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ACCELERATED ITERATION SCHEMES FOR TRANSONIC FLOW CALCULATIONS USING FAST POISSON SOLVERS

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Abstract

Reliable but slow methods for calculating transonic flows have been developed in recent years. These use central difference formulas in the subsonic (elliptic) zone, and upwind difference formulas in the supersonic (hyperbolic) zone. This report describes an improved iterative method for solving the resulting difference equations. Each iteration consists of two stages: in the first stage a direct method is used to solve Poisson's equation with the nonlinear terms treated as a forcing function; in the second stage the relaxation method is used to sweep out the errors in the supersonic zone. The combined method gives fast convergence, whereas the Poisson method by itself diverges when there is a region of supersonic flow, and relaxation by itself is very slow.

1. Introduction.

This report describes an improved iterative method for solving nonlinear partial differential equations of mixed type which appear in the calculation of transonic flows. In particular the equations to be considered are the transonic potential flow equation and the transonic small disturbance equation.

The potential flow equation can be derived from the Euler equations for compressible fluid flow by introducing the assumption that the flow is irrotational, so that we can define a potential ϕ. Then we find that in smooth regions of flow ϕ satisfies the quasilinear equation

$$\phi_{xx} + \phi_{yy} = \frac{1}{a^2} (u^2\phi_{xx} + 2uv\phi_{xy} + v^2\phi_{yy}) \tag{1.1}$$

In this equation u and v are the velocity components

$$u = \phi_x , \quad v = \phi_y \tag{1.2}$$

and a is the local speed of sound. Given the ratio of specific heats γ, and the stagnation speed of sound a_0, a can be determined from the energy equation

$$a^2 = a_0^2 - \frac{\gamma-1}{2} q^2 \tag{1.3}$$

in which q is the speed

$$q = (u^2 + v^2)^{1/2}$$

Equation (1.1) is elliptic when the local Mach number $q/a < 1$, and hyperbolic when $M > 1$. The boundary conditions are that the normal velocity vanishes at the body,

$$\frac{\partial \phi}{\partial n} = 0 \quad \text{on a given curve} \tag{1.4}$$

and that the flow is uniform with a prescribed speed at infinity.

Smooth transonic solutions are known to exist only in special cases [1]. We must therefore admit weak solutions with appropriate discontinuities [2]. Since an irrotational flow is isentropic the discontinuities are not true shock waves but isentropic jumps. If the normal component of mass flow and the tangential component of velocity are conserved across the discontinuities, and we also exclude discontinuous expansions, then they will be fairly good approximations to shock waves of moderate strength. The momentum deficiency across the discontinuities then provides an approximation to the wave drag [3].

We can ensure satisfaction of these jump conditions by treating the equation in the conservation form

$$\frac{\partial}{\partial x} (\rho \phi_x) + \frac{\partial}{\partial y} (\rho \phi_y) = 0 \tag{1.5}$$

where ρ is the density. If M_∞ is the Mach number of the free stream at infinity, then ρ can be determined from the local speed of sound by the relation

$$\rho^{\gamma-1} = M_\infty^2 a^2 \tag{1.6}$$

The form (1.5) also corresponds to the Bateman variational principle that

$$I = - \iint p \, dx \, dy$$

is stationary, where p is the pressure

$$p = \frac{\rho^\gamma}{\gamma M_\infty^2}$$

If the profile is given in the form

$$y = \tau f(x)$$

where τ is a sufficiently small parameter, we can expect the disturbances to be small. If we expand the solution in powers of τ under the assumption that $1 - M_\infty^2 \sim \tau^{2/3}$, and retain only the leading term, we can then obtain the transonic small disturbance equation [4]. A typical form is

$$\phi_{xx} + \phi_{yy} = A \phi_{xx} \tag{1.7}$$

where A is an approximation to the square of the local Mach number given by

$$A = M_\infty^2 (1 + (\gamma+1) \phi_x) \tag{1.8}$$

In this equation ϕ is the disturbance potential, so that the velocity components are $1+\phi_x$ and ϕ_y. The boundary condition at the body is transferred to the x axis, becoming

$$\phi_y = \frac{df}{dx} \text{ at } y = 0 . \tag{1.9}$$

Thus we have the double simplification that the upwind direction is known to be the x direction, and that the Neumann boundary condition no longer has to be satisfied on a curved profile.