APPLICATIONS OF ADJOINT BASED SHAPE OPTIMIZATION TO THE DESIGN OF LOW DRAG AIRPLANE WINGS, INCLUDING WINGS TO SUPPORT NATURAL LAMINAR FLOW

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Sustainable aviation is the challenge facing the aerospace community.
Outline of the Presentation

1. Introduction
   ✓ Challenge of Sustainable Aviation

2. Fundamentals of Aerodynamic Design
   ✓ Minimizing Aerodynamic Drag

3. Aerodynamic Design Methods
   ✓ Adjoint Based Aerodynamic Shape Optimization
   ✓ Adjoint Method with Transition Prediction

4. Results

5. Conclusion

Antony Jameson and Kui Ou
Aerodynamic Design
Introduction

**Challenge of Sustainable Aviation**
Challenge 1: Modern Efficiency

- Modern airplanes have vastly improved fuel efficiency.
- Relative fuel use per seat-km is reduced by 70%
Challenge 2: Growing Air Traffic

✓ Projected 5% annual growth in air traffic
✓ 3 fold increase in global air travel in the next 30 years

✓ Each long distance flight of B747 adds 400 tons of CO₂ to the atmosphere
✓ Fastest growing source of greenhouse gases
✓ Climate impact will overtake cars by the end of the decade
Challenge 3: Resolving Conflicting Objectives

Goal: develop technology that will allow a **tripling of capacity** with a **reduction in environmental impact**
Innovative Configurations and Optimum Aerodynamics

Challenge for aerodynamicists: maximize aerodynamic efficiency through innovative configurations and minimization of drag.
Fundamentals

Range Equation for a Jet Aircraft

\[
\dot{W}_f = \dot{W} = \frac{dW}{dt} = \frac{dx}{dt} \frac{dW}{dx} = V \frac{dW}{dx}
\]

\[
\dot{W}_f = -sfc \cdot T = -sfc \cdot D \cdot \frac{W}{L}
\]

\[
dx = V \frac{L}{D} \frac{1}{sfc} \frac{dW}{W}
\]

\[
R = V \frac{L}{D} \frac{1}{sfc} \log \frac{W_1}{W_0}
\]

\[
\begin{align*}
\text{aero} & \quad \text{prop} & \quad \text{struc}
\end{align*}
\]
Drag of Subsonic and Transonic Aircraft

\[ C_D = C_{Dp} + \frac{C_L^2}{\pi \cdot AR \cdot e} + \Delta C_{Dc} \]

This leads to the need:

✓ to minimize skin friction drag, maximize natural laminar flow
✓ to minimize transonic wave drag, minimize shock wave strength
✓ to minimize induced drag for a given lift, increase wing span
**Maximum Lift to Drag Ratio**

\[
\frac{L}{D_{\text{max}}} = \frac{\sqrt{\pi} \sqrt{e} \sqrt{AR}}{2 \sqrt{C_{D_p}}}
\]

where \( e \) is airplane efficiency factor, \( AR \) is wing aspect ratio, and \( C_{D_p} \) is the parasite drag coefficient.

The optimum aerodynamic design:

- Large wing span
- Extensive natural laminar flow
- Shock free wing

The last two items, i.e. natural laminar wing design and aerodynamic shape optimization are the topics today.
Hierarchy of Computational Aerodynamics

✓ Prediction of the flow past an airplane in the flight regimes
✓ Interactive calculations to allow rapid design improvement
✓ Automatic design optimization
Enable exploration of a large design space → free wing surface

Some modifications are too subtle to allow trial and error methods → changes smaller than boundary layer thickness

Progress in flow solvers and computer hardware have enabled sufficiently rapid turn around time → RANS-base design optimization under a minute using laptop computer
✓ The airplane aerodynamic performance can be greatly enhanced by tailoring airfoil shape.
✓ Inverse airfoil design problem has considerable theoretical and practical interest.
Let the profile P be conformally mapped to an unit circle C.

The surface velocity is \( q = \frac{1}{h} |\nabla \phi| \)
- where \( \phi \) is the potential in the circle plane,
- and \( h \) is the mapping modulus
  \( h = \left| \frac{dz}{d\sigma} \right| \)

Choose \( q = q_{\text{target}} \)

Solve for the mapping modulus
\[ h = \frac{1}{q_{\text{target}}} |\nabla \phi| \]
✓ Inverse design problem can be ill-posed
✓ Desired airfoil shape does not always exist
✓ A computationally efficient optimization method can dispense the inverse method
Optimization gradient

If $I$ is the cost function, and $F$ is the shape function, then

$$\delta I = G^T \delta F,$$

where

$$G = \frac{\partial I^T}{\partial F} - \psi^T \left[ \frac{\partial R}{\partial F} \right].$$

An improvement can be made with a shape change

$$\delta F = -\lambda G$$

where $\lambda$ is positive and small enough.

Optimization with steepest descent

$$\delta I = -\lambda G^2 < 0$$
Define the geometry through a set of design parameters

\[ f(x) = \sum \alpha_i b_i(x). \]

Then a cost function \( I \) is selected, for example, to be the drag coefficient or the lift to drag ratio, and \( I \) is regarded as a function of the parameters \( \alpha_i \).

\[ G = \frac{\partial I}{\partial \alpha_i} \approx \frac{I(\alpha_i + \delta \alpha_i) - I(\alpha_i)}{\delta \alpha_i}. \]

Make a step in the negative gradient direction by setting

\[ \alpha^{n+1} = \alpha^n + \delta \alpha, \]

where

\[ \delta \alpha = -\lambda G \]

so that to first order

\[ I + \delta I = I - G^T \delta \alpha = I - \lambda G^T G < I \]
Disadvantages

✗ Needs a number of flow calculations proportional to the number of design variables to estimate the gradient.

✗ Computational costs prohibitive as the number of design variables is increased.

✓ A more efficient method to compute sensitivity gradient is needed
## Automatic Design Based on Control Theory

- Regard the wing as a device to generate lift by controlling the flow
- Apply the theory of optimal control of PDEs [Lions ]
- Aerodynamic shape optimization via control theory
Abstract Description of the Adjoint Method: Cost Function

**Definition of a cost function**

\[ I = I(w, F) \]

Cost function are functions of flow variables \( w \) and wing shape \( F \)

**Total variation of the cost function**

\[ \delta I = \begin{bmatrix} \frac{\partial I^T}{\partial w} \end{bmatrix} \delta w + \begin{bmatrix} \frac{\partial I^T}{\partial F} \end{bmatrix} \delta F \]

A change in \( F \) results in a change in the cost function

\( \delta I = \delta I(\delta w, \delta F) \): each variation of \( \delta F \) causes a variation of \( \delta w \), which requires flows to be recomputed to obtain the gradient of improvement \( \rightarrow \) extremely expensive
Abstract Adjoint Concept: Constraint

**Flow governing equations are the constraints**

\[ R(w, F) = 0 \]

**Total variation of the governing equations**

\[ \delta R = \left[ \frac{\partial R}{\partial w} \right] \delta w + \left[ \frac{\partial R}{\partial F} \right] \delta F = 0 \]

**Lagrange multiplier**

\[ \delta R = 0 \quad \Rightarrow \quad \psi^T \delta R = 0 \]
Abstract Adjoint Description: Lagrange Multiplier

Augmentation of cost function

\[ \delta I = \delta I - \psi^T \delta R \]

Expansion and rearrangement of cost function

\[
\delta I = \left\{ \frac{\partial I^T}{\partial w} \delta w + \frac{\partial I^T}{\partial F} \delta F \right\} - \psi^T \left\{ \left[ \frac{\partial R}{\partial w} \right] \delta w + \left[ \frac{\partial R}{\partial F} \right] \delta F \right\} \\
= \left\{ \frac{\partial I^T}{\partial w} - \psi^T \left[ \frac{\partial R}{\partial w} \right] \right\} \delta w + \left\{ \frac{\partial I^T}{\partial F} - \psi^T \left[ \frac{\partial R}{\partial F} \right] \right\} \delta F
\]

\[ \delta I = \delta I(\delta w, \delta F) \]: cost variation is independent of \( \delta w \), i.e. no additional flow-field evaluations needed regardless of \( \delta F \) (e.g. infinitely dimensional) \( \rightarrow \) extremely effective, and attractive for design
Abstract Adjoint Description: Adjoint Equation

### Adjoint equation

\[
\frac{\partial I^T}{\partial w} - \psi^T \left[ \frac{\partial R}{\partial w} \right] = 0
\]

### Resulting cost function

\[
\delta I = \left\{ \frac{\partial I^T}{\partial w} - \psi^T \left[ \frac{\partial R}{\partial w} \right] \right\}^{0} \delta w + \left\{ \frac{\partial I^T}{\partial F} - \psi^T \left[ \frac{\partial R}{\partial F} \right] \right\} \delta F
\]

\[\delta I = \delta I(\delta F): \text{cost is associated with the additional solution of the adjoint equation}\]

\[\rightarrow \text{independent of design space dimension, which can be extremely large}\]
Abstract Adjoint Description: Optimization gradient

Optimization gradient

\[ \delta I = G^T \delta F, \]

where

\[ G = \frac{\partial I^T}{\partial F} - \psi^T \left[ \frac{\partial R}{\partial F} \right]. \]

An improvement can be made with a shape change

\[ \delta F = -\lambda G \]

where \( \lambda \) is positive and small enough.

Optimization with steepest descent

\[ \delta I = -\lambda G^2 < 0 \]

Descent towards a lower cost is hence guaranteed.
Advantages

✓ The advantage is that the cost function is independent of $\delta w$,
✓ The cost of solving the adjoint equation is comparable to that of solving the flow equations.
✓ When the number of design variables becomes large, the computational efficiency of the control theory approach over traditional approach
Smoothed Design Gradient

-modified sobolev gradient
  ✓ It is key for successful implementation of adjoint approach
  ✓ It enables the generation of smooth shapes
  ✓ It leads to a large reduction in design iterations
✓ The optimization process tends to reduce the smoothness of the trajectory at each step
✓ Sobolev gradient preserves the smoothness class of the redesigned surface
### Smoothed Design Gradient

<table>
<thead>
<tr>
<th>Smoothed Gradient</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Original:</strong> $\delta l = \int G(x) \delta F(x)dx$, $\delta F = -\lambda G$</td>
</tr>
<tr>
<td><strong>Smoothed:</strong> $\delta l = \int G(x) \delta F(x)dx$, $\delta F = -\lambda \bar{G}$</td>
</tr>
</tbody>
</table>

### Smoothing Equation

$$\bar{G} - \frac{\partial}{\partial x} \varepsilon \frac{\partial}{\partial x} \bar{G} = G$$

### Guaranteed Descent

<table>
<thead>
<tr>
<th>Guaranteed Descent</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Original Gradient:</strong> $\delta l = -\lambda \int G^2 dx$</td>
</tr>
<tr>
<td><strong>Sobolev Gradient:</strong> $\delta l = -\lambda \int (\bar{G}^2 + \varepsilon \left( \frac{\partial}{\partial x} \bar{G} \right)^2) dx$</td>
</tr>
</tbody>
</table>
Smoothed Design Gradient

✓ implicit smoothing may be regarded as a preconditioner
✓ allows the use of much larger steps for the search procedure
✓ leads to a large reduction in the number of design iterations needed for convergence.
Smoothed Design Gradient

Smoothing Equation using Second-order Central Differencing

\[ \tilde{G}_i - \epsilon (\tilde{G}_{i+1} - 2\tilde{G}_i + \tilde{G}_{i-1}) = \bar{G}_i, \quad 1 \leq i \leq n \]

\[ \bar{G} = PG \]

where \( P \) is the \( n \times n \) tridiagonal matrix such that

\[ P^{-1} = \begin{bmatrix}
1 + 2\epsilon & -\epsilon & 0 & . & 0 \\
\epsilon & . & . & . & . \\
0 & . & . & . & . \\
. & . & . & -\epsilon & . \\
0 & . & . & \epsilon & 1 + 2\epsilon
\end{bmatrix} \]

Then in each design iteration, a step, \( \delta F \), is taken such that

\[ \delta F = -\lambda PG \]
Outline of the Design Process

The design procedure

1. Solve the flow equations for $\rho, u_1, u_2, u_3$ and $p$.
2. Solve the adjoint equations for $\psi$.
3. Evaluate $\mathcal{G}$ and calculate Sobolev gradient $\overline{\mathcal{G}}$.
4. Project $\overline{\mathcal{G}}$ into an allowable subspace with constraints.
5. Update the shape based on the direction of steepest descent.
6. Return to 1 until convergence is reached.
Design Cycle

1. Flow Solution
2. Adjoint Solution
3. Gradient Calculation
4. Sobolev Gradient
5. Shape & Grid Modification
6. Repeat the Design Cycle until Convergence

Aerodynamic Shape Optimization
Prediction of Transition and Natural Laminar Flow
Optimization with Prediction of Transition Flows

Boundary-Layer Parameter

✓ $\sigma^*$
✓ $\theta$

Tollmien-Schlichting Instability

$$N_{TS} = f(Re_\theta, H_k, \frac{T_w}{T_e})$$

✓ determine the onset of critical point $Re_\theta \geq Re_{\theta_0}$
✓ compute Tollmien-Schlichting amplification factor $N_{TS}$
✓ transition occurred when $N_{TS} \approx 9$
Cross-Flow Instability

Crossflow Reynolds number:

\[ Re_{cf} = \frac{\rho_e w_{max} \delta_{cf}}{\mu_e} \]

Critical Reynolds number:

\[ Re_{critical} = 46 \frac{T_w}{T_e} \]

Amplification rate

\[ \alpha(Re_{cf}, \frac{w_{max}}{U_e}, H_{cf}, \frac{T_w}{T_e}) \]

Crossflow amplification factor

\[ N_{CF} = \int_{x_0}^{x} \alpha dx \]
Optimization with Prediction of Transition Flows

Viscosity for RANS Baldwin-Lomax Model

\[
\tau_{ij} = \left( \mu_{\text{laminar}} + \mu_{\text{turbulent}} \right) \left\{ \frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} - \frac{2}{3} \left[ \frac{\partial u_k}{\partial x_k} \right] \right\} \delta_{ij}
\]

Transition Prescription

✓ Create transition lines from \( N_{TS} \) and \( N_{CF} \)
✓ The wing surface and flow domain are divided into the laminar and turbulent regions
Optimization with Prediction of Transition Flows

Coupling Transition Prediction with Adjoint Solver

1. RANS solution with prescribed transition locations
2. Converge RANS solution sufficiently
3. Evoke transition prediction using converged flows
4. Compute $N_{TS}$ and $N_{CF}$
5. Iterate until transition location converges
6. Continue RANS with updated viscosity distribution
APPLICATION I: Drag Minimization
Application I: Viscous Optimization of Korn Airfoil

Mesh size = 512 x 64, Mach number = 0.75, $CL_{target} = 0.63$, Reynolds number = 20m
The Effect of Applying Smoothed Gradient

Application I: Drag Minimization
APPLICATION II: LOW SWEEP WING DESIGN
Application II: Low Sweep Wing Redesign using RK-SGS Scheme

Mesh size = 256x64x48, Design Steps = 15, Design variables = 127x33 = 4191 surface mesh points
APPLICATION III: NATURAL LAMINAR WING DESIGN
Airplane Geometry

(a) Wing-body Geometry

Mesh size = 256x64x48

(b) Wing-body Surface Mesh
Airplane Mesh

Mesh size = 256x64x48
Wing Planform and Sectional Profiles

(a) Cross Sectional Profiles

(b) Wing Planform Shape

Application III: Natural Laminar Wing Design
### Comparison of Turbulent and Laminar Drag

<table>
<thead>
<tr>
<th>CL</th>
<th>M=.40</th>
<th>M=.50</th>
<th>M=.60</th>
<th>M=.65</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Transitional</td>
<td>Turbulent</td>
<td>Transitional</td>
<td>Turbulent</td>
</tr>
<tr>
<td>1.1</td>
<td>–</td>
<td>–</td>
<td>277[226+51]</td>
<td>307[241+66]</td>
</tr>
</tbody>
</table>

**Application III: Natural Laminar Wing Design**
Transitional Flow Prediction: $M = 0.40$, $CL = 0.80$

Design Points: \( M = 0.40, \ CL = 0.4, 0.6, 0.9, 1.0 \)

At \( M = .40 \), the lower limit of the operating range is set by the appearance of suction peak. This suction peak appears at the lower surface when \( CL < .40 \), and at the upper surface when \( CL > .90 \).
Transitional Flow Prediction: $M=0.50$, $CL=0.80$

Design Points: \( M=0.50, \ CL=0.4, 0.6, 0.9, 1.0 \)

At \( M = .50 \), the lower limit of the operating range is set by the appearance of suction peak. This suction peak appears at the lower surface when \( CL < .40 \), and at the upper surface when \( CL > 1.0 \).
Design Points: M=0.60, CL=0.4, 0.6, 0.9, 1.0

At $M = .60$, the lower limit of the operating range is set by the appearance of suction peak at the lower surface, and the formation of shock at the upper surface. The suction peak appears at the lower surface when $CL < .40$, and the shock starts to form at the upper surface when $CL > .90$. 

Application III: Natural Laminar Wing Design
At $M = .65$, the lower limit of the operating range is set by the appearance of suction peak at the lower surface, and the formation of shock at the upper surface. The operating range has become too narrow to be useful. In particular, the suction peak appears at the lower surface when $CL \leq 40$, and the shock of moderate strength has already started to form at the upper surface when $CL = .50$. 

**Application III: Natural Laminar Wing Design**
The figure shows the operating boundaries within which favorable pressure distribution can be maintained.
Closing Remarks
Conclusions

### Key Features of Adjoint Based Approach

- Efficient computation on affordable computers
- Multi-point optimization
- Variety of cost functions: inverse design, drag minimization, or a combination

### Impact of Aerodynamic Shape Optimization

- Achieve highly optimized designs
- Subtle shape changes not achievable through trial and error
- Designer’s input is still important
- Shock free airfoil possible
- Efficient low sweep wing is possible
- Natural laminar flows wing possible over a wide range of operating conditions