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Engineering Notes

Design of Adjoint-Based Laws for Wing Flutter Control

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I. Introduction

An AIRPLANE, by its nature of being, is constructed so that it is as light as possible. The structural design is guided by static and dynamic factors. The more stringent constraints on the structural design are due to dynamic loads caused by aeroelastic interactions. One of the most commonly encountered problems in aeroelasticity is flutter [1], a term that is used to recognize the transfer of energy from unsteady aerodynamics associated with the surrounding fluid to the wing structure, resulting in rapidly divergent behavior. If flutter can be controlled at cruise speeds, we can design lighter wings and, consequently, more efficient airplanes. It is therefore in the aircraft designer's best interest to design innovative ways in which flutter can be controlled without making the resulting structure too heavy.

There are three important choices to make while designing active control strategies for suppressing flutter. The first is the choice of actuator. In this Note, the actuators we use are jets in the walls through which there is a small mass flow, either by way of blowing or suction. The second is to define a clear control objective. Finally, we need to design a control law that will make suitable state measurements and drive the actuators so that the desired control objective is achieved.

While different types of actuators can be designed for active flow control, zero-net-mass-flux (ZNMF) synthetic jets are gaining popularity as the actuator of choice. A ZNMF synthetic jet is popular for the main reason that it is formed entirely from the working medium of the flow. These eject and remove mass from the flow system through a narrow orifice periodically. This results in altering the momentum field around the orifice without adding or removing mass from the flow. The primary considerations in the design of synthetic jets are the size and positioning of the orifice and the time frequency of actuation. The design of synthetic jets and the physics of their interaction with a crossflow are discussed in detail by Glezer and Amitay [2].

Flow control for aerodynamics using synthetic jets has been studied experimentally by Amitay et al. [3], Tuck and Soria [4], Abe et al. [5], and Nishizawa et al. [6]. Numerical investigations were performed by Nae [7]. It should be noted that in all these experiments, the location and frequencies of the actuators were chosen a priori. The control implemented is therefore open-loop.

The study of closed-loop active flow control techniques is still in its primitive stages. This is because designing a closed-loop (feedback) control law requires understanding of the system dynamics. In spite of the fact that it is possible to obtain numerical solutions to the Navier–Stokes equations, understanding of the behavior of a flow-actuator system is extremely limited.

Feedback laws based on reduced-order models have been derived by Samimy et al. [8], Kumar and Tewari [9], and Cohen et al. [10]. The major drawback of these efforts is that the actuator dynamics are not modeled as part of the reduced-order description of the system.

All previous attempts at flow control have either involved designing simplistic controls for complex problems or complex feedback-based controls for simple problems. Problems like separation control, drag reduction, and control of the vortex-shedding frequency in the flow past a cylinder have all been controlled using open-loop controllers.

Closed-loop control has been demonstrated only on simplistic models derived from simulation or experiment.

An ideal flow control law should have the following properties:

1) The control law should be broadly applicable. We are looking for an algorithmic framework for generating flow control laws for a variety of problems. The development of such a framework would enable easy analysis and design of control laws for a variety of flow control problems.

2) The control law should be scientific and based on a realistic model of the fluid system.

3) The control law should be robust and account for variability in measurement, actuation, etc. This would mean that the control u should be feedback-based:

$$u = F(x) \tag{1}$$

where x is the current system state.

Our goal is therefore to develop feedback-based control laws that are derived from a realistic representation of the flow. We try to make sure that the framework is as generic as possible, lending easy extension to a variety of situations. We then discuss specific applications of the control law thus derived, including control of flutter.

The concept of flow control, as described in this Note, relies heavily on the adjoint method as developed by Jameson [11]. The method is well explained in Nadarajah [12]. The control law that is derived is based on a 2-D model.

II. Flutter Simulation

A. Flow Model: Euler Equations for Fluid Flow with Blowing at the Walls

In this Note, the fluid flow is modeled using the Euler equations. The Euler equations model the behavior of inviscid compressible fluids. They are

$$\frac{\partial \mathbf{w}}{\partial t} + \frac{\partial \mathbf{f_i}}{\partial x_i} = 0 \tag{2}$$

Here, x_i represents the Cartesian coordinate directions, $\mathbf{w}$ are the state variables, and $\mathbf{f_i}$ are the corresponding flux vectors, given by

$$\mathbf{w} = (\rho, \rho\mathbf{u}, \rho\mathbf{v}, \rho\mathbf{w}, \rho\mathbf{E}) \tag{3}$$

and

$$\mathbf{f_i} = (\rho\mathbf{u_i}, \rho\mathbf{u_i}\mathbf{u} + \delta_{\mathbf{i1}}P, \rho\mathbf{u_i}\mathbf{v} + \delta_{\mathbf{i2}}P, \rho\mathbf{u_i}\mathbf{w} + \delta_{\mathbf{i3}}P, \rho\mathbf{u_i}H) \tag{4}$$

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Here, ρ is the density, and u, v, and w are the velocities in the respective coordinate directions. The velocities are also described using Einstein notation as u_i. P is the pressure and H is the enthalpy. The steady-state Euler equations can be written in weak conservation form as follows:

$$\int_{\mathcal{B}} n_i \phi^T f_i(w)\,\mathrm{d}\mathcal{B} = \int_{\mathcal{D}} \frac{\partial \phi^T}{\partial x_i} f_i(w)\,\mathrm{d}\mathcal{D} \tag{5}$$

where ϕ is any test function. If a transformation is made from physical space x_i to computational space ε_i, defined by the mapping functions

$$K_{ij} = \left[\frac{\partial x_i}{\partial \varepsilon_j}\right], \qquad J = \det(K), \qquad K_{ij}^{-1} = \left[\frac{\partial \varepsilon_i}{\partial x_j}\right] \tag{6}$$

and

$$S = JK^{-1} \tag{7}$$

the Euler equations (5) become

$$\int_{\mathcal{B}_\xi} n_i \phi^T S_{ij} f_j(w)\,\mathrm{d}\mathcal{B}_\xi = \int_{\mathcal{D}_\xi} \frac{\partial \phi^T}{\partial \xi_i} S_{ij} f_j(w)\,\mathrm{d}\mathcal{D}_\xi \tag{8}$$

The boundary conditions for the case in which we have blowing or suction at the boundary can then be prescribed in terms of the blowing velocity as follows:

$$F_2 = (\rho q_n, \rho q_n u + S_{21}P, \rho q_n v + S_{22}P, \rho q_n w + S_{23}P, \rho q_n H) \tag{9}$$

where ρq_n is the prescribed mass flow at the boundary, initially set to zero in the design problem. Here, F_2 is the flux normal to the wall, as opposed to f_i, which is the flux in the ith coordinate direction.

B. Structural Model

The structural dynamic model is derived from the theory of elasticity, which relates the deformation and internal stresses of the structure to the external loads applied. A Lagrangian frame is used to describe the structure, as contiguous elements of the structure continue to remain contiguous unless structural failure occurs.

In the present Note we will first investigate the aeroelastic behavior and control of a 2-D airfoil whose schematics are shown in Fig. 1. A 2-D airfoil model can be shown to be a fair representation for flutter prediction, as shown by Theodorson and Garrik [13], of a straight wing of a large span by giving it the geometric and inertial properties of the cross section three-quarters of the way from the centerline to the wing tip. The equations of motion of this simple system can be shown to be as follows:

$$m\ddot{h} + S_\alpha \ddot{\alpha} + K_h h = -L \tag{10}$$

$$S_\alpha \ddot{h} + I_\alpha \ddot{\alpha} + K_\alpha \alpha = M_{ea} \tag{11}$$

where K_h and K_α are representative of the bending and torsional stiffness of the wing about its elastic axis (the elastic axis is the locus of points about which an applied force will not cause a rotation), m and I_α are the mass and moment of inertia of the wing section about the elastic axis, and S_α is the coupling term, which depends on the relative position of the center of gravity and the elastic axis.

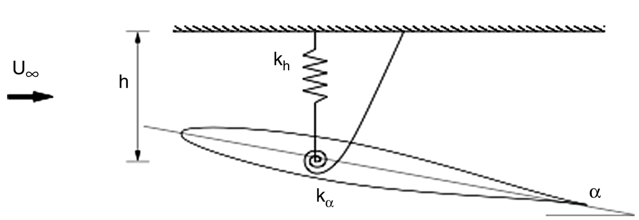

Fig. 1 Typical section-wing model geometry.

For the present Note we assume the structural properties to be fixed and we have some amount of control of the right-hand sides of the equations via blowing and suction. The objective is to find a suitable control law that will modify the aerodynamic terms so as to prevent flutter.

C. Computational Simulation

The flow is simulated by solving the unsteady Euler equations. The Euler equations are solved using a dual-time-stepping method, using a third-order backward difference formula in time, and a symmetric Gauss Seidel scheme for solving the inner iterations. The aforementioned flow simulation code is integrated with a two-degree-of-freedom structural model given for the 2-D simulation.

The aerodynamic and structural solvers are coupled by exchanging information at regular intervals during the convergence process. At the start of each iteration, the surface pressures are translated into nodal forces and the structural solver is called. The new displacement field is then translated to a movement of the computational fluid dynamics (CFD) mesh and then flow iterations are performed. While making the transfer one must make sure that the transfer of load is consistent and conservative [14].

The coupled aerostructural system is integrated using the Newmark scheme. The simulation techniques are discussed in detail in [15].

III. Derivation of Adjoint-Based Control Laws

A. System Linearization and Model Order Reduction

In Eqs. (10) and (11), the structural parameters are constant. The lift L and the moment M are complex nonlinear functions of the system state $\mathbf{w}, \alpha, \dot{\alpha}, h$, and $\dot{h}$. Moreover, $\alpha, \dot{\alpha}, h$, and $\dot{h}$ are themselves functions of the system state $\mathbf{w}$. Here, the state $\mathbf{w}$ is the vector consisting of all the Euler states at all finite volumes used in the simulation. Thus,

$$L = L(\mathbf{w}, \mathbf{u}) \tag{12}$$

$$M = M(\mathbf{w}, \mathbf{u}) \tag{13}$$

Linearizing about the nominal operating point, we get

$$L = \left(\frac{\partial L}{\partial \mathbf{w}}\right)^T \delta\mathbf{w} + \left(\frac{\partial L}{\partial \mathbf{u}}\right)^T \delta\mathbf{u} \tag{14}$$

$$M = \left(\frac{\partial M}{\partial \mathbf{w}}\right)^T \delta\mathbf{w} + \left(\frac{\partial M}{\partial \mathbf{u}}\right)^T \delta\mathbf{u} \tag{15}$$

It should be noted that for a simulation with one million finite volumes, the dimension of $\mathbf{w}$ is four million for a 2-D simulation and five million for a 3-D simulation. Thus, evaluating the above derivatives is a formidable computational challenge. It is also important to recognize that not all the derivatives are significant in the above representation. Consider, for example, a cell in the far field. The value of the state variables there is not going to change by much, however rapid the oscillations. Therefore, it is of very little use to evaluate these derivatives in our linearized model.

Instead, we choose to obtain a suitable reduced-order model that captures the essential physics. The most obvious reduction that we can obtain is in terms of α, $\dot{\alpha}$, h, and $\dot{h}$. We therefore work with a model of the form

$$L = L_\alpha \alpha + L_{\dot{\alpha}} \dot{\alpha} + L_h h + L_{\dot{h}} \dot{h} + \left(\frac{\partial L}{\partial \mathbf{u}}\right)^T \mathbf{u} \tag{16}$$

$$M = M_\alpha \alpha + M_{\dot{\alpha}} \dot{\alpha} + M_h h + M_{\dot{h}} \dot{h} + \left(\frac{\partial M}{\partial \mathbf{u}}\right)^T \mathbf{u} \tag{17}$$

Equations (16) and (17) assume that the nominal values of α, $\dot{\alpha}$, h, and $\dot{h}$ and $\mathbf{u}$ are zero, respectively. Thus, for the flutter control problem being studied, the following state vector is used:

$$\mathbf{x} = [\alpha \quad \dot{\alpha} \quad h \quad \dot{h}]^T \tag{18}$$

B. System Identification: Evaluation of Sensitivities

In our aerostructural model, the lift L and the moment M depend on the complete system state $\mathbf{w}$. However, using a full-order state model to design a controller is not feasible, given the extremely high dimensionality of the system. We therefore formulate a reduced-order model of the system. For this model to be complete, we need to evaluate the sensitivities with respect to the reduced-order state $\mathbf{x}$ and the control variables $\mathbf{u}$.

1. Sensitivities with Respect to the State Variables

The sensitivities of the lift and moment with respect to the state variables are evaluated in two different ways.

First, we use theoretical results from Theodorsen, as reported in [1]. Theodorsen theory assumes that the airfoil under consideration is thin and is oscillating in an incompressible flow. Under these considerations,

$$L_\alpha = \pi\rho v_\infty^2 c, \qquad L_{\dot{\alpha}} = \frac{\pi\rho v_\infty c^2}{4}$$
$$L_h = 0, \qquad L_{\dot{h}} = \pi\rho v_\infty c$$
$$M_\alpha = \frac{\pi\rho v_\infty^2 c^2}{4}, \qquad M_{\dot{\alpha}} = 0$$
$$M_h = 0, \qquad M_{\dot{h}} = \frac{\pi\rho v_\infty c^2}{4}$$

Here, ρ is the freestream density, v_∞ is the freestream velocity, and c is the chord of the airfoil.

In the least-squares method, we evaluate the sensitivities by studying the unforced response of a pitching airfoil and then estimating the sensitivities by a least-squares technique. We try to fit the data thus obtained to functions of the form

$$L = L_\alpha \alpha + L_{\dot{\alpha}}\dot{\alpha} + L_h h + L_{\dot{h}}\dot{h}$$
$$M = M_\alpha \alpha + M_{\dot{\alpha}}\dot{\alpha} + M_h h + M_{\dot{h}}\dot{h}$$

Our goal is to evaluate the sensitivities L_α, $L_{\dot{\alpha}}$, L_h, $L_{\dot{h}}$, M_α, $M_{\dot{\alpha}}$, M_h, and $M_{\dot{h}}$. We do this using a least-squares technique.

It can be seen from the simulation results that both techniques work quite well. The system identification by the least-squares technique works slightly better, in the sense that it achieves faster stabilization. This can be attributed to the fact that this represents the nonlinear system more closely.

2. Sensitivities with Respect to the Control Variables

We also need to evaluate the sensitivities of L and M with respect to the control variables $\mathbf{u}$, $\partial L/\partial \mathbf{u}$, and $\partial M/\partial \mathbf{u}$, respectively. We do this are using an adjoint method. In our case, the control variable is the normal mass flux at the wall ρq_n.

Let us assume that we are trying to find the sensitivities due to the control variables of a function I given by

$$I = \int_{\mathcal{B}_\xi} \mathcal{M}(w, \rho q_n)\, \mathrm{d}\mathcal{B}_\xi \tag{19}$$

Here, $\mathcal{M}$ is a function defined on the surface $\mathcal{B}_\xi$. $\mathcal{M}$ is a function of the state w and the control mass flux at the wall ρq_n. For example, when we are trying to compute $\partial L/\partial \mathbf{u}$, I is just the lift L. $\mathcal{M}$ is then given by

$$\mathcal{M} = P\hat{n}.\hat{k} \tag{20}$$

where $\hat{n}$ is the unit vector normal to the boundary, and $\hat{k}$ is the unit vector normal to the freestream. It can be seen that in this case, the pressure P is a function of the flow state $\mathbf{w}$, but not the surface mass flux ρq_n.

The next thing to be noted is that $\mathbf{w}$ and ρq_n are not arbitrary but related to each other by the flow equations used to model the system: in this case, the Euler equations. Thus, we also have a constraint given by the Euler equations (8) and (9). Since Eq. (8) is true for any test function ϕ, we can choose ϕ to be the adjoint variable ψ. We can then add the constraint given by the Euler equations (19) to form the augmented cost function given by

$$I = \int_{\mathcal{B}_\xi} \mathcal{M}(w, \rho q_n)\, \mathrm{d}\mathcal{B}_\xi - \int_{\mathcal{B}_\xi} n_i \psi^T S_{ij} f_j(w, \rho q_n)\, \mathrm{d}\mathcal{B}_\xi + \int_{\mathcal{D}_\xi} \frac{\partial \psi^T}{\partial \xi_i} S_{ij} f_j(w, \rho q_n)\, \mathrm{d}\mathcal{D}_\xi \tag{21}$$

Taking the first variation of the function I, we have

$$\begin{aligned}\delta I = &\int_{\mathcal{B}_\xi} \left(\frac{\partial \mathcal{M}}{\partial w}\delta w + \frac{\partial \mathcal{M}}{\partial \rho q_n}\delta(\rho q_n)\right) \mathrm{d}\mathcal{B}_\xi \\ &- \int_{\mathcal{B}_\xi} n_i \psi^T S_{ij} \left(\frac{\partial f_j}{\partial w}\delta w + \frac{\partial f_j}{\partial \rho q_n}\delta(\rho q_n)\right) \mathrm{d}\mathcal{B}_\xi \\ &+ \int_{\mathcal{D}_\xi} \frac{\partial \psi^T}{\partial \xi_i} S_{ij} \left(\frac{\partial f_j}{\partial w}\delta w + \frac{\partial f_j}{\partial \rho q_n}\delta(\rho q_n)\right) \mathrm{d}\mathcal{D}_\xi\end{aligned} \tag{22}$$

We can then choose our costate variable ψ so that it satisfies the adjoint equations

$$S_{ij}\left(\frac{\partial f_j}{\partial w}\right)^T \frac{\partial \psi}{\partial w} = 0, \quad \text{on } \mathcal{D}_\xi \tag{23}$$

and

$$\frac{\partial \mathcal{M}}{\partial w} = \psi^T \frac{\partial F_2}{\partial w}, \quad \text{on } \mathcal{B}_\xi \tag{24}$$

We also observe that

$$\frac{\partial f_j}{\partial \rho q_n} = 0, \quad \text{on } \mathcal{D}_\xi \tag{25}$$

The expression for the adjoint gradient then becomes

$$\begin{aligned}\delta I = &\int_{\mathcal{B}_\xi} \left(\frac{\partial \mathcal{M}}{\partial \rho q_n}\delta(\rho q_n)\right) \mathrm{d}\mathcal{B}_\xi \\ &- \int_{\mathcal{B}_\xi} \left(\psi_1 + \psi_2 u + \psi_3 v + \psi_4 w + \psi_5\left(E + \frac{P}{\rho}\right)\right)\delta\rho q_n\, \mathrm{d}\mathcal{B}_\xi\end{aligned} \tag{26}$$

It should be noted that the adjoint vector has the same number of dimensions as the flow vector. For a 2-D problem, the adjoint vector has four dimensions, and for a generic 3-D problem, the adjoint vector has five dimensions. The gradient is then modified to account for the fact that the nett mass flow through the boundaries is zero.

C. Flutter Control: Formulation of the Objective Function

We can define the flutter velocity as that point where we have sustained oscillations of the system. Let us define the state vector $\mathbf{x}$ as follows:

$$\mathbf{x} = [\alpha \quad \dot{\alpha} \quad \mathbf{h} \quad \dot{\mathbf{h}}]^{\mathbf{T}} \tag{27}$$

The control vector $\mathbf{u}$ is the vector of blowing/suction velocities at the wall. The dynamics of the system are represented by the equations derived in Sec. II. For the purposes of designing a controller, we model the lift L and the moment M using a reduced-order model as presented in Eqs. (16) and (17). The system model used to design a controller is then

$$m\ddot{h} + S_\alpha\ddot{\alpha} + K_h h = -\left(L_\alpha\alpha + L_{\dot{\alpha}}\dot{\alpha} + L_h h + L_{\dot{h}}\dot{h} + \left(\frac{\partial L}{\partial \mathbf{u}}\right)^T \mathbf{u}\right)$$

$$S_\alpha\ddot{h} + I_\alpha\ddot{\alpha} + K_\alpha\alpha = \left(M_\alpha\alpha + M_{\dot{\alpha}}\dot{\alpha} + M_h h + M_{\dot{h}}\dot{h} + \left(\frac{\partial M}{\partial \mathbf{u}}\right)^T \mathbf{u}\right)$$

This can be rephrased in state-space form as follows:

$$M\dot{\mathbf{x}} = \hat{A}\mathbf{x} + \hat{\mathbf{B}}\mathbf{u} \tag{28}$$

Here, the matrix $\hat{B}$ represents the sensitivities of the state vectors with respect to the control variables. This can be obtained by solving the adjoint equations. Inverting M, we get a system of the form

$$\dot{\mathbf{x}} = A\mathbf{x} + \mathbf{B}\mathbf{u} \tag{29}$$

It is possible to design a controller for the system (29) using linear quadratic regulator (LQR) techniques.

The objective of the problem is to control the system given by system (29), so that the final value of the state vector is given by

$$\mathbf{x}_\mathbf{f} = [\alpha_\mathbf{f} \quad \mathbf{0} \quad \mathbf{h}_\mathbf{f} \quad \mathbf{0}]^\mathbf{T} \tag{30}$$

If this is rephrased as an optimization problem, the objective would be to minimize the following function:

$$J = \frac{1}{2}\int_0^T ((\mathbf{x} - \mathbf{x}_\mathbf{f})^T Q(\mathbf{x} - \mathbf{x}_\mathbf{f}) + \mathbf{u}^T R\mathbf{u})\,\mathrm{d}t \tag{31}$$

where Q is a positive semidefinite weighting matrix and R is a positive definite matrix. In our case,

$$Q = I, \qquad R = \varepsilon I$$

where I is the identity matrix, and ε is a small positive constant. R is required to be positive definite, to ensure that the control computed is not of unreasonable magnitudes.

A feedback-control gain matrix can be derived for the flutter control problem by solving the Riccati equation [16]. The magnitude of control required at each actuator location is given by the control gain matrix K_{ss} The results are presented in the next section. It can be seen that this control law is successful in stabilizing the system.

IV. Results

The following experiments were conducted on a symmetric NACA 0012 airfoil at a freestream Mach number of 0.3. A 160×32 grid was used for the CFD simulation.

The structural properties were chosen as follows: $I_\alpha = 60$, $M = 60$, $K_h = 60$, $K_\alpha = 60$, and $S_\alpha = 30$. Our nominal rest point is $\alpha = 0°$ and $h = 0$.

As discussed in the last section, the adjoint method is used to find the gradients of lift and moment with respect to the control variables: namely, the blowing and suction velocities on the surface. It should be noted that this is done using a steady-flow assumption about the nominal rest point of the system.

A. Application of Feedback Control to the Nonlinear Flutter Problem

The uncontrolled and controlled aerostructural simulations are represented in Figs. 2 and 3. It should be noted that even though the feedback law is derived from a linearized model of the system, the control is applied to a complete nonlinear model. Two different methods are used to find the aerodynamic derivatives. It can be seen that the least-squares method does a better job than the Theodorsen method for flutter control. This is obvious, because this represents the nonlinear system more accurately. It should be noted that in this case, the actuation is continuous along the surface of the airfoil. The corresponding blowing/suction velocities are shown in Fig. 4. It should be noted that the freestream value of ρq_n in our simulation was 1. So the values of blowing and suction controls required are quite small. Moreover, we need zero control input at the equilibrium point, which is what we desire.

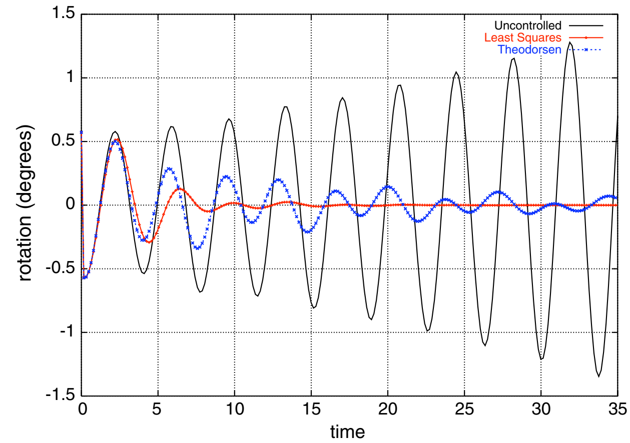

Fig. 2 Variation of angle of attack (degrees) with time: controlled and uncontrolled cases.

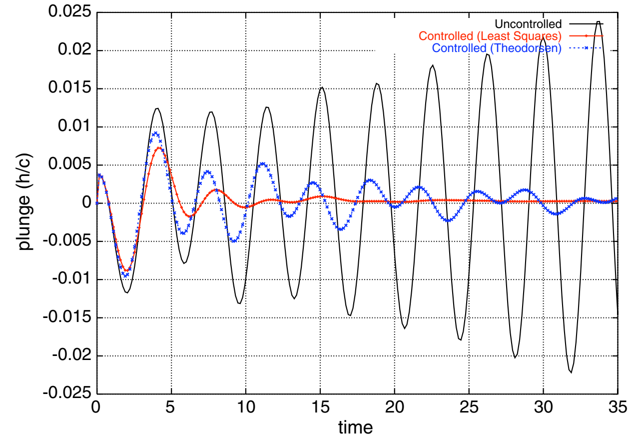

Fig. 3 Variation of plunge h/c with time: controlled and uncontrolled cases.

B. Reduction in the Number of Actuators

Our next step is to specialize the control law thus derived to work when the number of actuators is finite. It was found that flutter could be controlled with as few as four actuators: one each in the leading and trailing edges and one each in the middle of the upper and lower surfaces. The fact that there are only four actuation points is represented by zeroing-out the gradient everywhere except at these four locations. (Every location is represented by a small cluster of CFD cells to prevent numerical instability and damping of the actuation values.)

The entire procedure outlined in the previous section is then repeated to derive the feedback gain matrix K_{ss}. It can be seen that the matrix has nonzero values only at the desired locations of the controllers. Consequently, actuation is performed only at these sites. This is equivalent to controlling the problem with a finite number of actuators.

It can be seen from Fig. 5 that flutter is controlled successfully, even with a finite number of actuators. This is an important result, as it implies that this system can be implemented on a practical aerodynamic configuration.

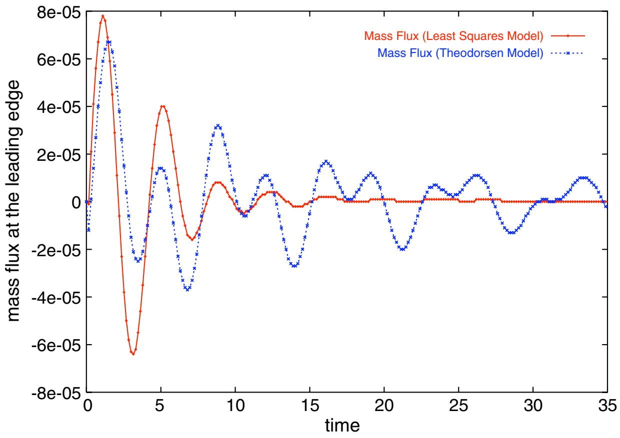

Fig. 4 Blowing/suction mass fluxes at the leading edge.

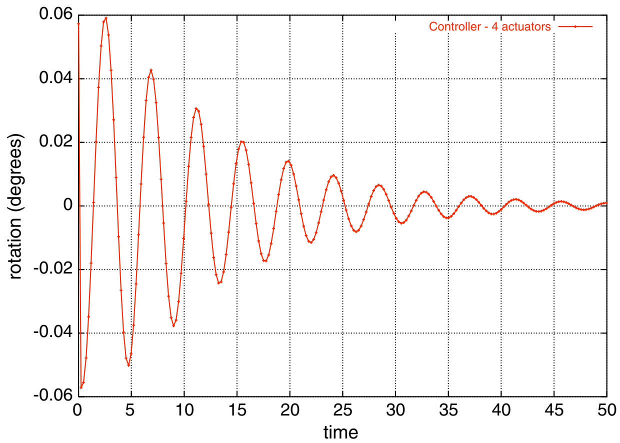

Fig. 5 Variation of angle of attack (degrees) with time: with four actuators.

Comparing this to the continuous-actuation case in Fig. 2, we can see that the damping rate is slower. This is to be expected, as the control authority is reduced with fewer actuators.

V. Conclusions

In this work, we developed a feedback algorithm for the control of flutter. We demonstrated the effectiveness of this control law in 2-D simulations. We also explored possibilities for the reduction in the number of actuators.

The control methodology described in this Note using blowing and suction is more flexible than using traditional oscillating trailing-edge control surfaces. We can choose to have the jets at strategic locations along the surface of the wing or airfoil. These locations can be chosen for optimum impact by doing an adjoint analysis.

The actuation performed using this algorithm is the smallest amount of actuation required for control purposes (as it is derived using an LQR algorithm).

Moreover, the actuation performed is based on feedback from the current flowfield, and an estimate of its future evolution is based on the reduced-order model derived. This is a scientific way of determining control input. The feedback nature of control ensures that control is robust.

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