NLF Airfoil and Wing Design by Adjoint Method and Automatic Transition Prediction

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This paper describes the application of optimization technique based on control theory for natural-laminar-flow airfoil and wing design in viscous compressible flow modeled by the Reynolds averaged Navier-Stokes equations. A laminar-turbulent transition prediction module which consists of a boundary layer method and two e^N -database methods for Tollmien-Schlichting and crossflow instabilities is coupled with flow solver to predict and prescribe transition locations automatically. Results of the optimization will demonstrate that an airfoil can be designed to have the desired favorable pressure gradient for laminar flow and the new airfoil can be redesigned for higher Mach number for performance benefits while still maintains reasonable amount of laminar flow. For 3D wing, a cost function which is used to minimize the drag and, at the same time, try to maintain the suction peak at 40 ~ 50% chord is used. The redesigned wing configuration will demonstrate an overall improvement not only at a single design point, but also at off-design conditions. The results prove the feasibility and necessary of incorporating laminar-turbulent transition prediction with flow solver in natural laminar-flow wing design.

I. Introduction

With the increasing of computational power, the CFD has become a routine tool used for aerodynamic analysis and provides reasonably accurate results. However, the ultimate goal in the design process is to find the optimum shape which maximizes the aerodynamic performance. The performance index might be the drag coefficient at a fixed lift, the lift-to-drag ratio, or matching the desired pressure distribution. In optimum shape design problems, the true design space is a free surface which has infinite number of design variables and will require $N + 1$ flow evaluations for N design variables to calculate the required gradients necessary for gradient-based optimization technique. Here we treat the wing as a device which controls the flow to produce lift with minimum drag and apply the theory of optimal control of systems governed by partial differential equations. By using the optimum control theory, we can find the Frechet derivative of the cost function with respect to the shape by solving an adjoint equation problem. The total cost, which is independent of number of design parameters, is one flow plus one adjoint evaluation and this makes this technique very attractive for the optimum shape design. After the Frechet derivative has been found, we can make an improvement by making a modification in a descent direction and the process repeats. Since this method was first proposed by Jameson,² it has been proved to be very effective in wing shape optimization.^{3,4}

In the flow calculation and shape optimization of laminar-flow airfoil by RANS equations, it is necessary to prescribe the locations where the flow transits from laminar to turbulent and apply the turbulence model in the turbulent flow regions. The transition locations are critical in order to obtain accurate results, e.g. the drag coefficient and lift-to-drag ratio, and those information are usually provided by the assumed transition locations based on the engineering judgement or experimental data if it is available. However, at the initial design stages, this information is usually not available and a direct numerical simulation at such high Reynolds number is not practical. Hence it is necessary to acquire the information of transition locations based on the solutions of RANS solver and transition prediction method which is much less expensive than direct numerical simulation.

In industrial design applications, the most widely used method for streamwise transition prediction is the e^N -database method. This is a method based on linear stability theory and experimental data. Linear

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stability theory states that the initial disturbances grow or decay linearly in steady laminar flow and the flow will remain laminar if the initial disturbances decay. In the derivation of linear stability equations, each flow variable is decomposed into a mean-flow term plus a fluctuation term and substitute into flow equations. Because the fluctuations are assumed to be small, their products can be neglect. With the addition of parallel flow assumption, a set of partial differential equations describing the grow or decay of the disturbances can be derived and detailed derivation can be found in.⁵ In the 1950s, van Ingen⁶ and Smith and Gamberoni⁷ independently used the results from the linear stability theory and compared them with experimental data of viscous boundary layers. They found that transition from laminar to turbulent frequently happened when the amplification of disturbances calculated from stability theory reached about 8100. This corresponds to e^N where N equals to 9 and this is the well known criterion for Tollmien-Schlichting instabilities. The present authors choose the e^N -database method for transition prediction because it has been proven⁸ to provide reasonably accurate transition locations on airfoils.

For a 3D swept wing, the pressure varies not only in the streamwise direction, but also in the spanwise direction. This variation of pressure in the spanwise direction consequently results in the development of secondary flow, or crossflow, in the boundary layer. The velocity profile of crossflow causes instability to develop in the boundary layer and provokes the transition of boundary layer from laminar to turbulent. This kind of instability is known as crossflow instability and much more difficult to predict than Tollmien-Schlichting instability. However, as streamwise instability, there exists some criteria that can be used at initial design stage and a similar N factor for crossflow, N_{CF} , can be calculated for crossflow transition prediction.

II. Transition Prediction

The first step in transition prediction using e^N -database method is to calculate viscous laminar boundary-layer parameters. In,^{9,10} RANS solvers were used to provide high accuracy boundary-layer parameters, e.g. displacement thickness, δ^* , and momentum thickness, θ , which are necessary for e^N -database method,

$$\begin{aligned}\delta^* &= \int_0^{\delta_e} \left(1 - \frac{U(y)}{U_e}\right) dy \\ \theta &= \int_0^{\delta_e} \frac{U(y)}{U_e} \left(1 - \frac{U(y)}{U_e}\right) dy\end{aligned}\tag{1}$$

where δ_e is the edge of boundary layer. For this method to be successful, the edge of boundary layer needs to be located. This can be achieved by first calculating boundary-layer edge velocity, U_e , with pressure distribution and isentropic relationship and, once the edge velocity is defined, the edge of boundary layer, δ_e , is located at the location where $U(y)$ intersects with U_e in the direction normal to the surface. After the edge of boundary layer has been located, the boundary-layer parameters can be calculated by Eq.(1). The use of a RANS solver to provide viscous data is straightforward; however, it is necessary to have large number of mesh points imbedded inside boundary layer and expensive grid adaptation may also be needed. To reduce the computational cost of resolving the boundary later and mesh adaptation, a compressible laminar boundary-layer method for swept, tapered wings¹¹ was chosen by the authors to produce highly accurate integral boundary-layer parameters for e^N -database method.

II.A. Streamwise Amplification Factor Calculation

With the availability of high quality boundary-layer parameters provided by the boundary-layer code, the next step toward transition prediction is to calculate amplification factor for Tollmien-Schlichtig waves, N_{TS} , based on boundary-layer parameters. This can be accomplished by using parametric fits to the amplification rates of TS waves and this has been done by Drela and Gleyzes et al.^{12,13} The current authors use the parametric fitting results from¹⁴ who introduces the ratio of wall temperature to the external temperature, T_w/T_e , as new parameter to account for the stabilizing effect of compressible boundary layer. The TS amplification factor can then be calculated by

$$N_{TS} = \int_{R_{e\theta_0}}^{R_{e\theta}} \frac{dn_{TS}}{dR_{e\theta}} dR_{e\theta},\tag{2}$$

where

$$\begin{aligned}
R_{e_\theta} &= \text{momentum thickness Reynolds number} \\
R_{e_{\theta_0}} &= \text{critical Reynolds number} = f(H_k, \frac{T_w}{T_e}) \\
\frac{dn_{TS}}{dR_{e_\theta}} &= g(H_k, \frac{T_w}{T_e}), \text{ and} \\
H_k &= \text{kinematic shape factor} = \frac{\int (1 - \frac{U}{U_e}) dy}{\int \frac{U}{U_e} (1 - \frac{U}{U_e}) dy}.
\end{aligned}$$

At each station, the above parameters are calculated and the critical point is reached when $R_{e_\theta} \geq R_{e_{\theta_0}}$. After the critical point is reached, Eq. (2) is used to integrate the amplification rate to give the amplification factor at the current station and transition is predicted when N_{TS} reaches about 9.

II.B. Crossflow Amplification Factor Calculation

For crossflow instability calculations, one of the most widely used methods is based on the work of Owen and Randall¹⁵ who suggest that crossflow Reynolds number

$$R_{cf} = \frac{\rho_e |w_{max}| \delta_{cf}}{\mu_e} \quad (3)$$

is the crucial parameter for crossflow instability. In the above definition, w_{max} is the maximum velocity in the crossflow velocity profile and δ_{cf} , the crossflow thickness, is the height where the crossflow velocity is about $1/10^{th}$ of w_{max} . Malik *et al.*¹⁶ state that the transition occurs when the critical Reynolds number

$$R_{crit} = 200 \left(1 + \frac{\gamma - 1}{2} M_e^2 \right) \quad (4)$$

is reached. Instead of simply using Eq. (4) as crossflow instability criterion, the parametric fitting results from¹⁴ are used in this work. The amplification rate, α , of crossflow instability can be expressed as

$$\alpha = \alpha \left(R_{cf}, \frac{w_{max}}{U_e}, H_{cf}, \frac{T_w}{T_e} \right) \quad (5)$$

Those parameters are calculated at each station and the amplification rate, α , is integrated

$$N_{CF} = \int_{x_0}^x \alpha dx \quad (6)$$

starting from x_0 to give the crossflow amplification factor at current station, where x_0 is the location at which crossflow Reynolds number exceeds its critical value

$$R_{cf0} = 46 \frac{T_w}{T_e}. \quad (7)$$

III. Transition Prescription

To simulate a flow around a wing which comprises both laminar and turbulent flows, it is necessary to divide the flow domain into laminar and turbulent subdomains and apply turbulence model in turbulent flow subdomain. The current turbulence model used in the RANS solver is the Baldwin-Lomax model²² with total viscosity defined as

$$\tau_{ij} = (\mu_{lam} + \mu_{turb}) \left\{ \frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} - \frac{2}{3} \left[\frac{\partial u_k}{\partial x_k} \right] \right\} \delta_{ij} \quad (8)$$

where μ_{lam} is the coefficient of laminar viscosity and μ_{turb} is the coefficient of eddy viscosity. The laminar-turbulent prescription is done by setting $\mu_{turb} = lt_switch(x) \mu_{turb}$, where $lt_switch(x)$ is the laminar-turbulent switch and its value depends on the location of x according to

$$lt_switch(x) = \begin{cases} = 0 & \text{if } x \in \text{laminar} \\ = 1 & \text{if } x \in \text{turbulent} \end{cases} \quad (9)$$

III.A. Transition Prescription on Surface

The first step in transition prescription is to split the airfoil surface into laminar and turbulent patches and this is achieved from the results of transition prediction module. The transition prediction module uses the pressure coefficients provided by the RANS solver as inputs, splits the airfoil into upper and lower surfaces from stagnation point, analyzes each surface separately, and the results are the transition locations on upper, x_{tran_upper} , and lower surface, x_{tran_lower} . Given the transition locations on upper and lower surfaces of airfoil, the lt_switch on the surface is set according to:

Upper surface:

$$\begin{aligned} x_{stag} \leq x < x_{tran_upper} &\Rightarrow lt_switch = 0 \\ x \geq x_{tran_upper} &\Rightarrow lt_switch = 1 \end{aligned}$$

Lower surface:

$$\begin{aligned} x_{stag} \leq x < x_{tran_lower} &\Rightarrow lt_switch = 0 \\ x \geq x_{tran_lower} &\Rightarrow lt_switch = 1 \end{aligned}$$

III.B. Transition Prescription in Flow Domain

With the laminar-turbulent patches defined on the surface of airfoil, the next step is to define laminar-turbulent regions in the flow field. This is done by projecting the turbulent patches into the flow field in the direction normal to airfoil surface and the extent of turbulent zones is defined at the edge, which is at a distance d_{edge} normal to the surface and can be controlled in the input file, of viscous layer. The result is turbulent subdomains surrounded by laminar zones and is shown schematically in Figure 1.

Figure 1. Schematic Diagram of Turbulent Subdomains Surrounded in Laminar Zones

IV. Coupling of Transition Prediction Module with RANS Solver

The flow and adjoint solver chosen in this research are based on those developed by Jameson^{17,18} and the flow solver solves the steady state RANS equations on structured meshes with multistep time stepping scheme. Rapid convergence to a steady state is achieved via variable local time stepping, residual averaging, and multi-grid scheme.

The RANS solver is coupled with transition prediction module which consists of laminar boundary-layer code and two transition prediction methods based on e^N -database method for Tollmien-Schlichting

and crossflow instabilities. The complete coupling of transition prediction module with RANS solver is summarized as following and shown schematically in Figure 2.

1. The RANS solver starts its flow iterations with prescribed transition locations setting far down stream on upper and lower surfaces of airfoil, e.g. 80% from the leading edge.
2. With this fixed transition locations, the RANS solver iterates until the density residual drops below certain level and the iteration on RANS solver is then suspended.

$$\frac{d\rho}{dt} \leq \frac{d\rho}{dt}_{limit}$$

3. The transition prediction module is called. The surface pressure distribution from RANS solver at current iteration is used as input for laminar boundary-layer code to calculate all the boundary-layer parameters which are necessary for two e^N -database methods.
4. With the calculated highly accurate boundary-layer parameters, Eq. (2) and (6) are used to calculate amplification factors for T-S and C-F instabilities and transition locations on both upper and lower surfaces can be determined. The calculated transition locations are then fed into RANS solver and transition prescriptions on airfoil surfaces and in flow domains are performed. This completes one iteration of transition prediction module.
5. The control of the program now returns back to the RANS solver and flow solver iterates again. With each successive flow iteration, the transition prediction module is called and the determination of transition locations becomes an iterative procedure. The is continued until the convergence criteria

$$|x_{tran}(k) - x_{tran}(k-1)| \leq \delta$$

is reached, where k is the current iteration and δ is a small value, and this condition is checked for N_{check} repeated times to prevent premature termination of transition prediction.

Figure 2. Coupling Structure of Flow Solver and Transition Prediction Module

V. NLF Airfoil and Wing Design Results

In this section, we first present the results of verification of boundary-layer code and transition locations tested on a benchmark case using the methodology described in section IV and then a natural-laminar-flow airfoil and wing design using Reynolds averaged Navier-Stokes equations will be demonstrated. The results demonstrate that it is necessary to prescribe the laminar-turbulent transition locations in order to obtain more realistic results, e.g. the drag coefficient and lift-to-drag ratio, in natural-laminar-flow wing design.

V.A. Verification of Boundary-Layer Parameters and Transition Locations

The accuracy of the boundary-layer parameters calculated by the QICTP¹¹ code is compared with the SWPTPR¹⁴ and DLR Tau codes,¹⁰ where the NLF(1)-0416 airfoil at specific flight condition was used as a test case. Figures 3 and 4 show the comparisons of calculated incompressible displacement thickness and momentum thickness, and they are both in good agreement.

Figure 3. Displacement Thickness, δ^* , on Upper Surface for NLF(1)-0416 Airfoil, $M_\infty = 0.3$, $Re_\infty = 4 \cdot 10^6$, $\alpha = 2.03^\circ$

Figure 4. Momentum Thickness, θ , on Upper Surface for NLF(1)-0416 Airfoil, $M_\infty = 0.3$, $Re_\infty = 4 \cdot 10^6$, $\alpha = 2.03^\circ$

The calculations of laminar boundary layer commonly terminate on the approach to flow separation

and this can be clearly seen on both figures. This early termination of the boundary layer calculation, in general, does not pose a problem for transition prediction because the calculated transition locations are usually located at upstream of the termination locations. In the case where the boundary-layer calculation does terminate before reaching the limiting N factor, the transition location is set at the location where boundary-layer calculation terminates, and this transition is classified as transition due to laminar separation.

The transition locations predicted with current method are compared with the experimental results from Somers¹⁹ and the results are in good agreements as can be seen from Table 1. In this case, the initial transition locations are set at 70% from the leading edge on both upper and lower surfaces of airfoil, and the transition prediction module is turned on after the density residual drops below a certain level. Figure 5 shows the convergence history of transition locations and it is clear that transition locations converge to their final values in about ten iterations after the transition prediction module is turned on.

Table 1. Comparison of Predicted Transition Locations with Experimental Results

	x_{tran_upper}	x_{tran_lower}
Current Method	0.348	0.587
Experiment	0.35	0.6

Figure 5. Convergence History of Transition Locations, $x_{tran,upper} = 0.348$, $x_{tran,lower} = 0.587$, for NLF(1)-0416 Airfoil, $M_\infty = 0.3$, $Re_\infty = 4 \cdot 10^6$, $\alpha = 2.03^\circ$

V.B. Natural-Laminar-Flow Airfoil Design

The design targets of this natural-laminar-flow airfoil are based on the specifications of the Honda lightweight business jet²⁰ at its cruise condition. The initial shape of the airfoil is designed by using the adjoint method with RANS equations¹⁸ and

$$I = \frac{1}{2} \oint_B (p - p_d)^2 dS$$

is used as the cost function. This corresponds to an inverse design problem and the shape of airfoil is modified to match the desired target pressure, p_d . The pressure coefficient of this designed airfoil at cruise condition is shown in Figure 6 and does demonstrate a reasonable amount of laminar flow on both surfaces. The convergence history of transition locations for the designed airfoil is shown in Figure 7 with the final transition locations located at 0.510 and 0.546 on upper and lower surface, respectively.

Figure 6. Pressure Distribution for Designed NLF Airfoil, $M_\infty = 0.69$, $Re_\infty = 11.7 \cdot 10^6$, $C_{l_{target}} = 0.26$

Figure 7. Convergence History of Transition Locations, $M_\infty = 0.69$, $Re = 11.7 \cdot 10^6$, $x_{tran,upper} = 0.51$, $x_{tran,lower} = 0.546$

With certain assumptions, a good estimate of range performance is provided by the Breguet range equation

$$R = \frac{VL}{D} \frac{1}{SFC} \log \frac{W_1}{W_2},$$

where V is the speed, L/D is the lift to drag ratio, SFC is the specific fuel consumption of the engines, W_1 is take-off weight, and W_2 is the landing weight. From aerodynamic point of view, this suggests that designer should try to increase the speed until the onset of drag rise in order to maximize range. The authors believe that the designed airfoil can be further optimized for a higher Mach number to improve the range parameter, $M_\infty L/D$, and still maintain a reasonable amount of laminar flow at the same time. The design Mach number is increased from 0.69 to 0.72 and the adjoint optimization technique is used to minimize drag and keep the same amount of lift. In this case, the adjoint method is mainly used to minimize the wave drag resulting from the existence of shock wave due to flying at higher Mach number. Figure 8 and 9 show the pressure distributions at new design Mach number before and after optimizations, respectively. As expected, there is a strong shock wave on the top of airfoil surface due to the increase of Mach number and this results in significant increase of wave drag. After 30 design cycles, the shock wave is completely eliminated and this greatly reduces the inviscid drag from 46 counts to 24 counts. The new designed airfoil has $M_\infty L/D = 33.4$, which is much better than the one flying at $M_\infty = 0.69$ with $M_\infty L/D = 31.4$.

Natural-laminar-flow airfoils may have undesirable characteristics, such as formation of shock waves, when flying at off-design conditions. The new designed airfoil is then tested at three off-design flight conditions to make sure that the new design does not exhibit undesirable characteristics. Figures 10-12 show the pressure distributions at those off-design conditions and they do demonstrate that the new design is satisfactory at both design and off-design conditions.

V.C. Natural-Laminar-Flow Wing Calculation

The wing used in the 3D computation is a semi-span, swept, tapered wing with taper ratio $\lambda = 0.278$. The leading and trailing edge of the wing are swept at $\Lambda_{LE} = 16.69^\circ$ and $\Lambda_{TE} = 1.67^\circ$, respectively, and cross sections are made up of airfoils designed at $M_\infty = 0.69$ from section V.B.

The mesh used in this computation is a C-type structured mesh with total number of 786432 cells in the flow domain. The wing is defined by 128 cells looping around the airfoil from the bottom of trailing edge to the top of trailing edge and has 33 airfoil sections along the span direction. To speed up the computation, the domain is divided into subdomains and a 3D RANS solver paralleled by MPI is used to solve the flow field to steady state. Figure 13 shows the distribution of mesh lines and divided subdomains used in this computation.

Three different target lift coefficients and their corresponding flight Mach numbers were studied. The target lift coefficients were achieved by constantly adjusting the angle of attack during flow iterations. Tables 2-4 summarize the comparisons of the aerodynamic coefficients for the results obtained from automatic transition prediction and 100% full turbulence for three cases studied here. Figures 14-15 show the contour plots of computed pressure coefficient on upper and lower surface, respectively, for $M_\infty = 0.69$ and $C_L = 0.26$ and it can be seen that the variations of pressure are mainly in the streamwise direction, but not much in chordwise direction.

In these calculations, the initial transition locations are set at 80% from the wing leading edge. For streamwise instability, $N_{TS} = 9$, which is well-known and accepted, was chosen as the limiting N factor. Depending on the levels of surface roughness, the N factor for crossflow instability varies in a wide range. Based on the results from Crouch and Ng²¹ and assumed surface roughness level, $N_{CF} = 8$ was chosen in this study. During the flow iteration, the density residual is monitored and transition prediction module is turned on after the residual drops below certain level. Each wing section is analyzed individually, the new transition location is calculated, and transition prescription is applied according to section III.

Figures 16-17 show the initial and final transition locations on upper and lower surface, respectively, for $M_\infty = 0.69$ and $C_L = 0.26$. Except at few inboard sections, the majority of transitions are due to Tollmien-Schlichting instability. In Figures 18-19, the contours of wall shear stress are shown and it can be clearly seen that there is a rise of shear stress downstream of transition lines.

Figure 20 shows the variations of drag coefficient as Mach number increases for both 100% turbulence and automatic transition prediction cases. Although the airfoils used for the wing section were designed at $M_\infty = 0.69$, the drag increases slowly until $M_\infty = 0.73$. Beyond this Mach number, there is a relatively large increase in drag due to the formation of shock waves on the upper surface. One of the most important

performance requirements for an executive jet is the the cruise efficiency, which can be measured by the range parameter $M \cdot L/D$. The range parameter as a function of Mach number for current wing is shown in Figure 21 and it does demonstrate a satisfactory characteristic around the designed Mach number. In fact, the range parameter keeps increasing until $M_\infty = 0.72$ before the formation of shock waves. It is also evident from Figures 20 and 21 that one does need to prescribe the transition locations in order to obtain more realistic results in laminar-turbulent flow calculations.

Table 2. Case 1: Comparison of Aerodynamic Coefficients , $M = 0.69, C_L = 0.26$

	C_L	$C_{D_{press}}$	$C_{D_{fric}}$	$C_{D_{tot}}$	L/D_{press}	L/D
Auto	0.258	58.0	39.5	97.5	44.54	26.49
100%	0.259	71.6	61.3	133	36.14	19.47

Table 3. Case 2: Comparison of Aerodynamic Coefficients, $M = 0.70, C_L = 0.38$

	C_L	$C_{D_{press}}$	$C_{D_{fric}}$	$C_{D_{tot}}$	L/D_{press}	L/D
Auto	0.379	89.1	40.3	129.4	42.59	29.31
100%	0.379	103.3	60.7	164.0	36.71	23.12

Table 4. Case 3: Comparison of Aerodynamic Coefficients, $M = 0.70, C_L = 0.50$

	C_L	$C_{D_{press}}$	$C_{D_{fric}}$	$C_{D_{tot}}$	L/D_{press}	L/D
Auto	0.50	130.2	40.4	170.7	38.33	29.25
100%	0.50	146.1	60.2	206.3	34.16	24.19

V.D. Natural-Laminar-Flow Wing Design

As for the 2D airfoil design case, the 3D wing can be designed for higher cruise Mach number to further improve the range performance. From Figure 20, it is clear that there is a sudden increase in drag at $M_\infty = 0.74$ due to the formation of relatively strong shock waves on the upper surface of the wing. The design target Mach number is then increased from $M_\infty = 0.69$ to $M_\infty = 0.74$ and the adjoint optimization technique is used to eliminate shock waves at new design Mach number. Figure 22 and 23 display the pressure distributions of the baseline NLF wing and redesigned configuration after 20 design cycles for full turbulence and automatic transition prediction cases, respectively. In both cases, the shock waves are completely eliminated and directly result in a reduction in drag.

The convergence history of drag minimization with automatic transition prediction is shown on Figure 24. The initial oscillations of drag coefficient are due to the formation of two relatively weak shock waves on the top of the wing and they are completely removed after 10 design iterations.

By eliminating shock waves at $M_\infty = 0.74$, the new designed wing does demonstrate an improvement in terms of drag coefficient. For wing design, one seeks not only an improvement at a single design point, but also requires the new design to perform not worse than the original design at off design conditions. Figure 25 shows the comparison of drag coefficient between original and new designed wing and the new wing clearly demonstrates an improvement over a wide range of cruising Mach number. The comparison of range parameter as a function of Mach number is shown on Figure 26 and an overall improvement is also evident.

It can be seen from figure 22 and 23 that the optimized wings have shifted their suction peaks forward during the drag minimization process. This is because only the drag coefficient, C_D , is used as the cost function, and this forward movement of suction peaks might result in early laminar-turbulent transition. To mitigate the adverse effects of forward movement of suction peaks, a new cost function which is used to minimize drag and, at the same time, try to maintain suction peaks at 40 ~ 50% chord is used. Figure 27

shows the pressure distribution with the new cost function and it is evident that the suction peaks are moved backward as compared with figure 23. Figure 28 and 29 show the comparisons of final transition lines calculated from two different cost functions for upper and lower surface. It can be seen that the new cost function results in a further delay of laminar-turbulent transition due to the effect of backward movement of suction peaks, and this delayed boundary-layer transition also results in further reduction in drag.

VI. Discussion and Conclusion

It can be seen from the results of this section that the predicted drag coefficient and lift-to-drag ratio are very different between full-turbulence and laminar-turbulent transition model. The difference in drag comes from the contributions of both pressure and skin friction drag. The higher skin friction drag in full turbulence case is due to the fact that the complete wing surface is submerged in high velocity gradient turbulent flow and high shear stress is applied to the complete wetted area of wing surface; in contrast, only part of the wing is subjected to high shear stress in laminar-turbulent case and this directly results in lower skin friction drag. The effect of turbulent boundary layer is not only on the skin friction drag, but also on the pressure drag as well. The existence of boundary layer creates pressure imbalance in the drag direction and greater imbalance of pressure is created if the flow is full turbulence than if the flow comprises both laminar and turbulent regions. This is the reason that there are also differences in pressure drag in Table 2-4.

For both 2D and 3D cases, the redesigned airfoil and wing configuration demonstrate satisfactory improvements not only at a single design point, but also at off-design conditions. The results show that it is feasible and necessary to incorporate the adjoint optimization technique with laminar-turbulent transition prediction in natural-laminar-flow wing design.

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Figure 8. Number of design iterations: 0, $M_\infty = 0.72$, $Re = 12 \cdot 10^6$, $C_{l_{target}} = 0.26$

Figure 9. Number of design iterations: 30, $M_\infty = 0.72$, $Re = 12 \cdot 10^6$, $C_{l_{target}} = 0.26$

Figure 10. Off-design Condition at $M_\infty = 0.69$, $C_{l_{target}} = 0.26$

Figure 11. Off-design Condition at $M_\infty = 0.70, C_{l_{target}} = 0.26$

Figure 12. Off-design Condition at $M_\infty = 0.71, C_{l_{target}} = 0.26$

Figure 13. Mesh Distribution and Divided Subdomains

Figure 14. Pressure Distribution on Upper Surface, $M = 0.69$, $C_L = 0.26$

Figure 15. Pressure Distribution on Lower Surface, $M = 0.69$, $C_L = 0.26$

Figure 16. Initial and Final Transition Locations on Upper Surface, $M = 0.69$, $C_L = 0.26$

Figure 17. Initial and Final Transition Locations on Lower Surface, $M = 0.69$, $C_L = 0.26$

Figure 18. Shear Stress, τ , Distribution on Upper Surface, $M = 0.69, C_L = 0.26$

Figure 19. Shear Stress, τ , Distribution on Lower Surface, $M = 0.69, C_L = 0.26$

Figure 20. C_D v.s. Mach number

Figure 21. Range parameter v.s. Mach number

IAI-NLF5

Mach: 0.740 Alpha:-0.015

CL: 0.258 CD: 0.01352 CM:-0.1669

Design: 20 Residual: 0.1833E-02

Grid: 257X 65X 49

Cp = -2.0

Tip Section: 92.3% Semi-Span

Cl: 0.210 Cd:-0.00372 Cm:-0.1050

Cp = -2.0

Root Section: 6.2% Semi-Span

Cl: 0.233 Cd: 0.02408 Cm:-0.1077

Cp = -2.0

Mid Section: 49.2% Semi-Span

Cl: 0.281 Cd: 0.00292 Cm:-0.1257

Figure 22. Full turbulence design for NLF 3D wing. Dashed lines and solid lines represent pressure distribution of the baseline NLF wing and redesigned configuration respectively

Figure 23. Automatic transition prediction design for NLF 3D wing. Dashed lines and solid lines represent pressure distribution of the baseline NLF wing and redesigned configuration respectively

Figure 24. Convergence history of the NLF wing cost function

Figure 25. Comparison of drag coefficient as a function of Mach number between the baseline and redesigned NLF wing

Figure 26. Comparison of range parameter as a function of Mach number between the baseline and redesigned NLF wing

IAI-NLF5

Mach: 0.740 Alpha:-0.281

CL: 0.260 CD: 0.00998 CM:-0.1763

Design: 20 Residual: 0.2216E-02

Grid: 257X 65X 49

Cp = -2.0

Tip Section: 92.3% Semi-Span

Cl: 0.205 Cd:-0.00525 Cm:-0.1112

Cp = -2.0

Root Section: 6.2% Semi-Span

Cl: 0.235 Cd: 0.02290 Cm:-0.1201

Cp = -2.0

Mid Section: 49.2% Semi-Span

Cl: 0.284 Cd: 0.00134 Cm:-0.1397

Figure 27. Redesign of 3D wing with new cost function. Dashed lines and solid lines represent pressure distribution of the baseline NLF wing and redesigned configuration respectively

Figure 28. Comparison of final transition lines on upper surface

Figure 29. Comparison of final transition lines on lower surface