q_n) d\mathcal{B}_\xi \\ &+ \int_{\mathcal{D}_\xi} \frac{\partial \psi^T}{\partial \xi_i} S_{ij} f_j(w, \rho q_n) d\mathcal{D}_\xi \ . \end{aligned} \tag{B.2}$$

Taking the first variation of the Cost Function we have

$$\begin{aligned} \delta I &= \int_{\mathcal{B}_\xi} \left(\frac{\partial \mathcal{M}}{\partial w} \delta w + \frac{\partial \mathcal{M}}{\partial \rho q_n} \delta(\rho q_n) \right) d\mathcal{B}_\xi \\ &- \int_{\mathcal{B}_\xi} n_i \psi^T S_{ij} \left(\frac{\partial f_j}{\partial w} \delta w + \frac{\partial f_j}{\partial \rho q_n} \delta(\rho q_n) \right) d\mathcal{B}_\xi \\ &+ \int_{\mathcal{D}_\xi} \frac{\partial \psi^T}{\partial \xi_i} S_{ij} \left(\frac{\partial f_j}{\partial w} \delta w + \frac{\partial f_j}{\partial \rho q_n} \delta(\rho q_n) \right) d\mathcal{D}_\xi \ . \end{aligned} \tag{B.3}$$

Choosing our computational co-ordinate systems so that $\xi_2 = 0$ corresponds to the wing surface, we have

$$F_2 = \begin{bmatrix} \rho q_n \\ \rho q_n u + S_{21} P \\ \rho q_n v + S_{22} P \\ \rho q_n w + S_{23} P \\ \rho q_n E + q_n P \end{bmatrix} . \tag{B.4}$$

Therefore,

$$\begin{aligned} \delta F_2 &= \begin{bmatrix} 1 \\ u \\ v \\ w \\ E + \frac{P}{\rho} \end{bmatrix} \delta(\rho q_n) \\ &+ q_n \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ -u & 1 & 0 & 0 & 0 \\ -v & 0 & 1 & 0 & 0 \\ -w & 0 & 0 & 1 & 0 \\ (\gamma - 1)\left(u^2 + v^2 + w^2\right) - \gamma E & -(\gamma-1)u & -(\gamma-1)v & -(\gamma-1)w & \gamma \end{bmatrix} \delta(w) \\ &+ (\gamma - 1) \begin{bmatrix} 0 \\ S_{21} \\ S_{22} \\ S_{23} \\ 0 \end{bmatrix} \begin{bmatrix} \frac{1}{2}\left(u^2 + v^2 + w^2\right) & -u & -v & -w & 1 \end{bmatrix} \delta(w) \end{aligned} \tag{B.5}$$

We then have the adjoint equations

$$S_{ij} \frac{\partial f_j}{\partial w}^T \frac{\partial \psi}{\partial w} = 0 \text{ , on } \mathcal{D}_\xi \ , \tag{B.6}$$

and

$$\frac{\partial \mathcal{M}}{\partial w} = \psi^T \frac{\partial F_2}{\partial w} \text{ , on } \mathcal{B}_\xi \ . \tag{B.7}$$

We also observe that

$$\frac{\partial f_j}{\partial \rho q_n} = 0 \text{ , on } \mathcal{D}_\xi \ . \tag{B.8}$$

The expression for the adjoint gradient then becomes

$$\begin{aligned} \delta I &= \int_{\mathcal{B}_\xi} \left(\frac{\partial \mathcal{M}}{\partial \rho q_n} \delta(\rho q_n) \right) d\mathcal{B}_\xi \\ &- \int_{\mathcal{B}_\xi} \left(\psi_1 + \psi_2 u + \psi_3 v + \psi_4 w + \psi_5 \left(E + \frac{P}{\rho} \right) \right) \delta \rho q_n d\mathcal{B}_\xi \ . \end{aligned} \tag{B.9}$$

The gradient is then modified to account for the fact that the nett. mass flow through the boundaries is zero.

From equation (B.7) and (B.9) it is clear that the Adjoint Gradient depends only on the Flow and Adjoint Variables at the Boundaries. Therefore this is clearly a case of Feedback Control, where the feedback is the values of the state variables at the boundary.