Unsteady Transonic Flows Supporting Non-unique Solutions

Antony Jameson, Kui Ou, Jeff Thomas, Earl Dowell, Kenneth Hall, John Vassberg

Stanford University
Duke University
Boeing Company

USNCCM, July 22nd, 2013
Outline of the Presentation

1. Introduction
   ✓ History of Non-unique Transonic Solution

2. Non-unique Transonic Solutions
   ✓ Viscous Unsteady Flows
   ✓ Comparison with Euler Solutions
   ✓ Discussion of Mechanism

3. Harmonic Balance Solutions

4. Conclusion
Introduction: A History of Transonic Flows
Supporting Non-unique Solutions
Non-uniqueness transonic flow solution was first documented by Jameson and Steinhoff at the GAMM Workshop, Stockholm, September 1979. The solution to the full potential equation was obtained using Jameson’s FCPOT method.

(a) symmetric solution  (b) shock waves  (c) asymmetric solution

Figure: Joukowski airfoil at Mach 0.840, $\alpha = 0.000$
The first non-unique transonic solution for the Euler equation was presented by Jameson at the Honolulu AIAA conference, June 24-26, 1991. The study showed that even when the production of entropy by shock waves is properly included in the mathematical model (Euler equation), non-uniqueness may still occur.

**Figure:** (a) J-75 airfoil at Mach 0.750, $\alpha = -0.200$, and (b) GAW75-06-15 Airfoil at Mach 0.750, $\alpha = -2.250$
The question of non-unique transonic flows was re-examined by Hafez and Guo (1999) who formed both lifting and non-lifting solutions for a 12 percent thick symmetric airfoil with parallel sides from 25 to 75 percent chord in a Mach range from 0.825 to 0.843.

The question was further pursued in detail in a series of studies by Kuz’mín and Ivanova (2004, 2006) who confirmed the results of Hafez and Guo, and also showed that airfoils with positive curvature everywhere could support non-unique solutions.

Unsteady simulations of flow past airfoils exhibiting transonic non-uniqueness were investigated by Caughey in 2004.
Four Airfoils Supporting non-unique Transonic Solutions: NU4, JF1, JB1, JC6

- In non-lifting transonic flow they exhibit a transition from a solution with two supersonic zones on each surface below a certain critical Mach number to a situation with one supersonic zone on each surface above the critical Mach number.

- In the region of instability solutions with positive lift are found in which there is a single supersonic zone on the top surface and two supersonic zones on the lower surface.

- Solutions with negative lift which are the mirror images of the solutions with positive lift.
Non-unique Transonic Solutions: Unsteady Viscous Flows
✓ Simulation results based on unsteady Navier-Stokes equations (modified version of Jameson’s UFLO103).
✓ Reynolds number of 6,000,000 has been used for all viscous flows
NU4 airfoil is a consequence of a shape optimization study for symmetric airfoils in transonic flow, in which an attempt was made to find a 12 percent thick airfoil with a shock free solution at Mach 0.84. NU4 has an almost shock free solution at its design Mach number, but also allows a lifting and non-lifting solution at zero angle of attack.
✓ The $C_l - \alpha$ sweep shows that NU4 airfoil assumes unique solution at Mach 0.830.

✓ As the speed further increases, non-unique transonic solutions are observed, i.e. multiple $C_l$ solutions are supported for a given Mach number and $\alpha$.

✓ As the Mach number increases, the range of $\alpha$ that can support non-unique solutions widens, and the non-unique solutions also become more stable.
Non-unique transonic solutions can appear when multiple shocks are formed on the airfoil surface.

If the shocks are far enough apart and do not interact with each other for a given range of disturbance, the solution remains unique, i.e. $M = 0.830$ case.

If the shocks are closely spaced, and the perturbation is sufficient to cause the shocks to interact / coalesce, it can lead to non-unique flows, i.e. $M = 0.837$ case.
✓ The reverse occurs as the speed further increases
✓ The range of $\alpha$ supporting non-unique flows becomes narrower
✓ At sufficiently high Mach number, the transonic flows become unique again
At $M=.841$, small change in $\alpha$ causes two closely spaced shocks to coalesce into one single shock.

The newly formed flow supporting the single shock is stable even after the original $\alpha$ is restored.

At $M=.848$, the airfoil admits only a single shock on each airfoil surface.

Without interaction of multiple shocks, the transonic flows remain unique.
JF1 airfoil consists of a parallel sided slab closed by a semi-circular nose and two parabolic arcs at the rear. Depending on the extent of the parabolic arcs a Mach range exists in which lifting solutions can be found at zero angle of attack.
(a.1) $M=0.831$, $\alpha = 0^\circ$

(b.1) $M=0.837$, $\alpha = 0^\circ$

(c.1) $M=0.831$, $\alpha = 0^\circ$

(d.1) $M=0.837$, $\alpha = 0^\circ$

(e.1) $M=0.831$, $\alpha = 0^\circ$

(f.1) $M=0.837$, $\alpha = 0^\circ$
Non-unique Transonic Flows: RANS Solution vs Euler Solution
JB1 Airfoil M.827

(a) UFLO103

(b) FLO82

Jameson, Ou, Thomas, Dowell, Hall, Vassberg

Non-unique Transonic Flows
JB1 Airfoil M.827

(a) UFLO103 M=.827, $\alpha = 0^\circ$

(b) FLO82 M=.827, $\alpha = 0^\circ$

---

Jameson, Ou, Thomas, Dowell, Hall, Vassberg

Non-unique Transonic Flows
JF1 Airfoil M.835

(a) UFLO103

(b) FLO82
(a) UFLO103 $M=0.835$, $\alpha = 0^\circ$

(b) FLO82 $M=0.835$, $\alpha = 0^\circ$

---

JF1 Airfoil

MACH 0.835  ALPHA 0.000

CL 0.0774  CD 0.0602  CM -0.0351

GRID 384X 128  NSTEP 1  RES 0.346E+02

---

1.2 0.8 0.4 0.0 -0.4 -0.8 -1.2 -1.6 -2.0

Cp +++++++

---

JF1 AIRFOIL

MACH 0.83500  ALPHA 0.00000

CL 0.00000  CD 0.046047  CM 0.00000

GRID 384X 64  NDES 0  RES 0.568E-13  GMAX 0.100E-05

---

1.2 0.8 0.4 0.0 -0.4 -0.8 -1.2 -1.6 -2.0

Cp ++++++++

---

JAMESON, OU, THOMAS, DOWELL, HALL, VASSBERG

Non-unique Transonic Flows
NU4 Airfoil M.840

(a) UFLO103

(b) FLO82

Jameson, Ou, Thomas, Dowell, Hall, Vassberg
Non-unique Transonic Flows
NU4 Airfoil M.840

(a) UFLO103 M=0.840, $\alpha = 0^\circ$

(b) FLO82 M=0.840, $\alpha = 0^\circ$

NU4 AIRFOIL

MACH 0.840    ALPHA 0.000
CL 0.0749    CD 0.0156    CM -0.0187
GRID 384X 128    NSTEP 1    RES 0.986E+01

0.1E+01 0.8E+00 0.4E+00 -0.2E-15 -0.4E+00 -0.8E+00 -0.1E+01 -0.2E+01 -0.2E+01

CP

+++++++++++++++++++++

NU4 AIRFOIL

MACH 0.840    ALPHA 0.000
CL 0.038750    CD 0.004184    CM -0.012122
GRID 640X 128    NCYC 2000    RES 0.320E-13

1.2 0.8 0.4 0.0 -0.4 -0.8 -1.2 -1.6 -2.0

CP

Jameson, Ou, Thomas, Dowell, Hall, Vassberg

Non-unique Transonic Flows
JC6 Airfoil M.847

(a) UFLO103

(b) FLO82

Jameson, Ou, Thomas, Dowell, Hall, Vassberg

Non-unique Transonic Flows
JC6 Airfoil M.847

(a) UFLO103 M=.847, $\alpha = 0^\circ$

(b) FLO82 M=.847, $\alpha = 0^\circ$

(a) UFLO103 M=.847, $\alpha = 0^\circ$

(b) FLO82 M=.847, $\alpha = 0^\circ$
Non-unique Transonic Flows: Discussion
Effects of Disturbance from the Singular Point

- $\Delta M$

- $+\Delta M$

Non-unique Transonic Flows
Effects of Disturbance from the Singular Point

Jameson, Ou, Thomas, Dowell, Hall, Vassberg

Non-unique Transonic Flows
Non-unique Transonic Flows: Harmonic Balance
Zero Degree Angle-of-Attack Unsteady Response

Hafez Airfoil / Copied from Caughey (2004)
Unsteady Lift Response, $M_{\infty} = 0.84$

Unsteady Flow past Flat Hafez I Airfoil

JC6 Airfoil / OVERFLOW
Unsteady Lift Response, $M_{\infty} = 0.847$

Lift Coefficient, $c_l$
Drag Coefficient, $c_D$
Moment Coefficient, $C_m$
Angle of Attack, $\alpha$

Nondimensional Time, $t'_\infty = tU_\infty/c$
JC6 Airfoil Transonic Non-Uniqueness

JC6 Airfoil Harmonic Balance/OVERFLOW Unsteady Zeroth Harmonic Pressure Distributions, $M_\infty = 0.847$.

$\bar{\omega} = 0.1, \bar{\alpha}_1 = 0.001$ Degree

$\bar{\omega} = 0., \bar{\alpha}_1 = 0.03$ Degree

$\bar{\omega} = 0.001, \bar{\alpha}_1 = 0.03$ Degree

$\bar{\omega} = 0.1, \bar{\alpha}_1 = 0.03$ Degree
JC6 Airfoil Transonic Non-Uniqueness

JC6 Airfoil Harmonic Balance/OVERFLOW Unsteady First Harmonic Pressure Distributions (Real Part), $M_\infty = 0.847$.

$\bar{\omega} = 0.1, \bar{\alpha}_1 = 0.001$ Degree

$\bar{\omega} = 0.1, \bar{\alpha}_1 = 0.03$ Degree

$\bar{\omega} = 0.001, \bar{\alpha}_1 = 0.03$ Degree

$\bar{\omega} = 0.001, \bar{\alpha}_1 = 0.03$ Degree

Jameson, Ou, Thomas, Dowell, Hall, Vassberg

Non-unique Transonic Flows
Closing Remarks
Some closing remarks:

- Non-unique transonic flows were previously found for steady Euler solutions.
- Non-unique transonic flows for unsteady NS solutions are also found to exist.
- The existence of boundary layers modify the conditions for which the non-unique solutions appear, compared to the Euler solutions.
- The non-unique transonic solutions exist over a range of Mach numbers.
- The non-unique solutions appear when weak double shocks appear on the airfoil.
- Non-unique solutions generate larger lift coefficients, than the unique counter-part.

- When the separation of the two shocks are sufficiently small such that the shock movement due to the disturbance leads to the coalescence of the two shocks, a new stable flow is established.
- The flow with the coalesced single shock remains stable after the removal of the disturbance, resulting in non-unique solution for the same flow conditions.
- For a given perturbation, non-unique solution does not appear when the shocks are sufficiently separated from each other, or
- when the flows involve only single shock on each airfoil surface.