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Optimal Control of LCOs in Aero-Structural Systems

Having your (nonlinear) cake and eating it too

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Aero-elastic systems, by their very nonlinear nature, frequently encounter Limit Cycle Oscillations (LCOs). An LCO occurs when the system being described reaches a periodic steady state where there is a constant exchange of energy between the various degrees of freedom, in this case: the Aero and Structural components. These LCOs might be benevolent at times, but are not so in most cases. In this paper we study Active and Passive Control Strategies for LCOs and explore the possibility of harnessing the nonlinearities to our benefit. After defining a control approach based on Optimal Control Theory, we study the specific case of a nonlinear panel interacting with a supersonic flow and undergoing LCOs, and tailor the system parameters to achieve the desired control objectives.

I. Introduction

Aerospace Systems, by their very nature, are constructed to be as light as possible. The push for more efficient systems plateaus at the point where further reduction in structural weight is accompanied by a reduction in the structural stiffness, which might pose restrictions on the operating envelope of the vehicle. It is observed, that in the air vehicles of the current age, the most stringent constraints on the dynamic stability of structures are imposed by Aero-Elastic loads.

One of the most frequently encountered phenomena in Aero-Structural interactions amongst Aerospace systems is the occurence of Limit Cycle Oscillations (LCOs). An LCO occurs when the system being described reaches a periodic steady state where there is a constant exchange of energy between the various degrees of freedom, in this case: the Aero and Structural components. LCOs represent the nonlinear nature of the system being observed. These might prove harmful if the amplitudes of oscillation are fairly high.

Some widely studied LCO problems in Aero-Elastic systems are that of an airfoil with a cubic nonlinear restoring force and a nonlinear panel interacting with supersonic flow. Beran et al.[1] have developed sohpisticated computation algorithms for studying LCOs in general and have studied these problems in great detail.

The nonlinear nature of a system, however, is akin to a knife that cuts both ways. Ways have been studied where the nonlinear nature of a system can be exploited to our advantage. For instance, Blackburn et al.[2] show that combined translational and rotational motion can be used to propel a bluff body in quiescent flow. It can be envisioned therefore that the nonlinear nature of Aero-Elastic systems can be harnessed so that the resulting behavior is beneficial to us. Such studies have a lot of potential in the design of Micro Air Vehicles, whose operation is inspired by the flight of insects.

A good first step in this direction is the identification of suitable control objectives and the determination of optimal control strategies for the same. Such control strategies could be passive, where the system parameters are tweaked so as to achieve open-loop optimal performance, or they could be active, where the system state is monitored periodically and suitable control input is delivered actively. Such active control systems are essentially state feedback systems and can be shown to be theoretically equivalent to Optimal open-loop systems.[3]

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The control algorithms employed in this work are based on the Algorithms for Shape Optimization developed for Aerodynamic Systems by Jameson and his co-workers over the last few years.[4] The major highlight of these algorithms are the establishment of an Adjoint Method whereby the sensitivities of a predefined cost function with respect to all the design variables, whose numbers could potentially range in the thousands to millions, can be calculated with the same level of computational complexity as the State equations themselves. The cost function, in this case, is a property of the LCO degrees of freedom at all instants of time. The sensitivities calculated relate the system response, viz. the behaviour of the LCO dof's, to unit changes in the design variables in a direction that minimizes (or maximizes) the chosen cost function.

The algorithms that are developed are tested on a problem where a nonlinear panel interacts with a supersonic flow. Like we discussed earlier, our optimization/control problem can be framed so as to minimize or maximize the oscillation amplitudes. Minimization of LCO amplitudes is a problem that is of relevance in the aerospace industry, viz. flutter suppression, etc. The enhancement of the amplitude of oscillation is a problem that is relevant to the design of MAVs and is studied herein. This is important because, at the conditions under which MAVs are operated, there are some very unique nonlinear phenomena that occur that can be exploited to our advantage.

II. Formulation of the Control Problem

A. The Nonlinear Dynamical equations governing the LCO problem

The nonlinear dynamical equations of an autonomous system can be written as

$$\frac{d\mathbf{x}}{dt} = \mathbf{f}(\mathbf{x}, \lambda). \tag{II.1}$$

Here t is the time, $\mathbf{x}(t)$ is an N_f dimensional array of real variables, and $\mathbf{f}$ is an N_f array of nonlinear functions dependent on $\mathbf{x}$ and λ, an N_d array of design parameters. Time is scaled by the LCO period, T, so as to yield a set of evolutionary equations in terms of the scaled system time, s.

$$\frac{d\mathbf{x}}{ds} = T\mathbf{f}(\mathbf{x}, \lambda). \tag{II.2}$$

This equation set is then solved using the cyclic method due to Beran et. al.[1] This is done by expressing the time derivative in (II.2) as a Second Order Central difference of the state vector at that instant in time. M such time instances are chosen to form the collocated vector $\mathbf{X}$, which is a collection of the values of $\mathbf{x}$ at M different time instances.

$$\mathbf{X} = (\mathbf{x_1}, \mathbf{x_2}, \ldots, \mathbf{x_M}). \tag{II.3}$$

The LCO equations are then

$$\mathbf{G}(\mathbf{X}, T, \lambda) = 0. \tag{II.4}$$

B. Passive Performance Enhancement by Optimization

As discussed earlier, different control objectives are sought for different physical problems. The one potentially most relevant to the design of MAVs is that of LCO amplitude amplification, and will be studied here. A suitable cost function for the case of LCO amplification would then be

$$I = \left(\frac{1}{2}\mathbf{X}^T Q \mathbf{X}\right)^{-1} + \frac{1}{2}\lambda'^T R \lambda', \tag{II.5}$$

where

$$\lambda' = \lambda - \bar{\lambda}, \tag{II.6}$$

where $\bar{\lambda}$ is an average value of the design parameter, and λ' is the deviation about the average. Q and R are suitable symmetric integration matrices. Minimizing the above cost function seeks to maximize the LCO amplitude. The first term in the expression for the cost function, (II.5) ensures that the LCO amplitude is maximized over the entire time period, and the second term ensures that the values of the design variables don't grow boundlessly.

Taking a variation of the cost function I described in (II.5), we get

$$\delta I = -\frac{\mathbf{X}^T Q \delta \mathbf{X}}{(\mathbf{X}^T Q \mathbf{X})^2} + \lambda'^T R \delta\lambda'. \tag{II.7}$$

Variation of the constraint function described by (II.4) yields

$$\delta \mathbf{G} = \frac{\partial \mathbf{G}}{\partial \mathbf{X}} \delta \mathbf{X} + \frac{\partial \mathbf{G}}{\partial T} \delta T + \frac{\partial \mathbf{G}}{\partial \lambda'} \delta\lambda' = 0. \tag{II.8}$$

We now subtract $\mathbf{\Psi}^T \delta \mathbf{G} = 0$ from (II.7), where $\mathbf{\Psi}$ is an arbitrary co-state vector. This co-state vector spans all the LCO states at all instants of time. We then re-arrange to seperate variations of solution variables from the variation of design variables as follows

$$\delta I = -\frac{\mathbf{X}^T Q \delta \mathbf{X}}{(\mathbf{X}^T Q \mathbf{X})^2} - \mathbf{\Psi}^T \frac{\partial \mathbf{G}}{\partial \mathbf{X}} \delta \mathbf{X} - \mathbf{\Psi}^T \frac{\partial \mathbf{G}}{\partial T} \delta T + \left(\lambda'^T R - \mathbf{\Psi}^T \frac{\partial \mathbf{G}}{\partial \lambda'} \right) \delta\lambda'. \tag{II.9}$$

The co-state vector $\mathbf{\Psi}$ is then chosen to eliminate the first three terms in the RHS of (II.9) by satisfying

$$-\frac{\mathbf{X}^T Q \delta \mathbf{X}}{(\mathbf{X}^T Q \mathbf{X})^2} - \mathbf{\Psi}^T \frac{\partial \mathbf{G}}{\partial \mathbf{X}} \delta \mathbf{X} - \mathbf{\Psi}^T \frac{\partial \mathbf{G}}{\partial T} \delta T = 0. \tag{II.10}$$

Equation (II.10) is called the Adjoint equation. With this selection δI is expressed only in terms of variations in the design variables as follows

$$\delta I = \left(\lambda'^T R - \mathbf{\Psi}^T \frac{\partial \mathbf{G}}{\partial \lambda'} \right) \delta\lambda', \tag{II.11}$$

leading to the identification of the gradient $\mathbf{g}$ in the optimization problem:

$$\mathbf{g}^T = \left(\lambda'^T R - \mathbf{\Psi}^T \frac{\partial \mathbf{G}}{\partial \lambda'} \right), \tag{II.12}$$

$$\delta I = \mathbf{g}^T \delta\lambda' \tag{II.13}$$

It should be noted that this is the first time that a co-state and an Adjoint based Sensitivity analysis procedure have been developed for LCO dofs.

III. The Nonlinear Elastic Panel Undergoing LCOs

The supersonic flow past a nonlinearly deforming panel is an interesting problem that has been studied in the past by Dowell et al.,[5] and is especially relevant now in the context of Performance Enhancement and Control System design for MAVs, that expect to harness inherent nonlinear advantages of the Aero-Structural interactions. This also serves as a good multi disciplinary model on which our algorithms can be tested. The nonlinear nature of the system arises because of the large scale deformations of the structure, which are modelled by Von Karman's large-deflection plate equations.[6] and.[7]

The equations of motion for the structural displacement of the system are as follows:

$$\frac{\partial^2 M_x}{\partial x^2} - F_x \frac{\partial^2 w}{\partial x^2} + \rho h \frac{\partial^2 w}{\partial t^2} = q - F_{AZ}, \tag{III.1}$$

$$\frac{\partial F_x}{\partial x} = 0, \tag{III.2}$$

where

$$\begin{aligned} F_x &= \frac{Eh}{1-\nu^2} \left[\frac{\partial u}{\partial x} + \frac{1}{2} \left(\frac{\partial w}{\partial x} \right)^2 \right], \\ M_x &= D \frac{\partial^2 w}{\partial x^2}, \\ D &= \frac{Eh^3}{12(1-\nu^2)}. \end{aligned} \tag{III.3}$$

Here M_x and F_x are the moments and forces respectively in the x direction, E is the Young's modulus, h is the thickness of the structure, ρ is the density and ν is the Poisson's ratio. F_{AZ} is the applied force in the z direction, q is zero and u and w are the displacements in the x and z directions respectively.

The aero component comes from the forcing term F_{AZ} in (III.1). This is modeled using Piston theory as follows

$$P - P_\infty = \frac{2q}{\sqrt{M_{\infty^2} - 1}} \left[\frac{\partial w}{\partial x} + \left(\frac{M^2 - 2}{M^2 - 1} \right) \frac{1}{U} \frac{\partial w}{\partial t} \right]. \tag{III.4}$$

Here P_∞ is the freestream pressure and P is the pressure on the upper surface of the panel undergoing oscillations.

The above Partial Differential Equations are discretized in a grid and values of the displacement w_i and velocity s_i are collected in a state vector $\mathbf{x}$:

$$\mathbf{x} = [s_1, w_1, \ldots, s_N, w_N]. \tag{III.5}$$

Now, equations (III.1), (III.3) and (III.4) are simplified and non-dimensionalized:

$$\begin{aligned} x &\equiv x/L, \\ \tau &\equiv tU/L, \\ w &\equiv w/h, \\ \lambda &\equiv 2qL^3/\beta D, \\ \mu &\equiv \rho L/\rho_m h, \end{aligned} \tag{III.6}$$

In the case of the passive performance enhancement method, λ is assumed to take on an average value of $\bar{\lambda}$ throughout the length of the panel, with a perturbation λ'. The simplification involved assumes that the perturbation λ' is relatively small. Having made this assumption, and non-dimensionalizing, equations (III.1), (III.3) and (III.4) can be written in the form of equation (II.2) and the sensitivity analysis of Section B applies directly to the panel problem.

IV. Results

The equations described in the previous section were set up computationally and solved using a steepest descent method. The mean value of $\bar{\lambda}$ as specified in equation (II.6) is set to 4288.75. First of all, a comparison was made between the gradients calculated using the Adjoint Method and the Gradients calculated using the Finite Difference Method. The results are presented in Figure IV.1. As can be observed from the figure, the two curves are almost spot on. The difference in the numerical values of these gradients is shown in Figure IV.2.

The infinity norm of the gradient is plotted in figure IV.3. As can be seen the changes in the gradient become smaller and smaller with increasing iteration numbers.

The constant property panel and the optimized panel are allowed to undergo LCO and their maximum deflection curves are compared in Figure IV.4. We wanted the amplitude of the oscillations to increase and it can be seen that this is exactly what happens.

Finally, the values of the perturbations in λ are plotted in Figure IV.5. It can be seen that these perturbations are fairly large, which reinforces the need to use the full set of equations described in the previous section. Another important observation that can be made is the fact that the optimizer, in order to passivley enhance the oscillation amplitudes, increases the flexibility at the $3/4 \quad th$ point which is where the amplitude of oscillation is maximum.

V. Conclusions

A new methodology for optimally controlling limit-cycle oscillations has been developed and applied to the classical panel flutter problem. The purpose of the work is to show the potential for either using LCOs productively or controlling the destructive growth of LCOs. The algorithms developed were demonstrated on a panel LCO amplitude enhancement problem.

To achieve optimal control, sensitivities of an LCO objective function, with respect to passive structural properties are computed using an adjoint formulation expressed over the entire LCO. Numerical experiments

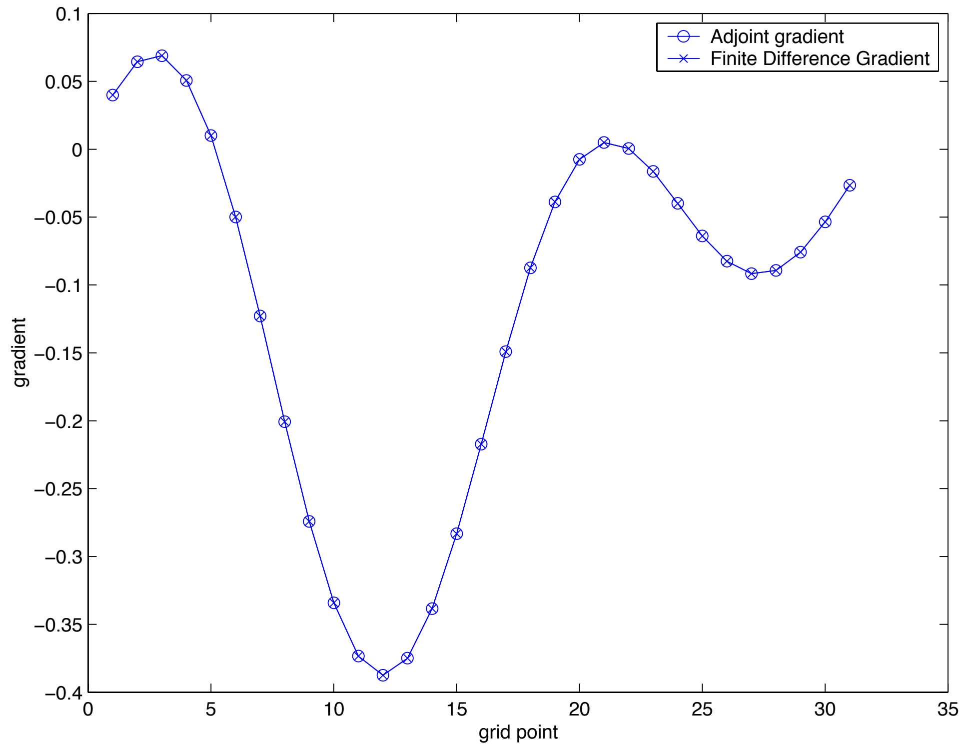

Figure IV.1. Comparison of adjoint and finite difference gradients

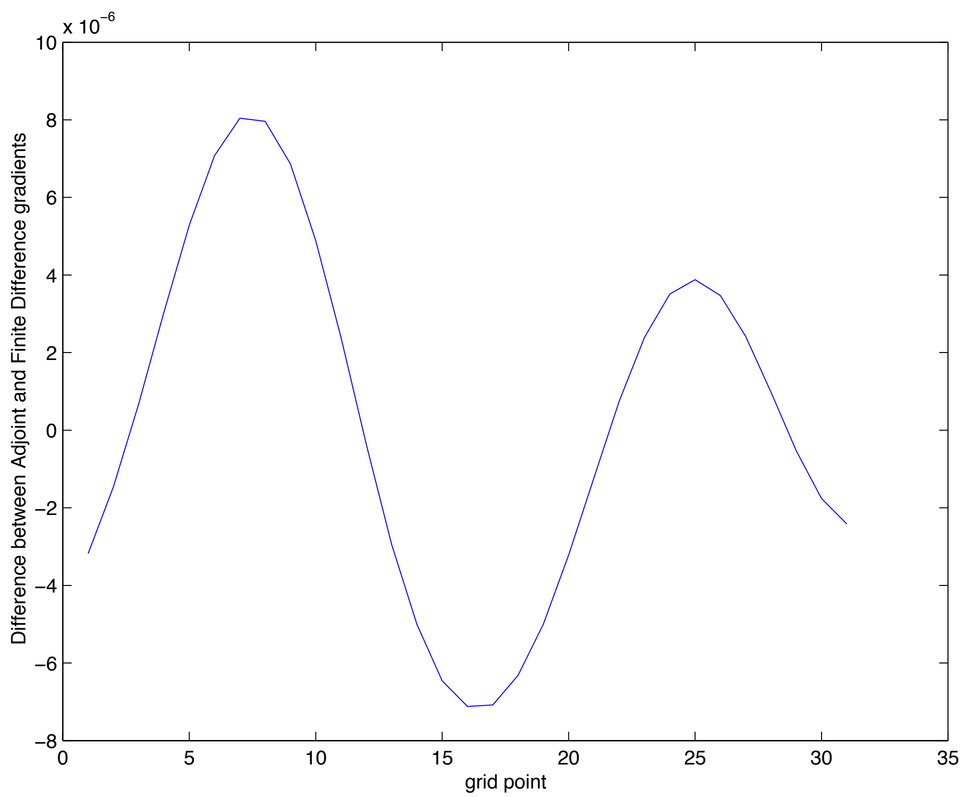

Figure IV.2. Difference between the adjoint and finite difference gradients

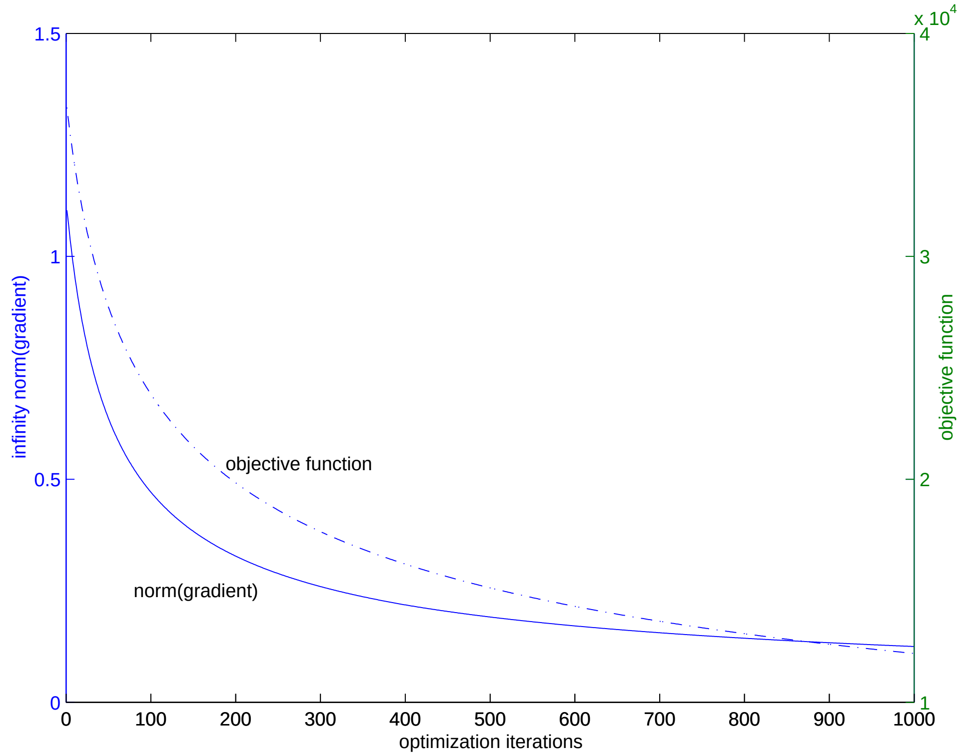

Figure IV.3. Gradient Convergence

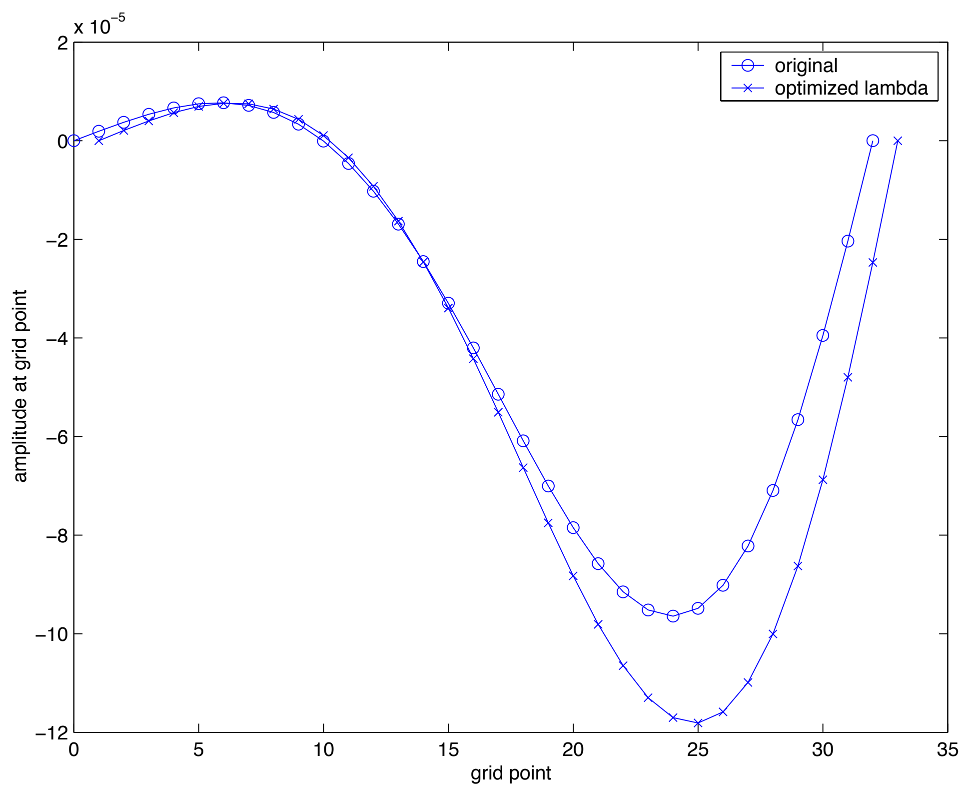

Figure IV.4. Comparison of maximum deflection curves of the base and enhanced panels

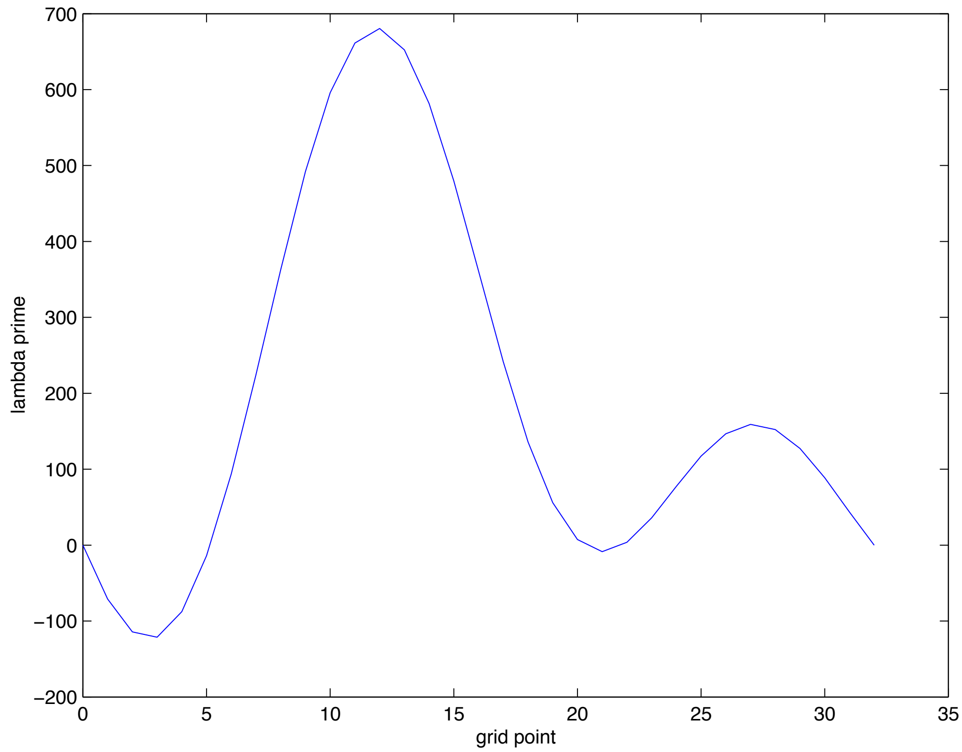

Figure IV.5. Values of λ' vs. x on the optimized Panel

verified that the optimization process was convenient and that the sensitivities computed with the adjoint formulations matches well with conventional techniques.

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