Influence of Shape Parameterization on Aerodynamic Shape Optimization

John C. Vassberg

Boeing Technical Fellow
Advanced Concepts Design Center
Boeing Commercial Airplanes
Long Beach, CA 90846, USA

Antony Jameson

T. V. Jones Professor of Engineering
Dept. of Aeronautics & Astronautics
Stanford University
Stanford, CA 94305-3030, USA

Von Karman Institute
Brussels, Belgium
9 April, 2014

LECTURE SERIES OUTLINE

- **INTRODUCTION**
- **THEORETICAL BACKGROUND**
  - SPIDER & FLY
  - BRACHISTOCHRONE
- **SAMPLE APPLICATIONS**
  - MARS AIRCRAFT
  - RENO RACER
  - GENERIC 747 WING/BODY
- **DESIGN-SPACE INFLUENCE**

LECTURE-3 OUTLINE

- **AIRFOIL ANATOMY**
  - TRUE LEADING EDGE & MLL CHORD
  - AIRFOIL STACK - WING GEOMETRY
- **DESIGN-SPACE PARAMETERIZATION**
  - BEZIER FAMILY
  - FREE SURFACE
  - B-SPLINES
- **SAMPLE OPTIMIZATIONS**
  - NACA0012-ADO AIRFOIL
  - ONERA-M6 WING
  - ADO-CRM WING

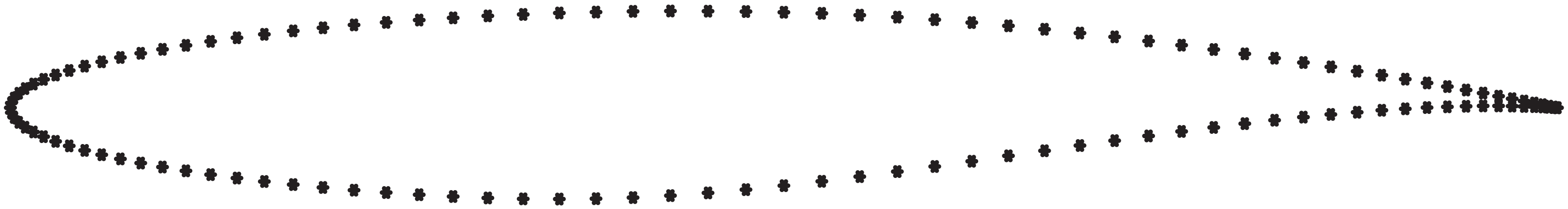
AIRFOIL ANATOMY

- **AIRFOIL DEFINITION**
  - PLANAR - NOT 3D SPACE CURVES
  - MLL CHORD
  - UPPER & LOWER SURFACE CONTOURS
  - LEADING- & TRAILING-EDGE PTS
    - $TE_{Base} \geq 0, \;\; TE = \frac{1}{2}(TE_U + TE_L)$
- **AIRFOIL STACK**
  - MINIMAL SET OF DEFINING STATIONS
  - ASSEMBLED & SURFACED IN WRP
  - TRANSFORMED TO FRP

AIRFOIL ANATOMY

- **ESTIMATING TRUE LEADING EDGE**
  - IDENTIFY DISCRETE LE
  - 3-POINT CIRCLE FIT
  - CONSTRUCT TRUE MLL CHORD
- **AIRFOIL PROPERTIES**
  - LEADING-EDGE RADIUS
  - THICKNESS & CAMBER
  - INFLECTION POINTS

AIRFOIL ANATOMY

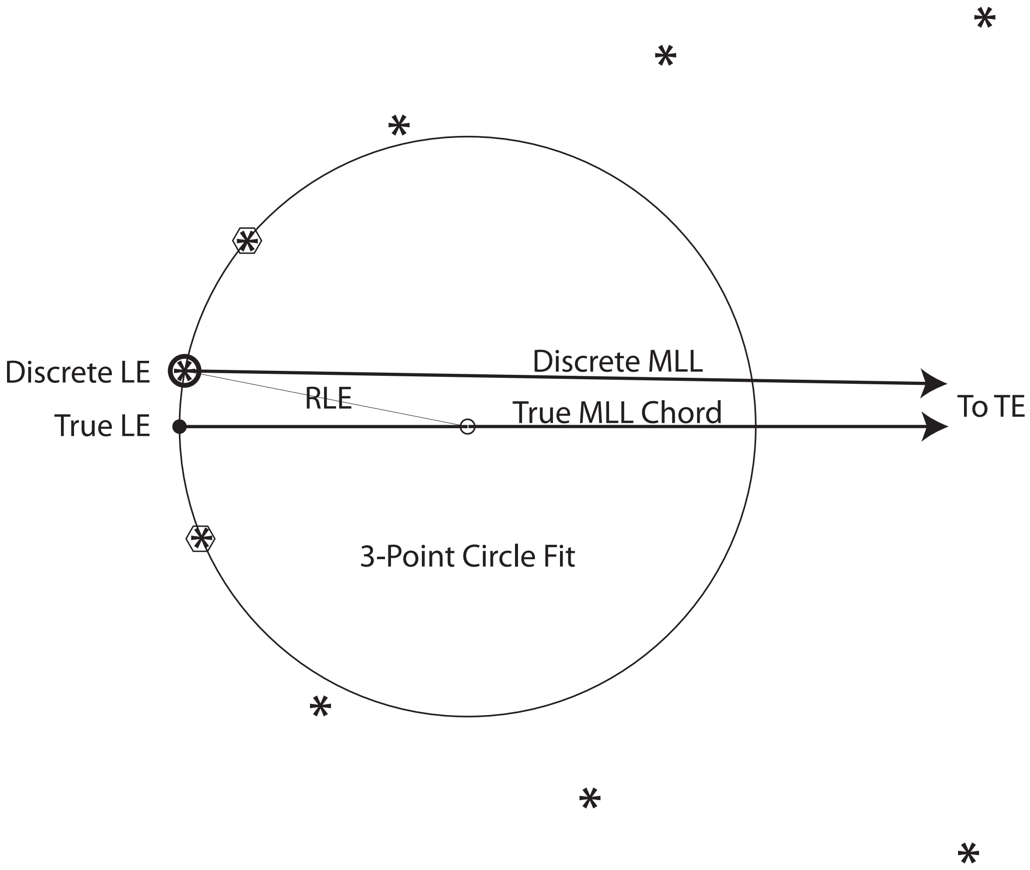


RAE 2822 Airfoil
Coordinates

RAE2822 Airfoil Discrete Coordinates.

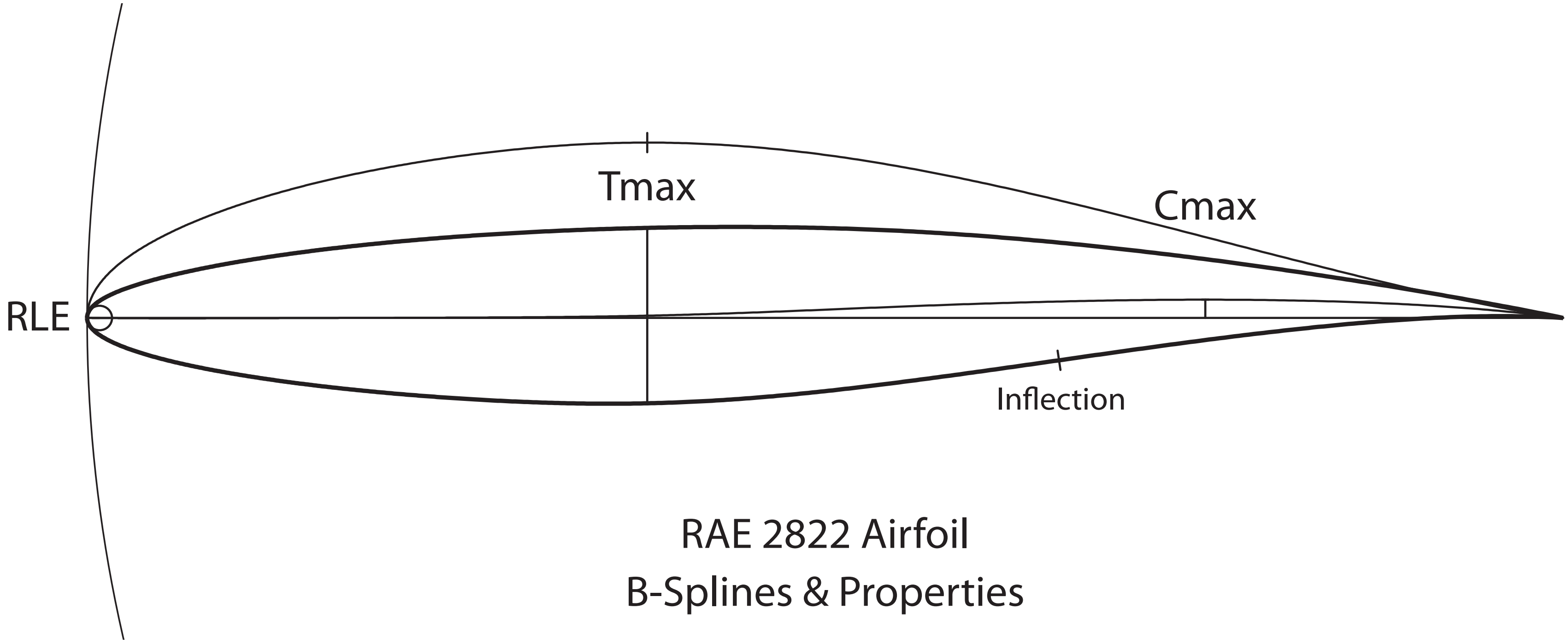
http://aerospace.illinois.edu/m-selig/ads/coord/rae2822.dat

AIRFOIL ANATOMY



Estimate of True Leading-Edge Point.

$$
\begin{aligned}
R_{LE} &= 0.008554, \\
(X,T)_{Tmax} &= (0.379526,\ 0.121108), \\
(X,C)_{Cmax} &= (0.757536,\ 0.012641), \\
(X,Y,\theta)_{Inflect} &= (0.65848,\ -0.02927,\ 8.29056^\circ).
\end{aligned}
$$



RAE2822 LS-Fit B-Splines & Geometric Properties.

DESIGN-SPACE PARAMETERIZATION

- **AERODYNAMIC CONSIDERATIONS**
  - STREAMWISE CURVATURE CONTINUITY
  - SPANWISE CONTINUITY
- **DESIGN CONSIDERATIONS**
  - LOCAL CONTROL
- **CUBIC CURVES - OPTIMUM BALANCE**
  - SERIES OF CUBIC BEZIER CURVES
  - CUBIC B-SPLINES

DESIGN-SPACE PARAMETERIZATION

- **BEZIER FAMILY**
  - NACA0012-ADO EQN.
  - LEAST-SQUARES FIT
  - DEGREE ELEVATION
- **FREE SURFACE**
- **CUBIC B-SPLINES**
  - RAE2822 LEAST-SQUARES FIT
  - THICKNESS & CAMBER
  - LEADING-EDGE RADIUS
  - OSCULATING CIRCLE

DESIGN-SPACE PARAMETERIZATION

Abbott and von Doenhoff give the NACA0012 equation as:

$$y_N(x) = \pm\frac{0.12}{0.2}\left(0.2969\sqrt{x} - 0.1260x - 0.3516x^2 + 0.2843x^3 - 0.1015x^4\right)$$

Note: Blunt Trailing Edge.

Nadarajah suggests changing the coefficient of the x^4 term such that a sharp trailing-edge is recovered at $x = 1$. The resulting analytic equation defining the NACA0012-ADO airfoil shape is:

$$y_A(x) = \pm\frac{0.12}{0.2}\left(0.2969\sqrt{x} - 0.1260x - 0.3516x^2 + 0.2843x^3 - 0.1036x^4\right)$$

Note: Sharp Trailing Edge.

NACA0012-ADO BEZIER

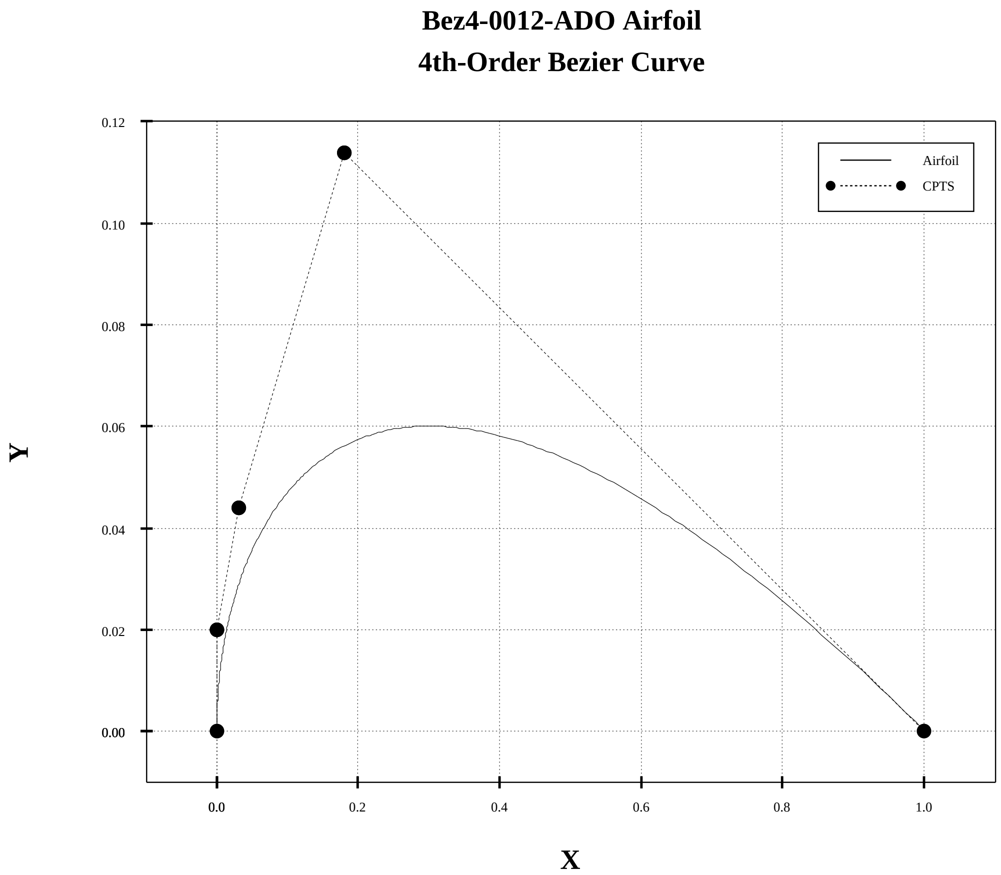
Table I:
Bez4-0012-ADO Control Points.

n	$xcpt_n - FIT$	$ycpt_n$-Fit
0	0.0000000	0.0000000
1	0.0000000	0.0256211
2	0.0308069	0.0438166
3	0.1795085	0.1135797
4	1.0000000	0.0000000

$$I = \int_0^1 [y_F(u) - y_A(x(u))]^2 \, du.$$

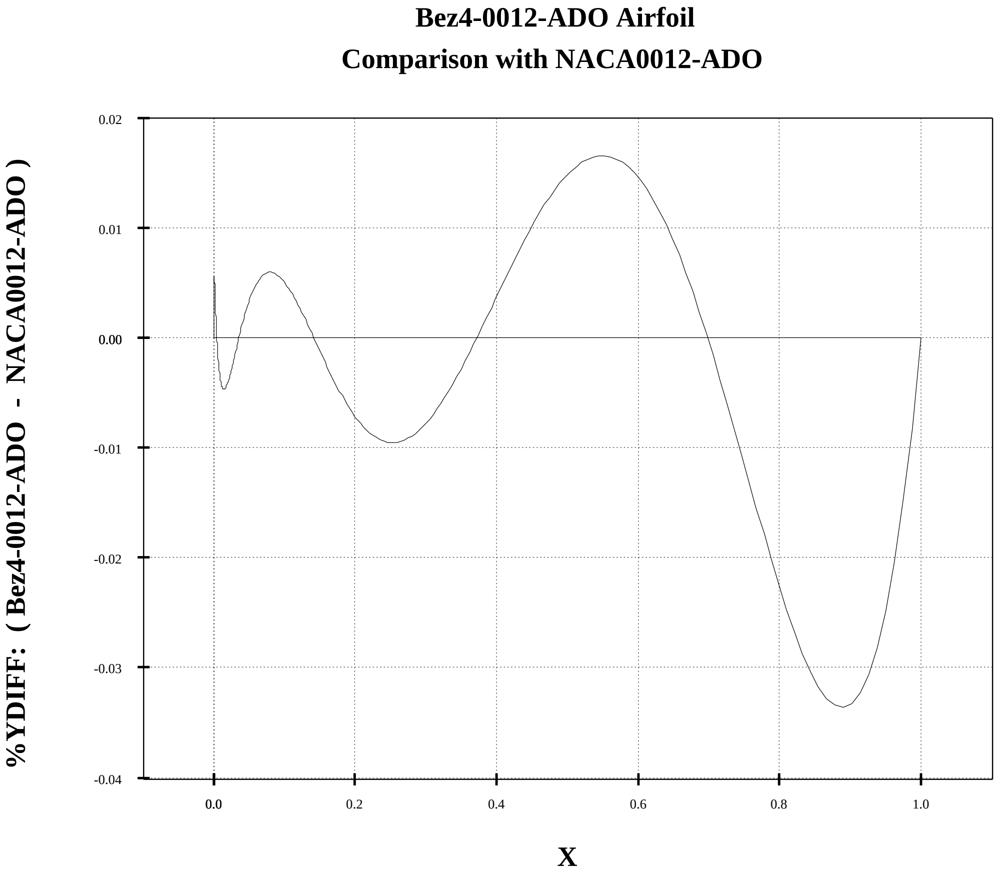
$$I_{min} \doteq 0.9497 * 10^{-8}.$$

NACA0012-ADO BEZIER



Bez4-0012-ADO Airfoil & 4^{th}-Order Bezier Control Points.

NACA0012-ADO BEZIER



ΔY [Bez4-0012-ADO - NACA0012-ADO] Airfoils.

BEZIER DEGREE ELEVATION

Elevating a K^{th}-order Bezier curve to $(K+1)^{st}$-order has control points given by the following recursive formula.

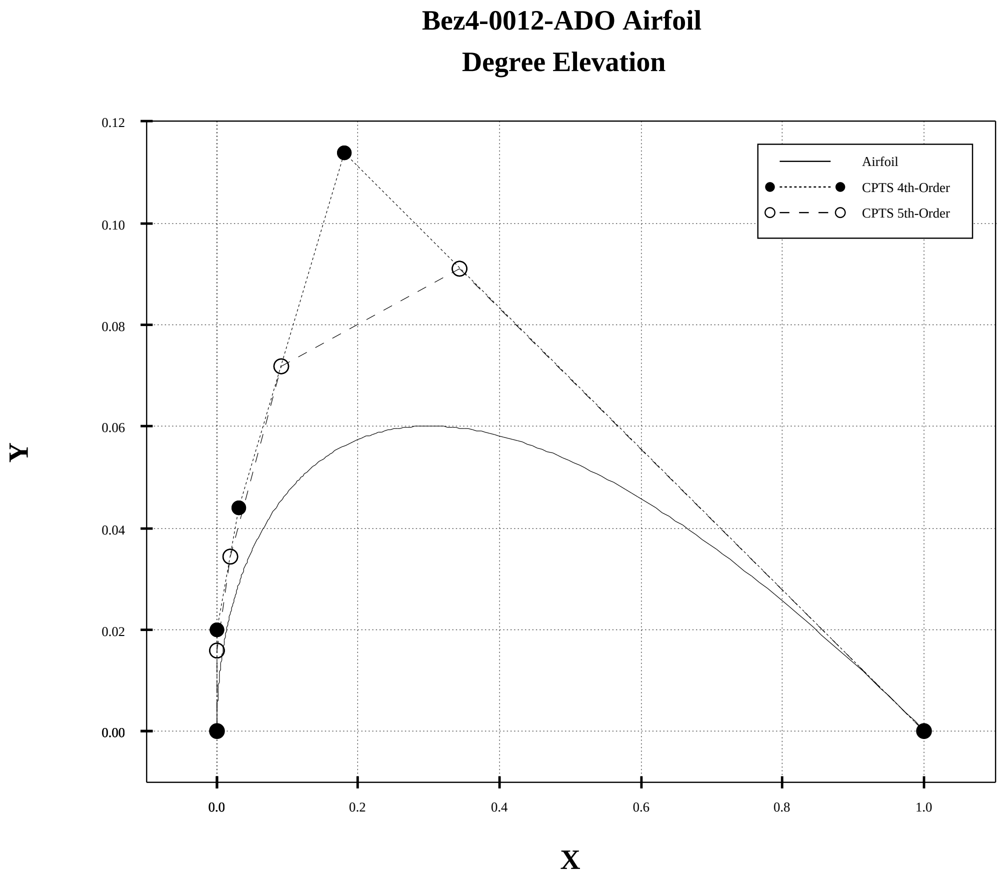
$$\mathcal{B}_k^{(K+1)} = \left(\frac{k}{K+1}\right)\mathcal{B}_{k-1}^{(K)} + \left(\frac{K+1-k}{K+1}\right)\mathcal{B}_k^{(K)};$$

$where\; 0 \leq k \leq K+1.$

$\mathcal{B}^{(K)}$ and $\mathcal{B}^{(K+1)}$ represent control points of the K^{th}-order and $(K+1)^{st}$-order Bezier curves, respectively.

While $\mathcal{B}_{-1}^{(K)}$ and $\mathcal{B}_{K+1}^{(K)}$ do not exist, their factors are zero.

BEZIER DEGREE ELEVATION



Degree Elevation of Bez4-0012-ADO from 4^{th} to 5^{th} Order.

CUBIC B-SPLINES

Third-order B-Splines of 33 control points define each surface. The $xcpt$ coordinates are preset by a cosine distribution.

$$\begin{aligned} xcpt_0 &= 0, \\ xcpt_n &= \frac{1}{2}\left[1 - cos\left(\frac{n-1}{31}\,\pi\right)\right], \quad 1 \le n \le 32. \end{aligned}$$

Since the leading- and trailing-edge points are pinned, the first and last control points have $ycpt_0 = 0$, and $ycpt_{32} = \pm\frac{1}{2}TE_{Base}$.

Curvature continuity at the LE requires $ycpt^u_1 = -ycpt^l_1$.

The remaining $ycpt$ coordinates of each B-Spline are defined with a least-squares fit of their corresponding grid points.

B-SPLINE DERIVATIVES

- **FUNCTIONS**

$$x(t)\ ,\quad y(t)\ ,\quad t(x)\ ,$$

$$T(t) = [y_u(t) - y_l(t)]\ ,\quad C(t) = \frac{1}{2}[y_u(t) + y_l(t)]$$

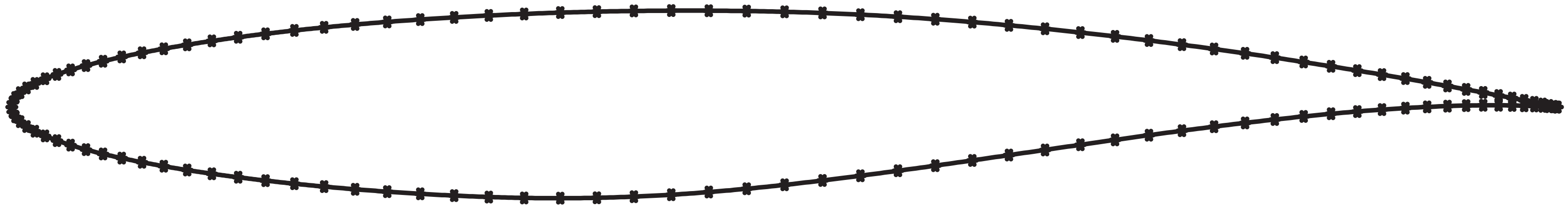
- **DERIVATIVES**

$$\dot{x}(t)\ ,\quad \dot{y}(t)\ ,\quad \ddot{x}(t)\ ,\quad \ddot{y}(t)\ ,\quad \frac{dy}{dx}(t) = \frac{\dot{y}}{\dot{x}}\ ,\quad \dot{T}(t)\ ,\quad \dot{C}(t)$$

- **CURVATURE**

$$\mathcal{K}(t) = \frac{[\dot{x}\ddot{y} - \ddot{x}\dot{y}]}{\left[\dot{x}^2 + \dot{y}^2\right]^{3/2}}\ ,\quad \rho(t) = \frac{1}{\mathcal{K}(t)}$$

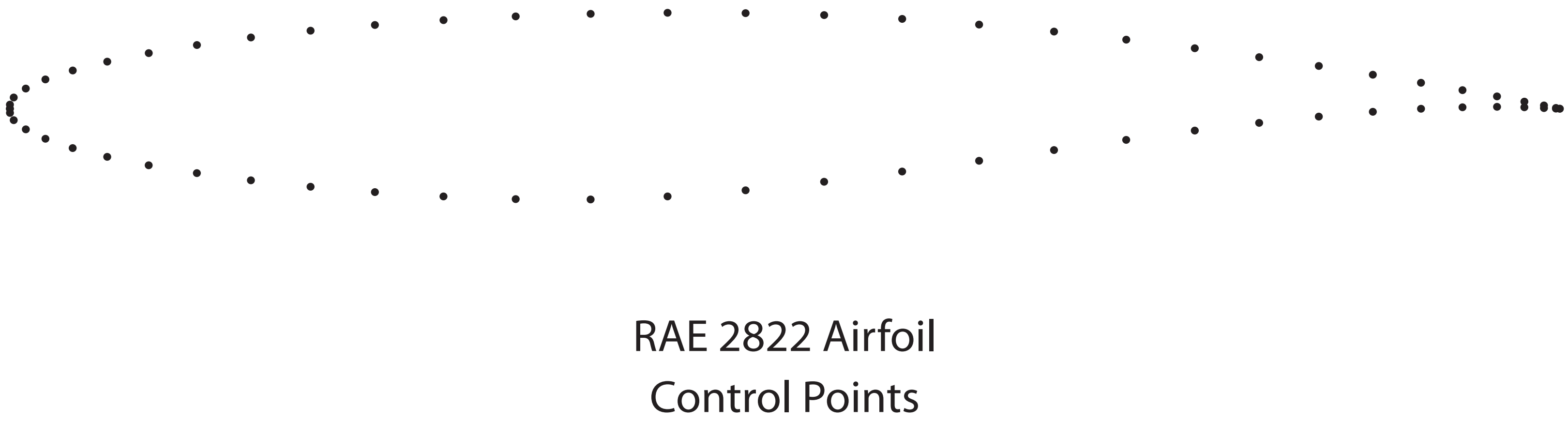
RAE2822 CUBIC B-SPLINES



RAE 2822 Airfoil
Coordinates & B-Splines

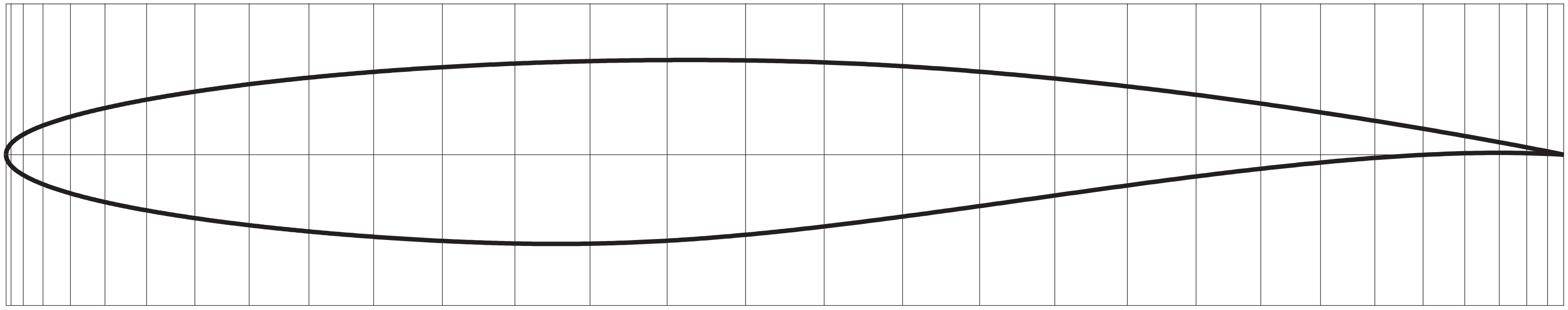
RAE2822 Coordinates with Least-Squares-Fit B-Splines.

RAE2822 CUBIC B-SPLINES



RAE2822 Airfoil Control Points.

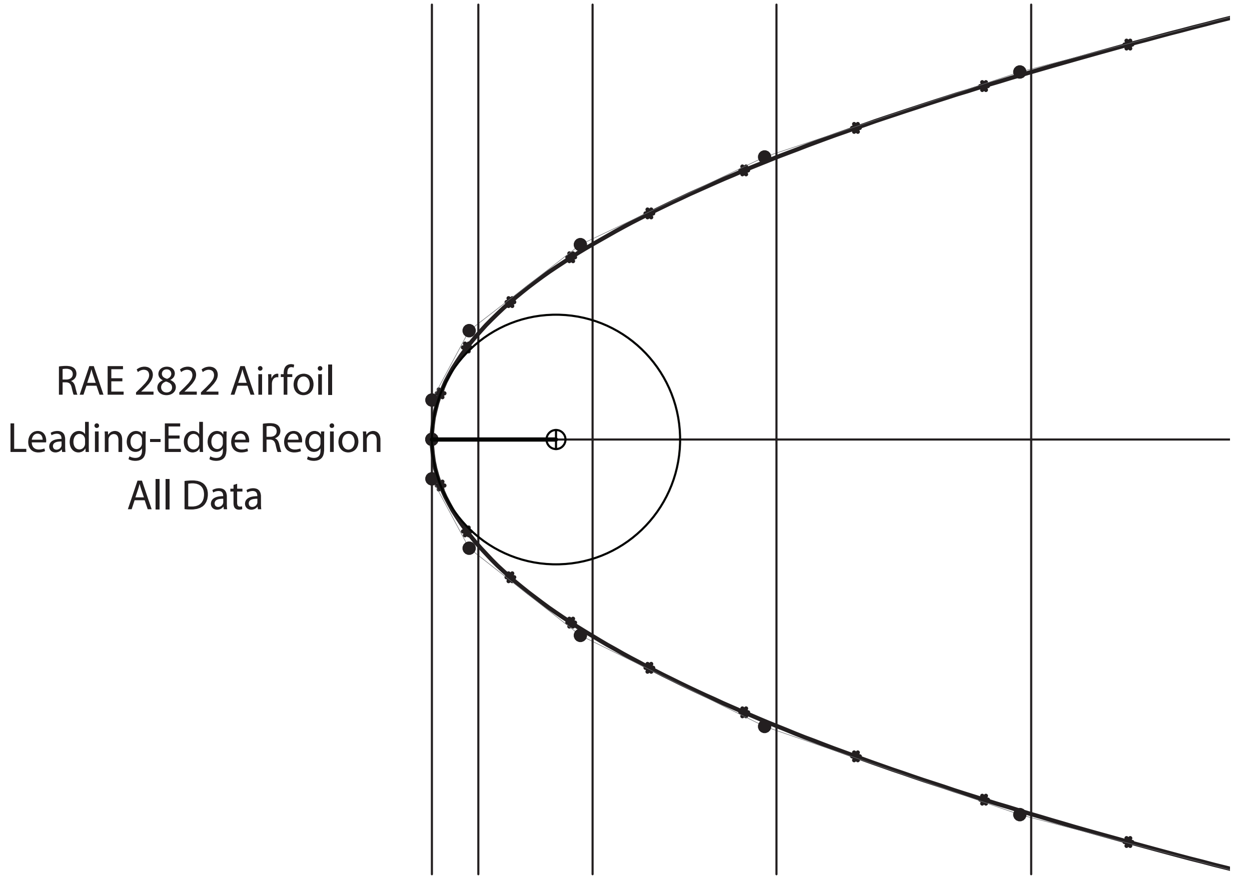
RAE2822 CUBIC B-SPLINES



RAE 2822 Airfoil
Curve Segments

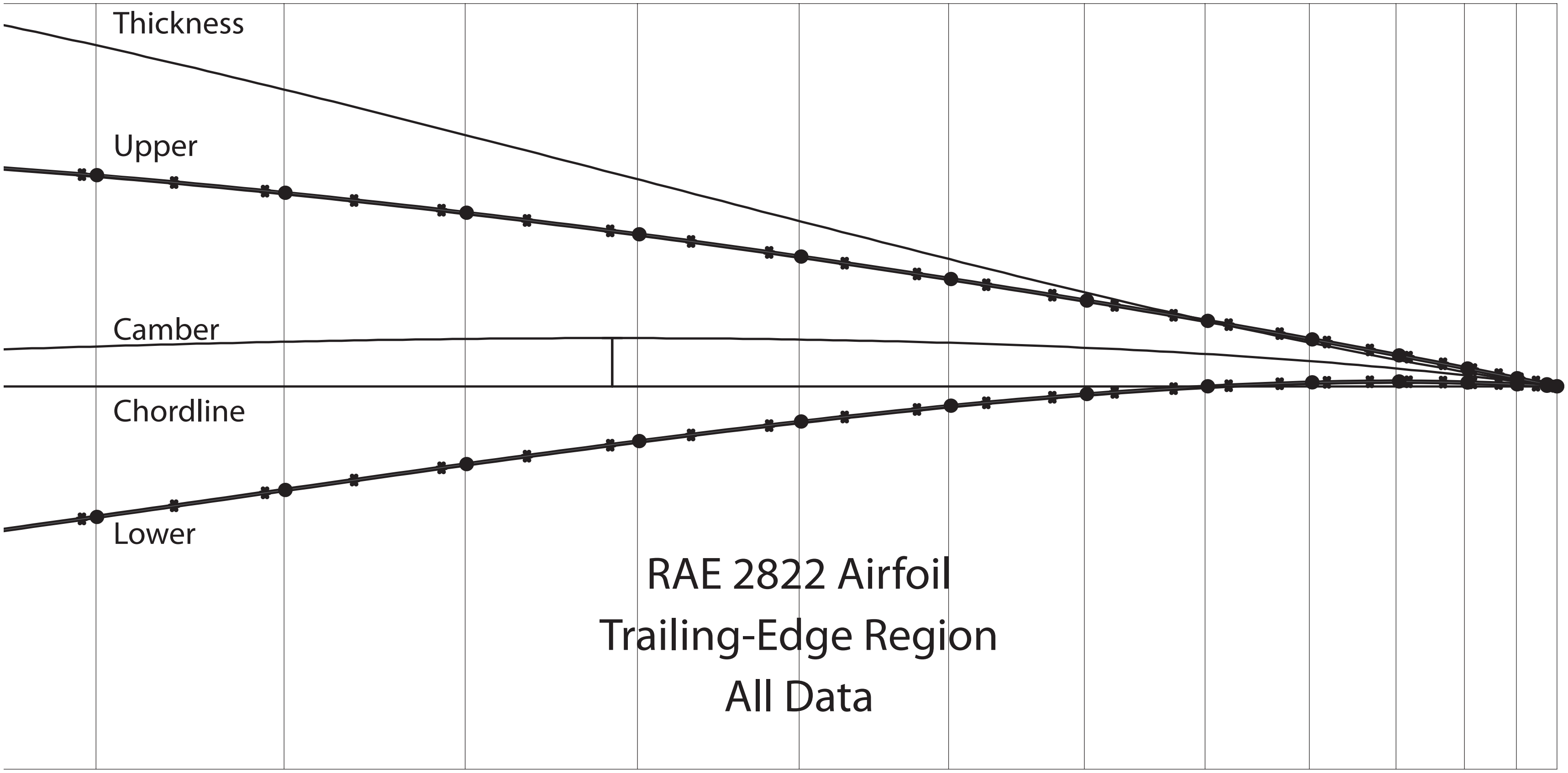
RAE2822 B-Spline Curve Segments.

RAE2822 CUBIC B-SPLINES



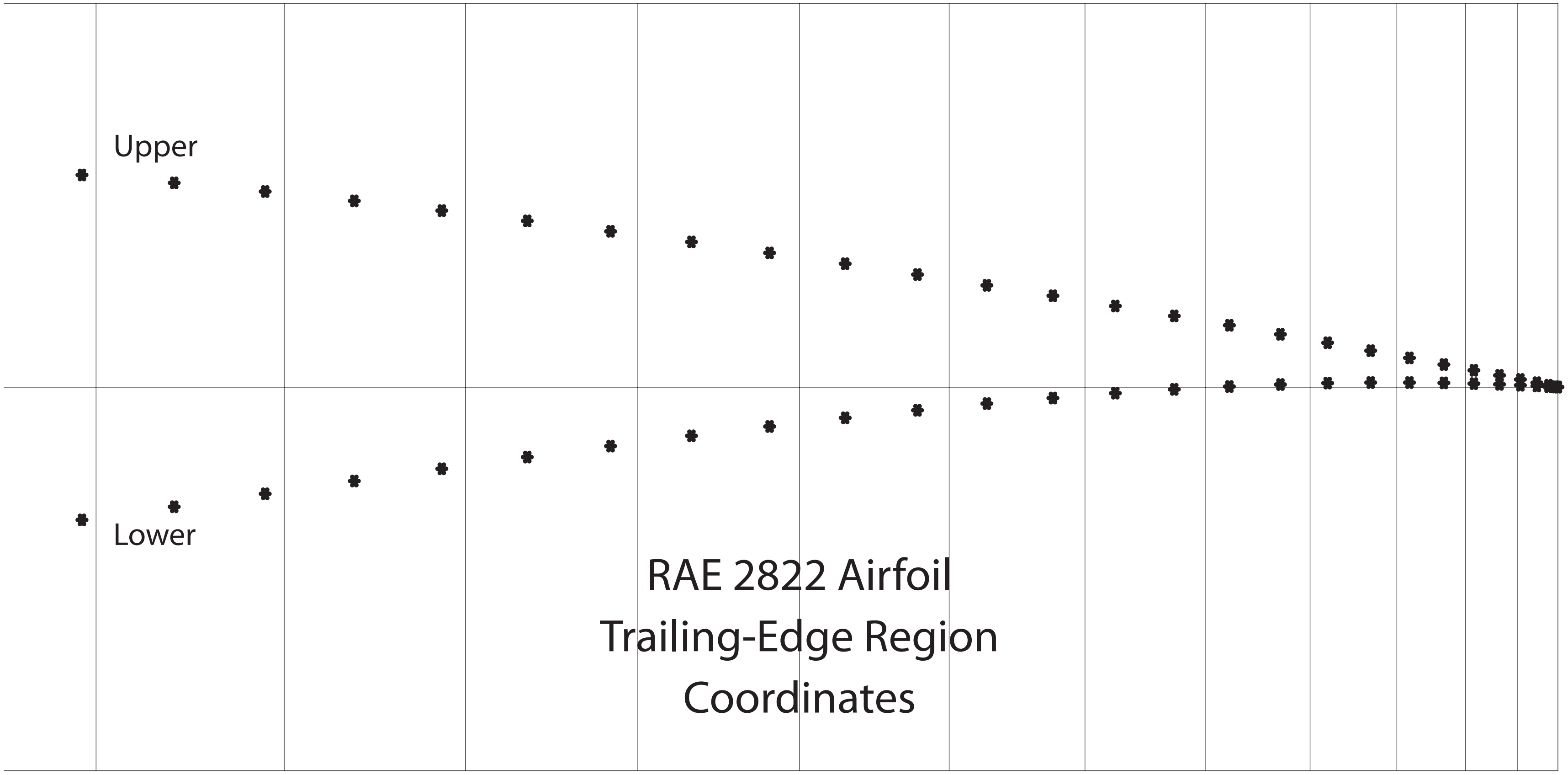
RAE2822 Coordinates, B-Splines & Leading-Edge Radius.

RAE2822 CUBIC B-SPLINES



RAE2822 Coordinates, B-Splines, Thickness & Camber near TE.

RAE2822 CUBIC B-SPLINES

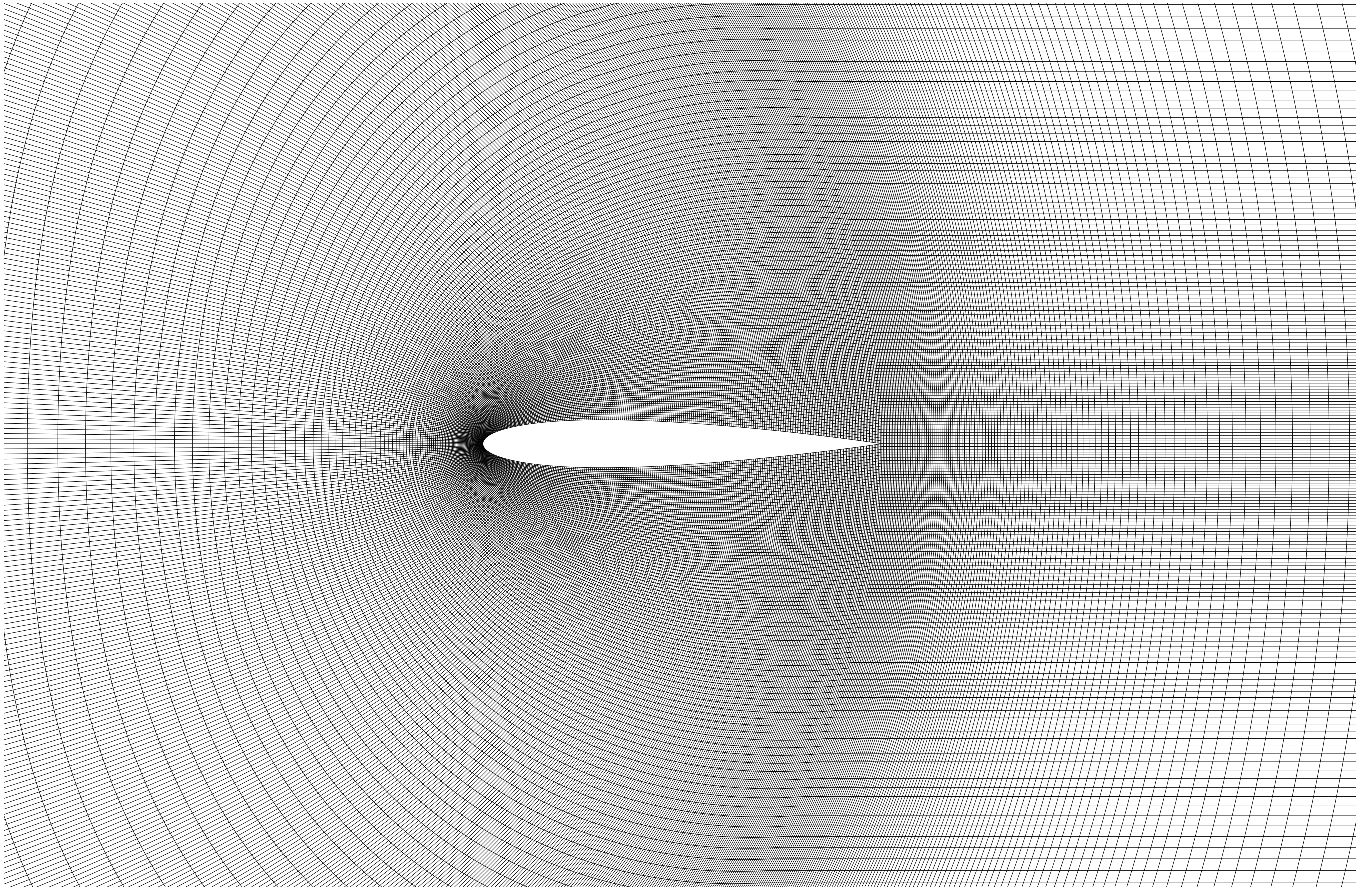


RAE2822 Coordinates & Curve-Segment Grid near TE.

OPTIMIZATION & CFD METHODS

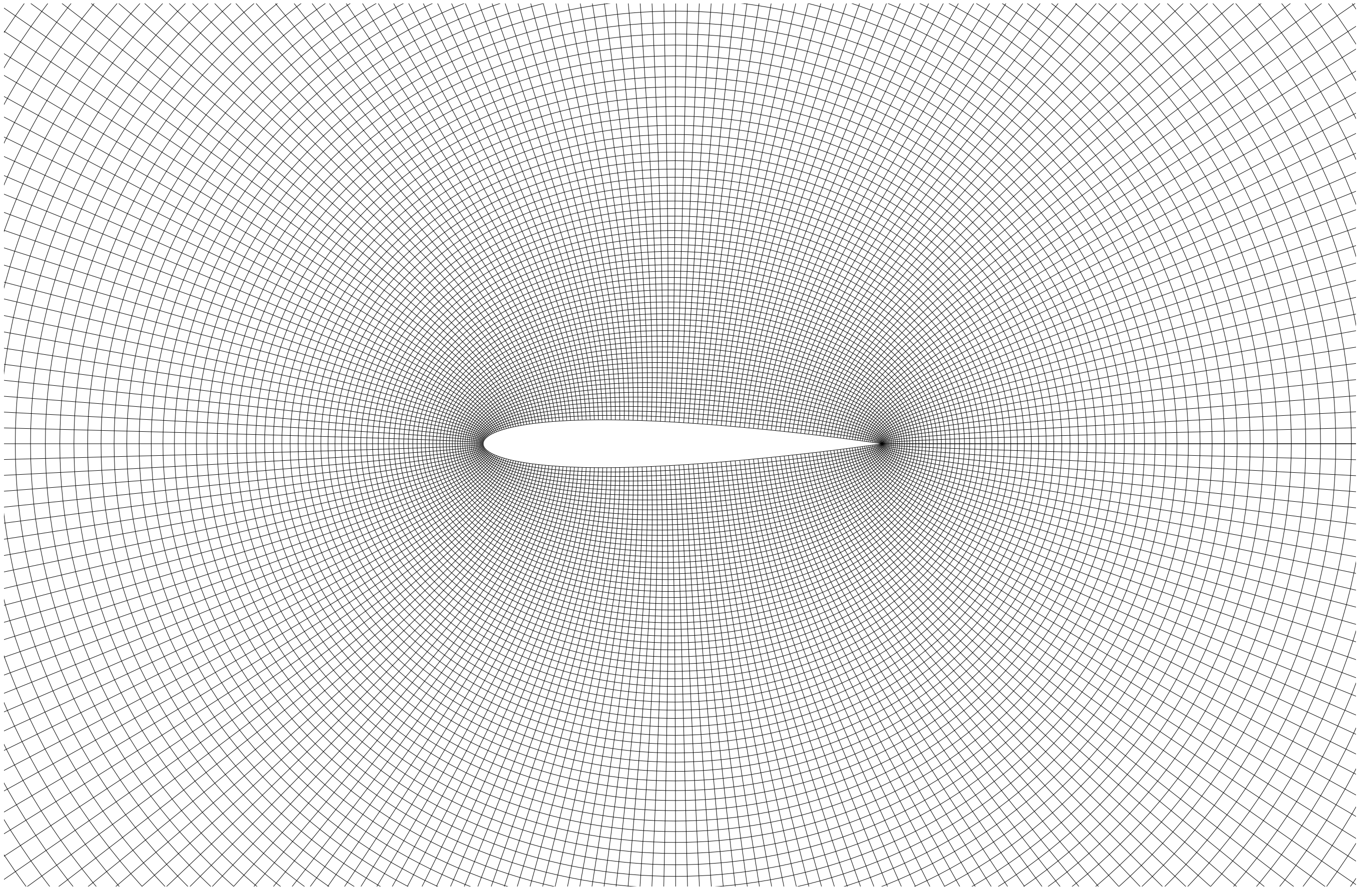
- **MDOPT & CMA-ES**
  - BEZIER
  - NON-GRADIENT OPTIMIZATIONS
  - OVERFLOW
- **SYN83 & SYN107**
  - FREE SURFACE & B-SPLINES
  - GRADIENT-BASED OPTIMIZATIONS
- **FLO82 CROSS ANALYSIS**
  - RIGOROUS GRID-CONVERGED PROCESS
  - RICHARDSON EXTRAPOLATION

SYN83 GRID



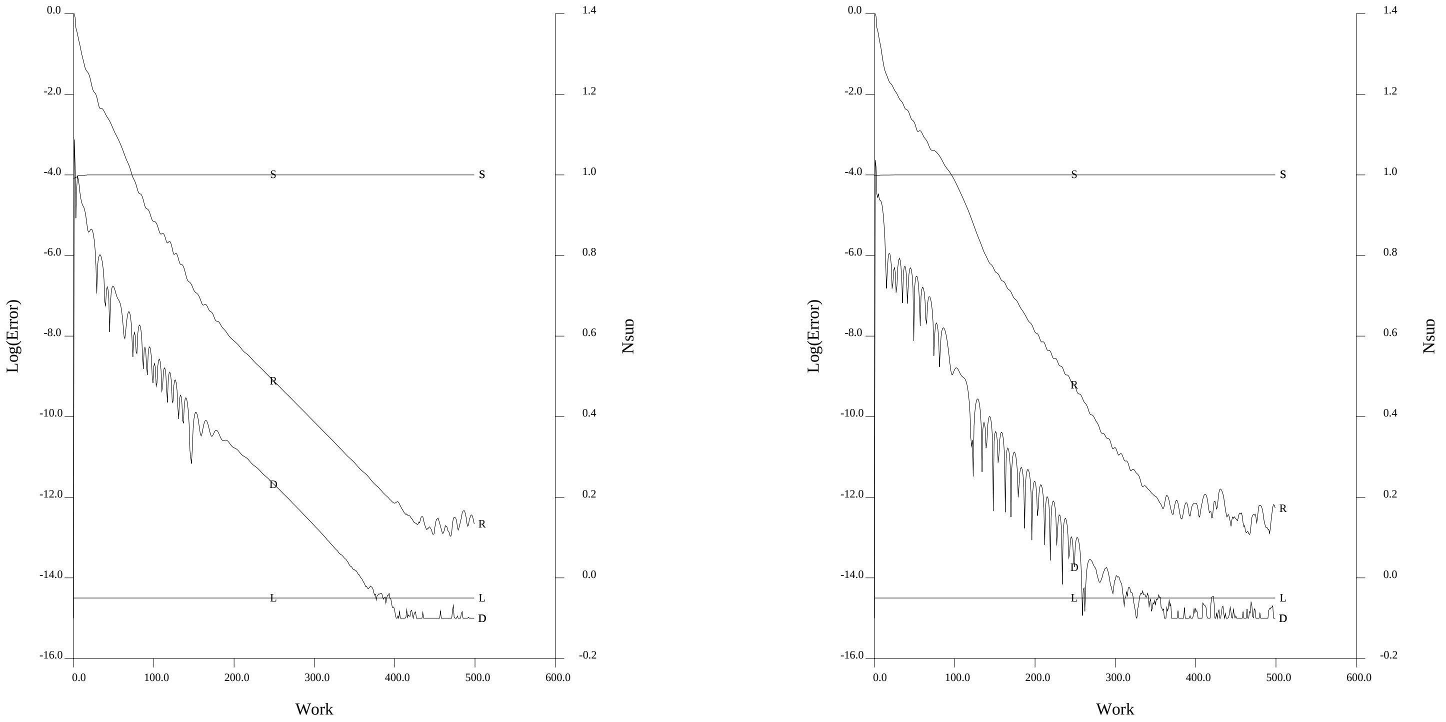
Close-up view SYN83 C-mesh about NACA0012-ADO.

MDOPT, CMA-ES & FLO82 GRID



Close-up view NACA0012-ADO 256x256 O-mesh.

FLO82 CONVERGENCE

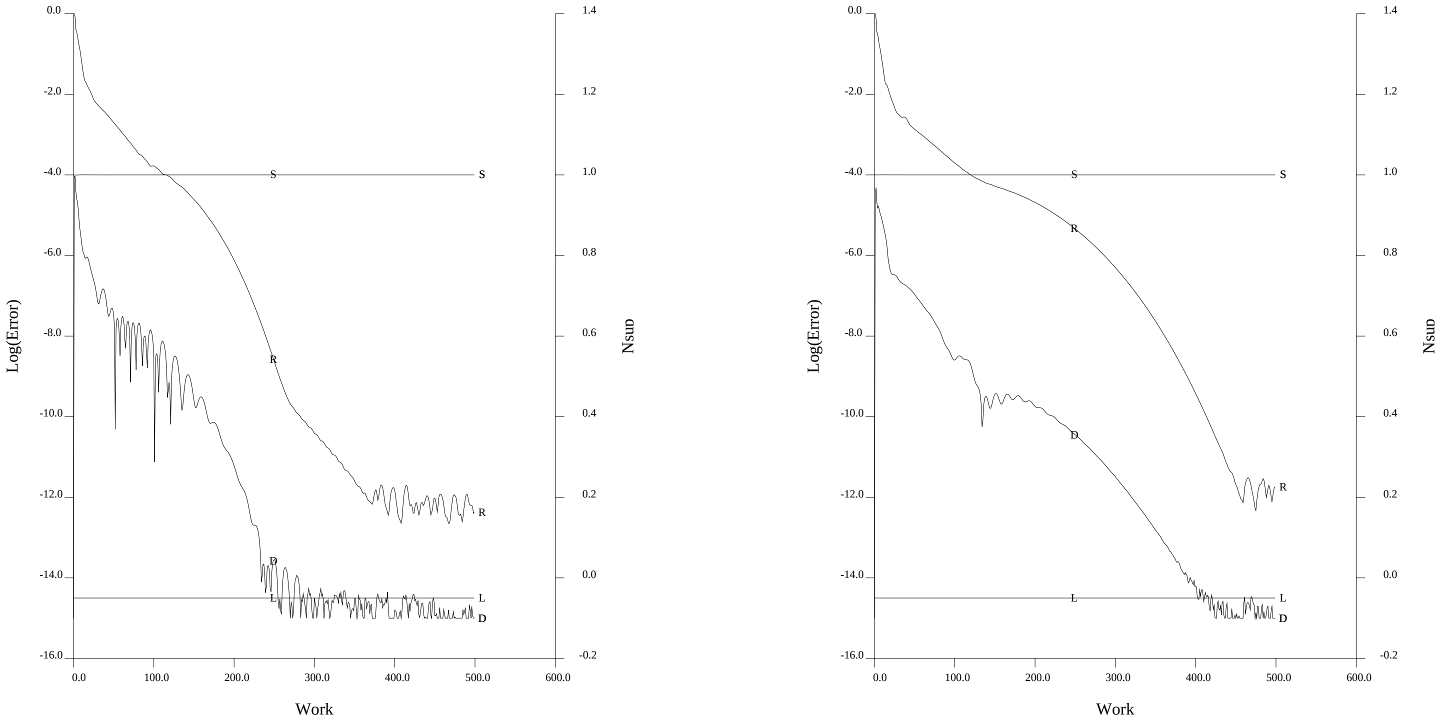


```
NACA0012-ADO-Tx1.01
MACH      0.850   ALPHA  0.0000000
CL  0.000000000   CD  0.048749723
RES1 0.102E+00   RES2 0.223E-13   RED   -12.66
WORK     499.00   RATE     0.9432
GRID    256X  256   NSUP     4302
```

```
NACA0012-ADO-Tx1.01
MACH      0.850   ALPHA  0.0000000
CL  0.000000000   CD  0.048819743
RES1 0.152E+00   RES2 0.823E-13   RED   -12.27
WORK     499.00   RATE     0.9450
GRID    512X  512   NSUP    17184
```

FLO82 Convergence Histories, $M = 0.85$, $\alpha = 0^\circ$.

FLO82 CONVERGENCE



```
NACA0012-ADO-Tx1.01
MACH      0.850    ALPHA  0.0000000
CL  0.000000000    CD  0.048848155
RES1 0.249E+00    RES2 0.106E-12    RED   -12.37
WORK     499.00    RATE      0.9445
GRID    1024X 1024    NSUP    68714
```

```
NACA0012-ADO-Tx1.01
MACH      0.850    ALPHA  0.0000000
CL  0.000000000    CD  0.048857545
RES1 0.417E+00    RES2 0.756E-12    RED   -11.74
WORK     499.00    RATE      0.9473
GRID    2048X 2048    NSUP    274830
```

FLO82 Convergence Histories, $M = 0.85$, $\alpha = 0^\circ$.

NACA0012-ADO OUTLINE

- **MODEL PROBLEM**
- **PREVIOUS 3-PHASE STUDY**
  - Vassberg, *et.al.*, 2011
- **CMA-ES RESULTS**
- **SYN83 RESULTS**
- **FLO82 CROSS ANALYSIS**
- **RELATED NACA0012-ADO STUDIES**
  - Bisson & Nadarajah, 2014
  - Carrier, *et.al.*, 2014

NACA0012-ADO MODEL PROBLEM

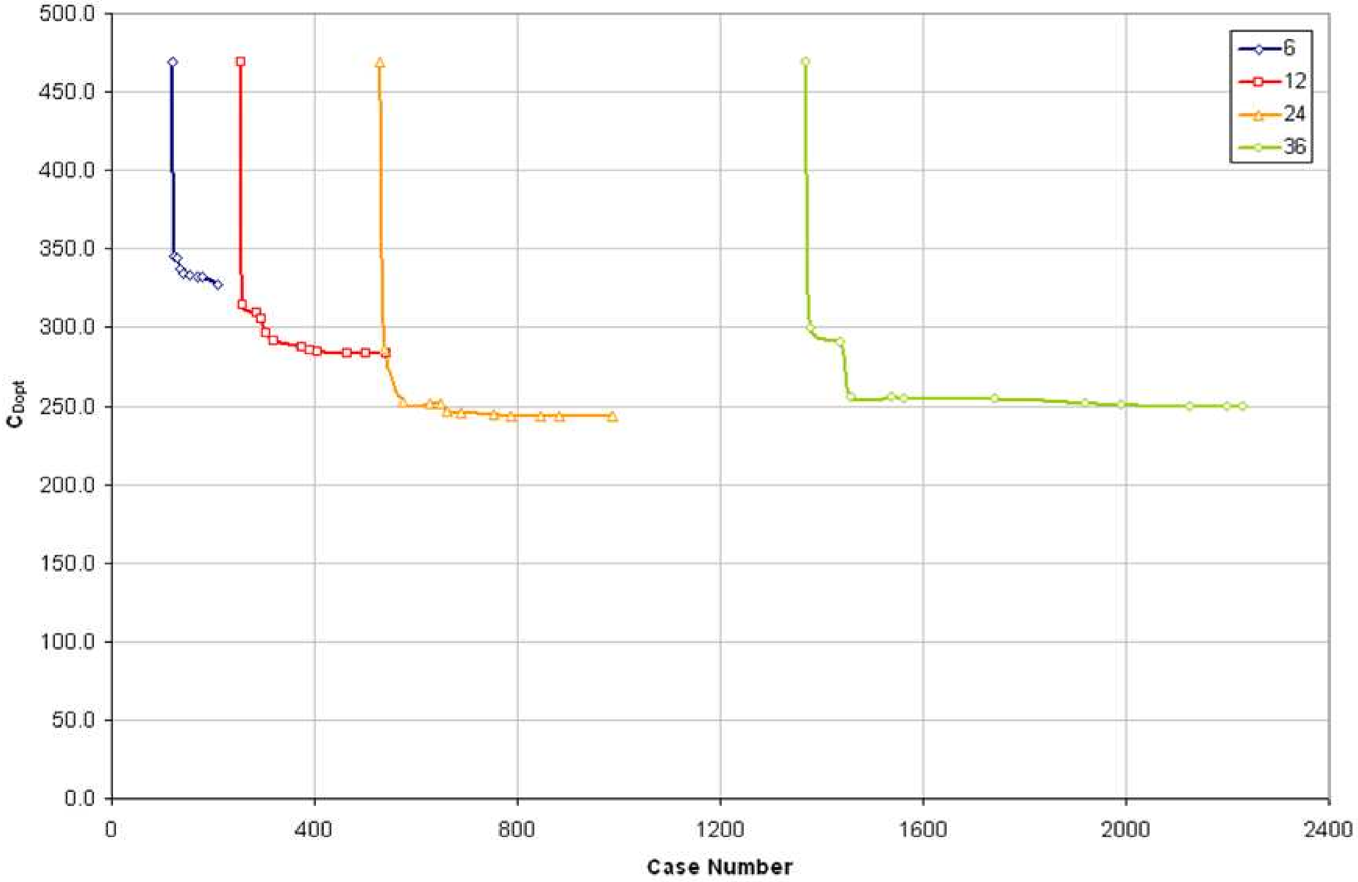
The objective of this optimization is to mimimize the drag of a symmetric airfoil, for an inviscid transonic flow at the condition of $M = 0.85$, and $\alpha = 0^\circ$, subject to the geometric constraint:

$$y_{Optimum}(x) \geq y_{Baseline}(x) \ \ ; \ \ 0 \leq x \leq 1.$$

Note that the flow physics of this model problem is such that the only true source of drag is that associated with any shocks that may arise.

Vassberg crafted this with anticipation that a shock-free design at these flow conditions and under these geometric constraints is unachievable.

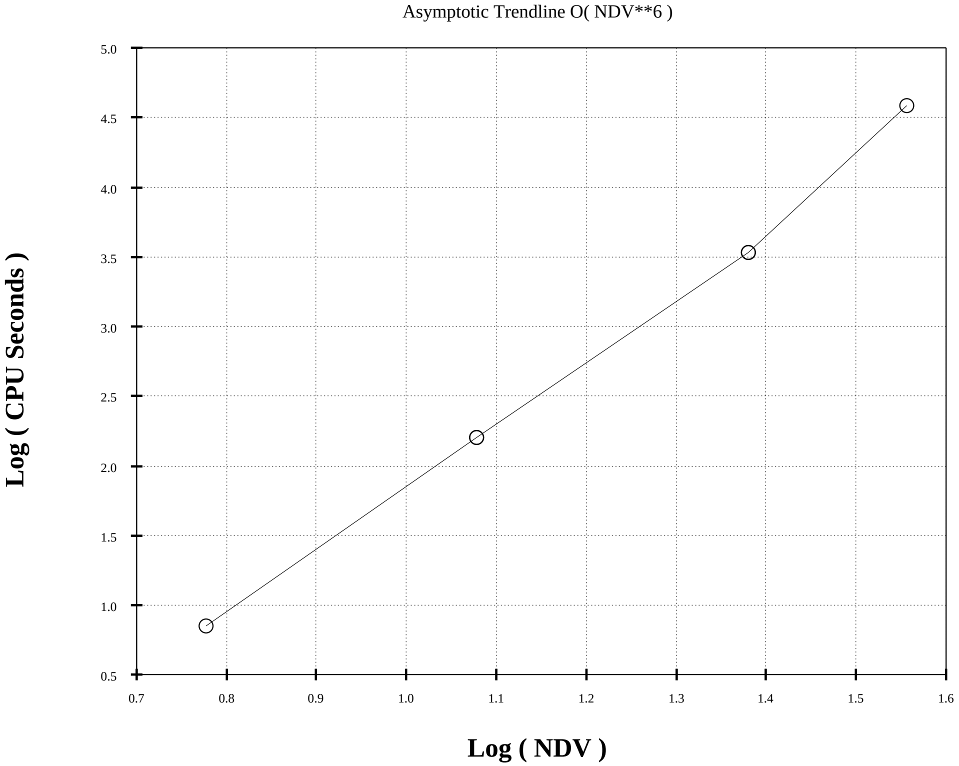
MDOPT RESULTS - PHASE-I



MDOPT Drag Histories, Phase-I.

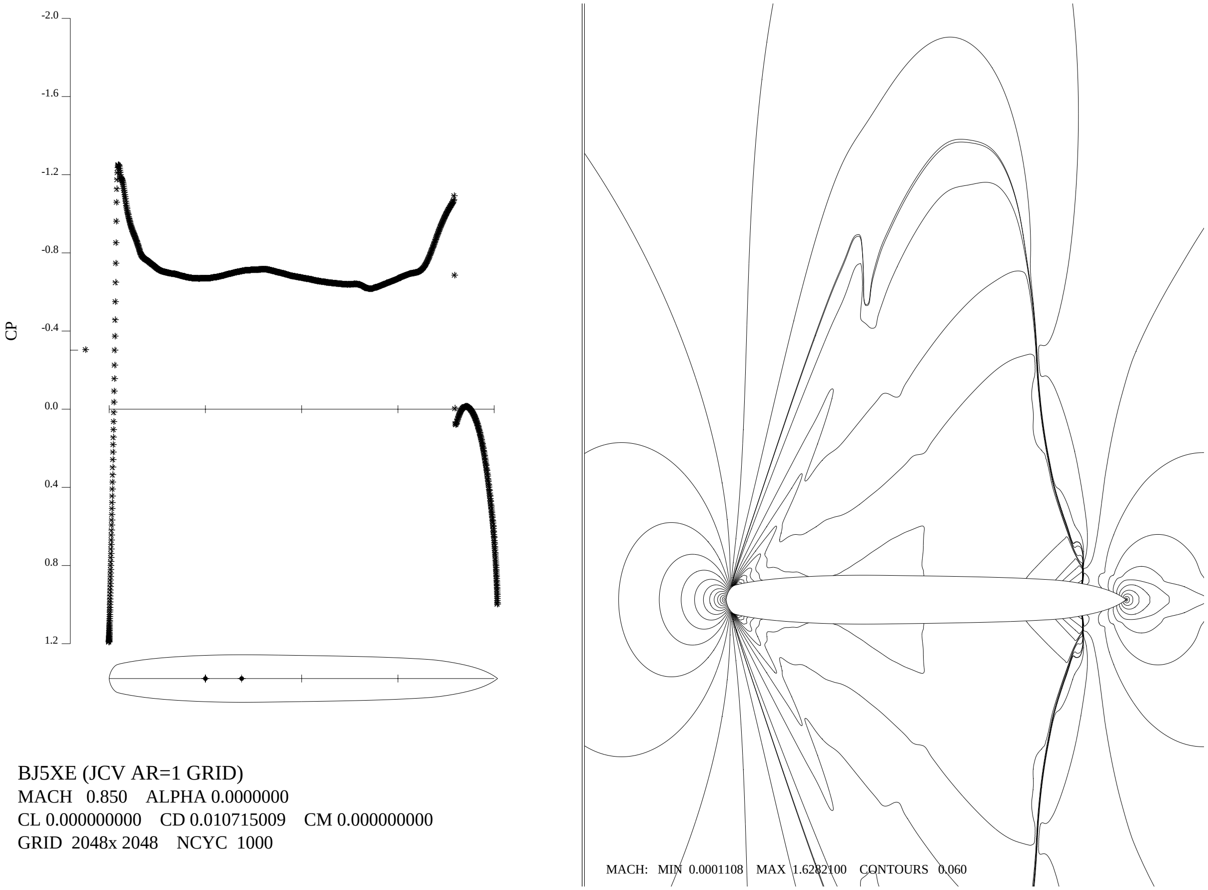
Table II: DOE Response-Surface Build Times.

NDV	$NCoef$	No. DOE Cases	CPU (sec)
6	21	121	7
12	78	256	157
24	300	529	3,341
36	666	1,369	38,042



MDOPT Response-Surface Build-Time Trendline.

SYN83 RESULTS - PHASE-II

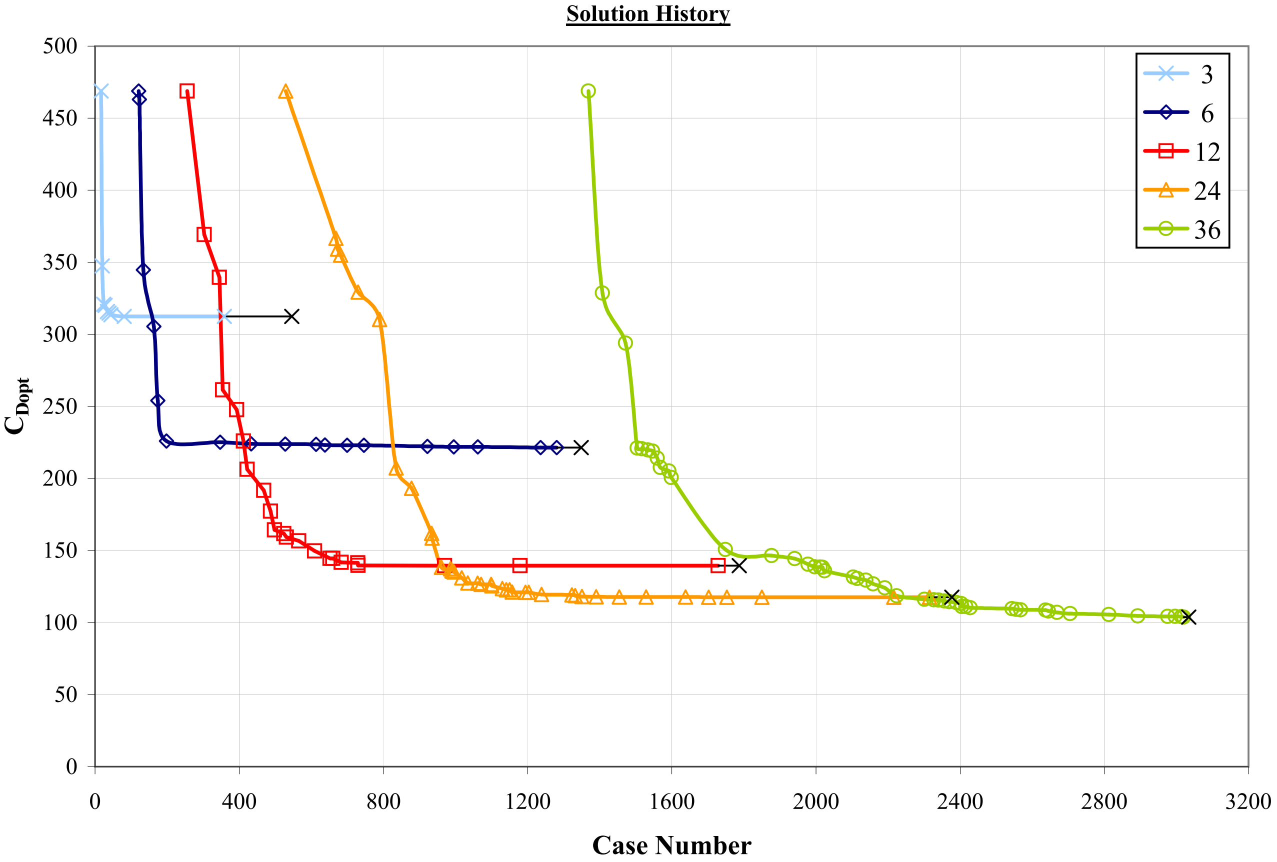


```
BJ5XE (JCV AR=1 GRID)
MACH   0.850    ALPHA 0.0000000
CL 0.000000000    CD 0.010715009    CM 0.000000000
GRID  2048x 2048    NCYC  1000
```

```
MACH:   MIN  0.0001108    MAX  1.6282100    CONTOURS   0.060
```

FLO82 Solution, BJ5XE Airfoil, $M = 0.85$, $\alpha = 0^\circ$.

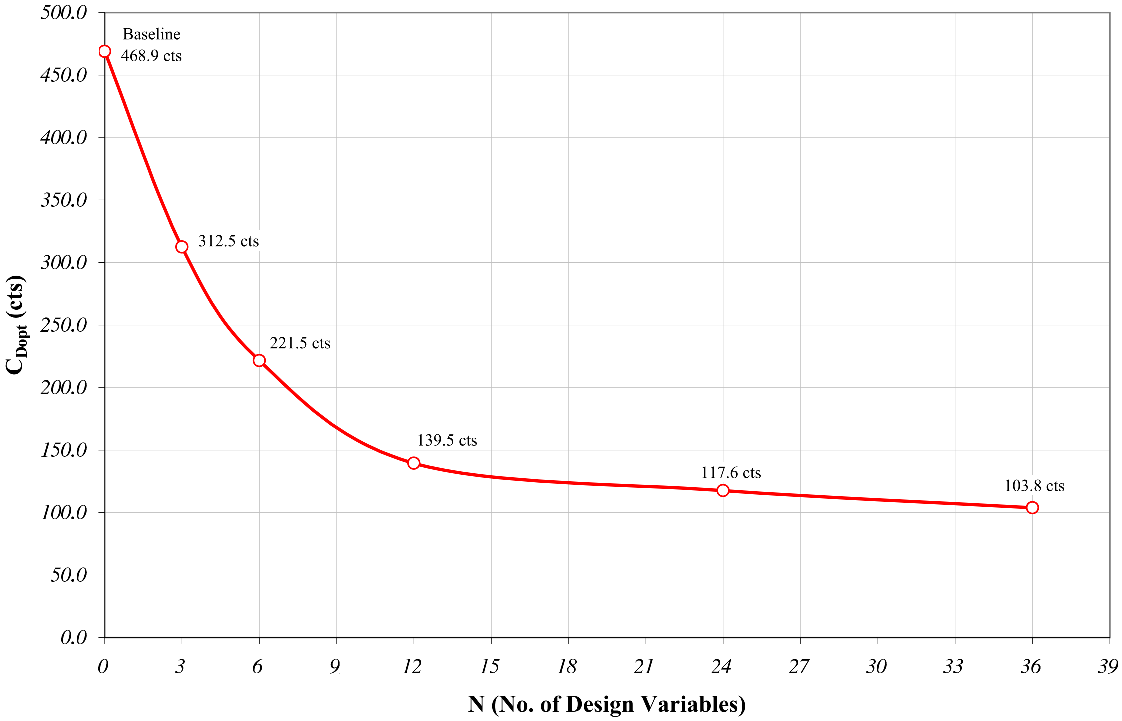
MDOPT RESULTS - PHASE-III



MDOPT Drag Reduction Histories, Phase-III.

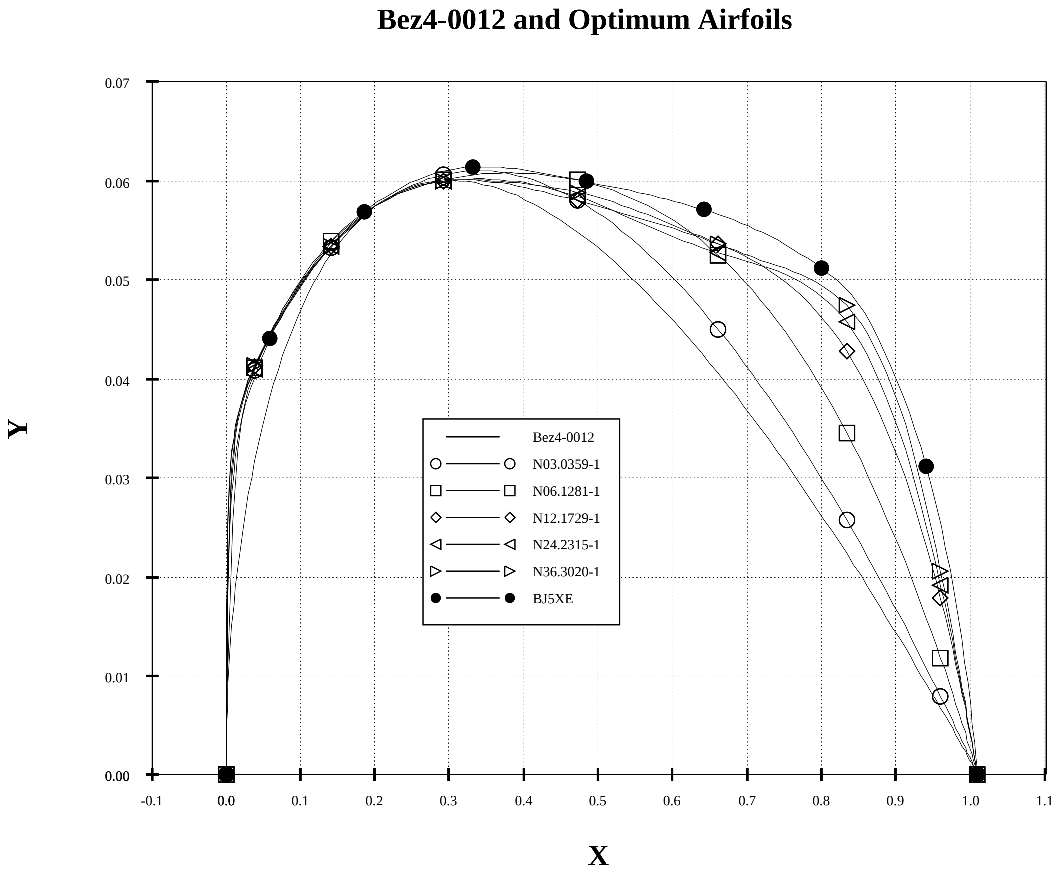
Table III: Phase-III MDOPT Results.

NDV	No. Cases	Optimum Case	C_{Dopt}
0	1	-	468.9
3	546	359-1	312.5
6	1,349	1281-1	221.5
12	1,787	1729-1	139.5
24	2,377	2315-1	117.6
36	3,034	3020-1	103.8



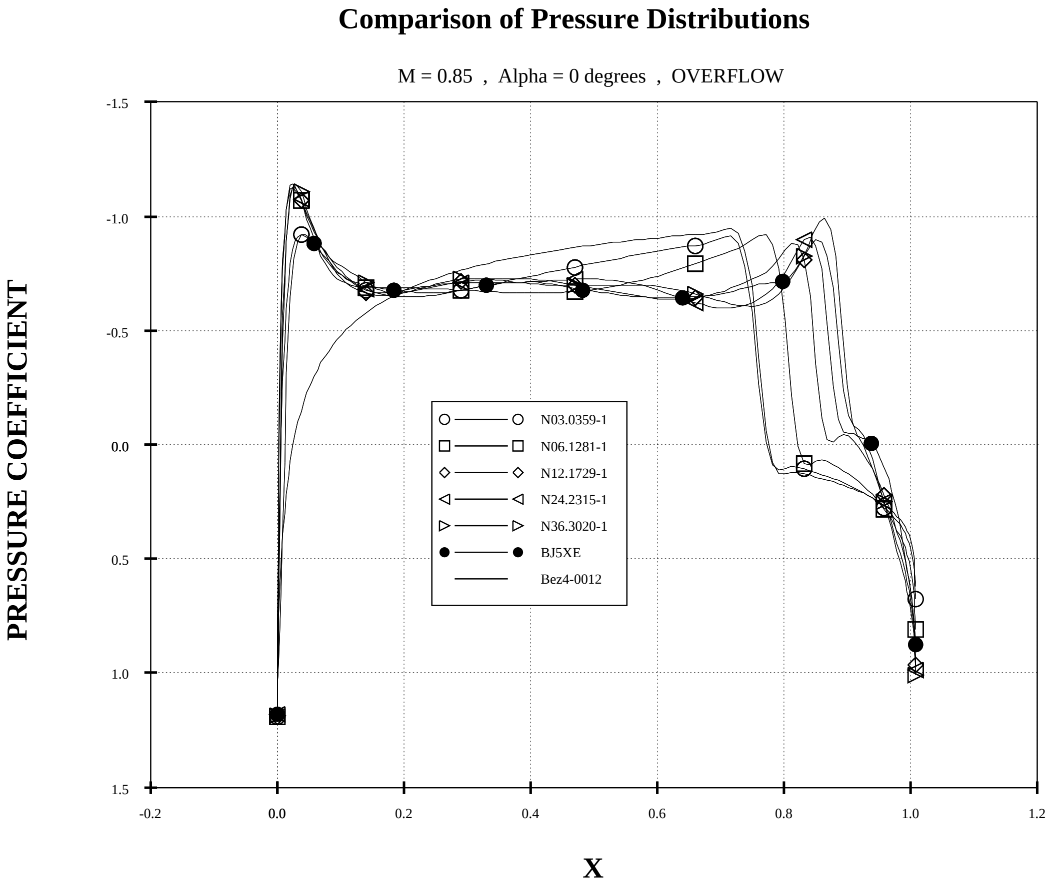
Optimum Realized Drags, $NDV = [0, 3, 6, 12, 24, 36]$, Phase-III.

MDOPT RESULTS - PHASE-III



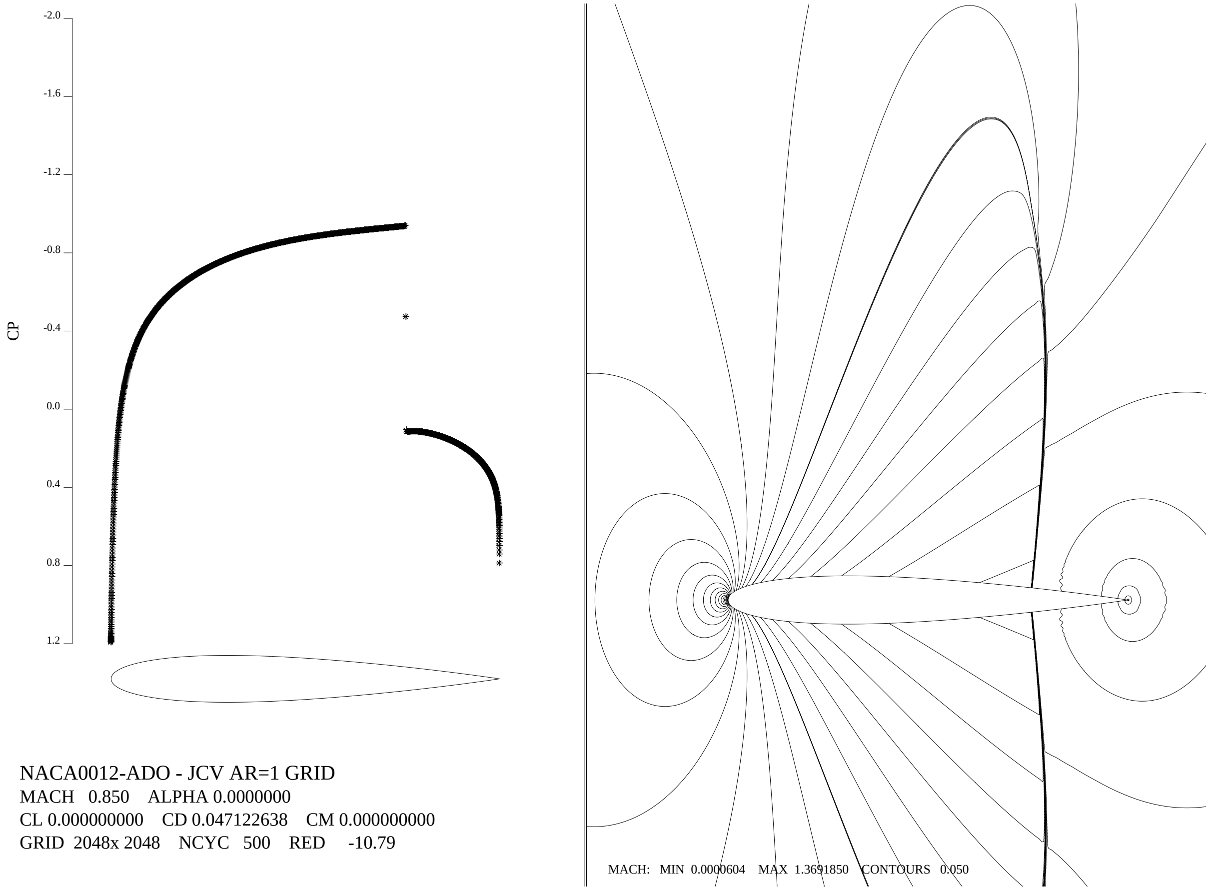
Bez4-0012, MDOPT Optimums & BJ5XE Airfoils.

MDOPT RESULTS - PHASE-III



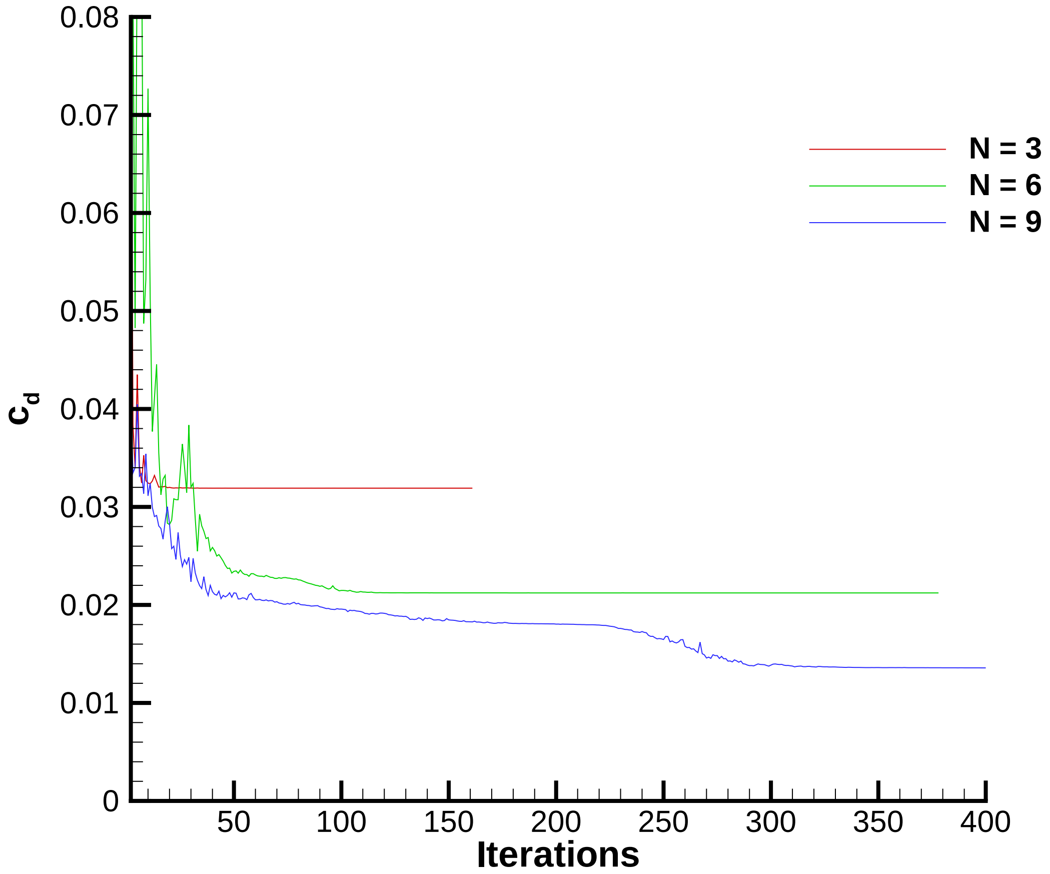
Bez4-0012, MDOPT Optimums & BJ5XE Pressures.

NACA0012-ADO BASELINE



FLO82 Solution, NACA0012-ADO Airfoil, $M = 0.85$, $\alpha = 0^\circ$.

CMA-ES RESULTS



CMA-ES Convergence Histories, [3,6,9]-Spaces.

Table IV: CMA-ES/OVERFLOW Results (C_d in $counts$).

N	Iterations	Population	Total Runs	C_{Dopt}	ΔC_d
0	-	-	1	483.70	-
3	161	7	1,127	319.17	-164.53
6	378	9	3,600	212.27	-271.43
9	400	10	4,000	135.74	-347.96

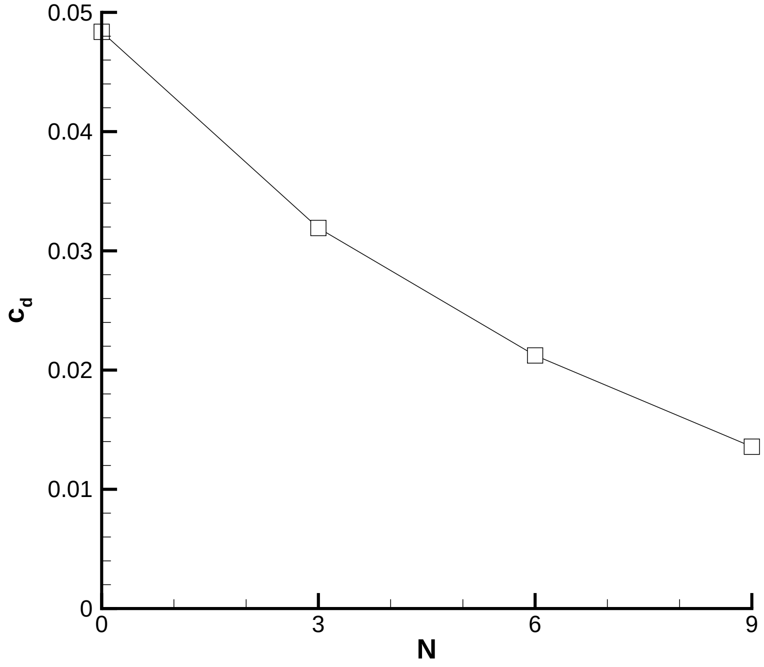
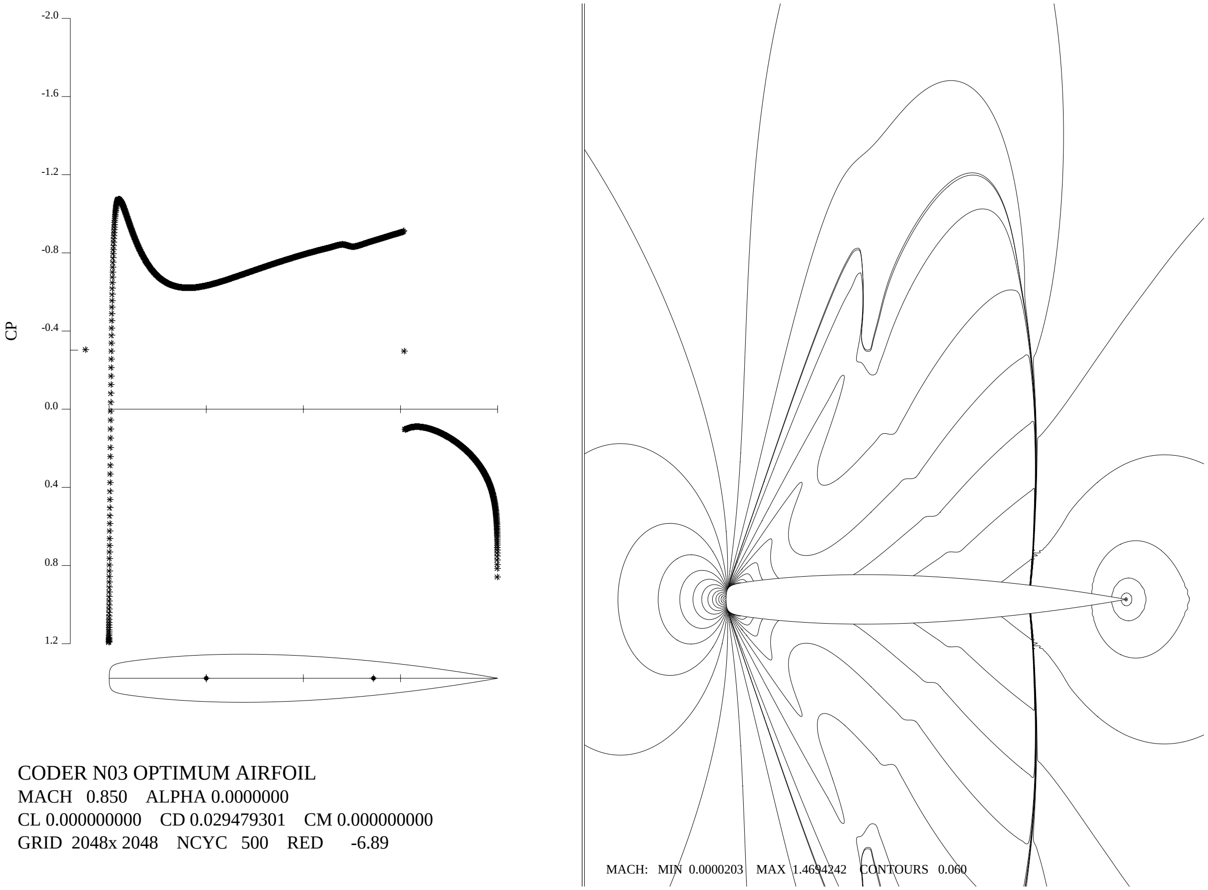


Table V:
CMA-ES Optimum Design
Perturbation Vectors in Bezier Space

N	$ycpt_{147}$	$ycpt_{258}$	$ycpt_{369}$
3	0.054596	-0.050248	0.026009
6	0.000000	0.111226	-0.168611
	0.182871	-0.152965	0.085558
9	0.000000	0.094056	-0.084422
	0.049105	0.085296	-0.237675
	0.295685	-0.239163	0.123469

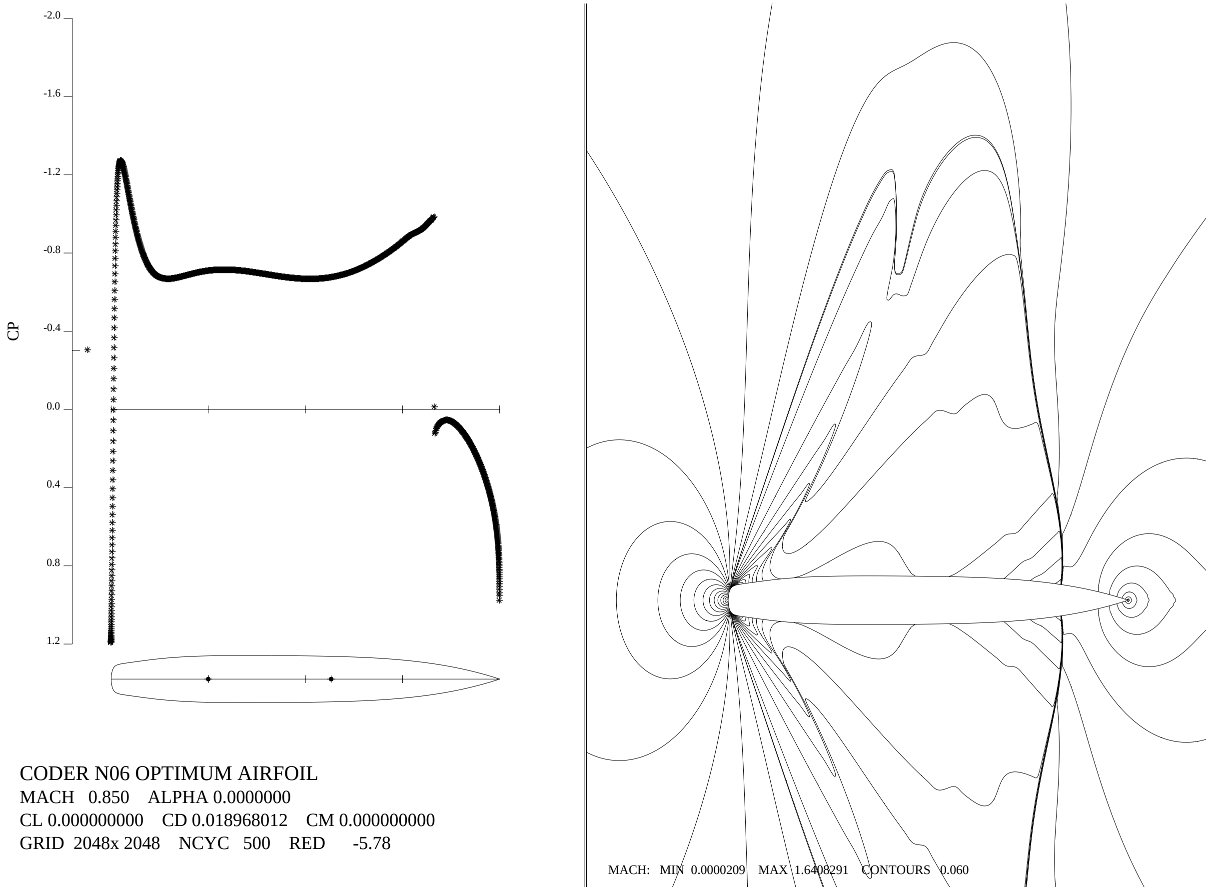
Optimum Drag Levels, Bezier Design-Space Family.

CMA-ES RESULTS



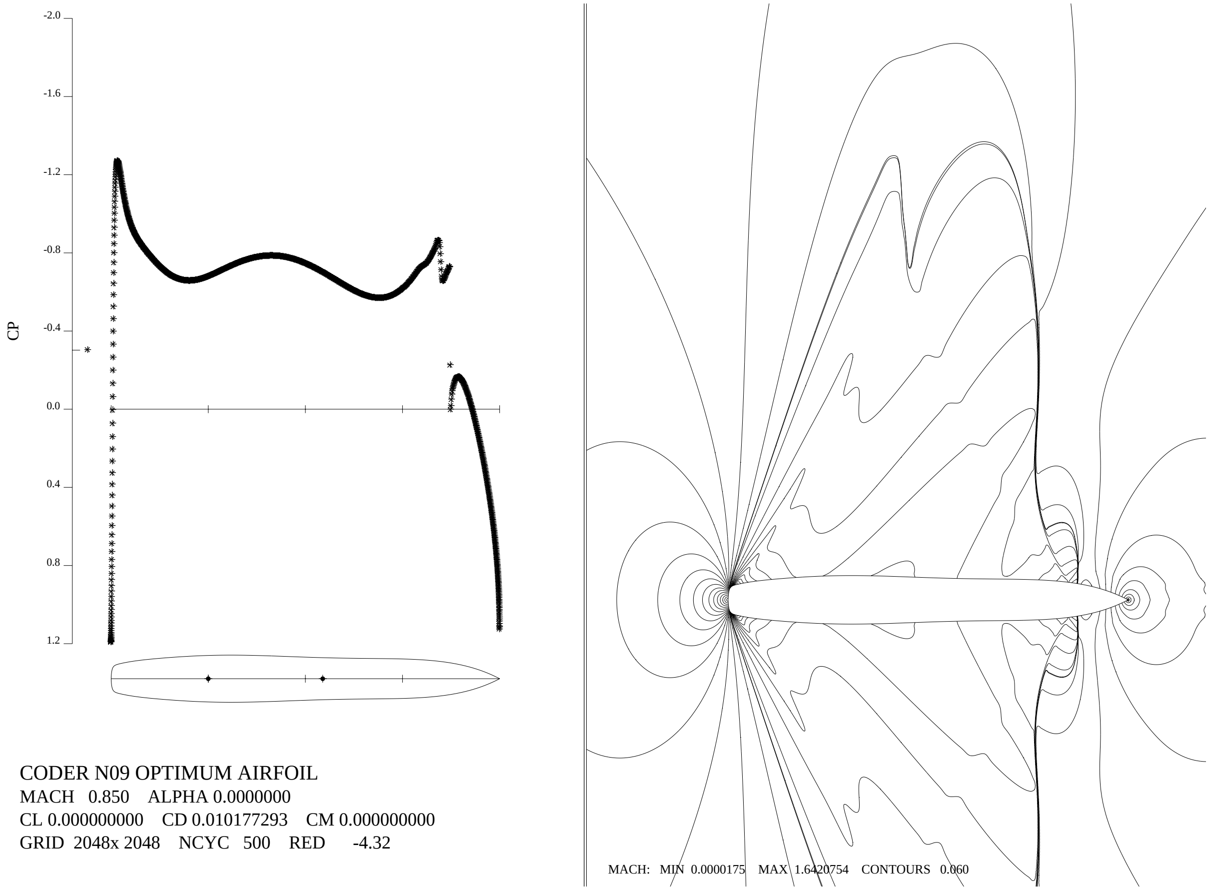
FLO82 Solution, CMAES-N03 Airfoil, $M = 0.85$, $\alpha = 0^\circ$.

CMA-ES RESULTS



FLO82 Solution, CMAES-N06 Airfoil, $M = 0.85$, $\alpha = 0^{\circ}$.

CMA-ES RESULTS



CODER N09 OPTIMUM AIRFOIL
MACH 0.850 ALPHA 0.0000000
CL 0.000000000 CD 0.010177293 CM 0.000000000
GRID 2048x 2048 NCYC 500 RED -4.32

MACH: MIN 0.0000175 MAX 1.6420754 CONTOURS 0.060

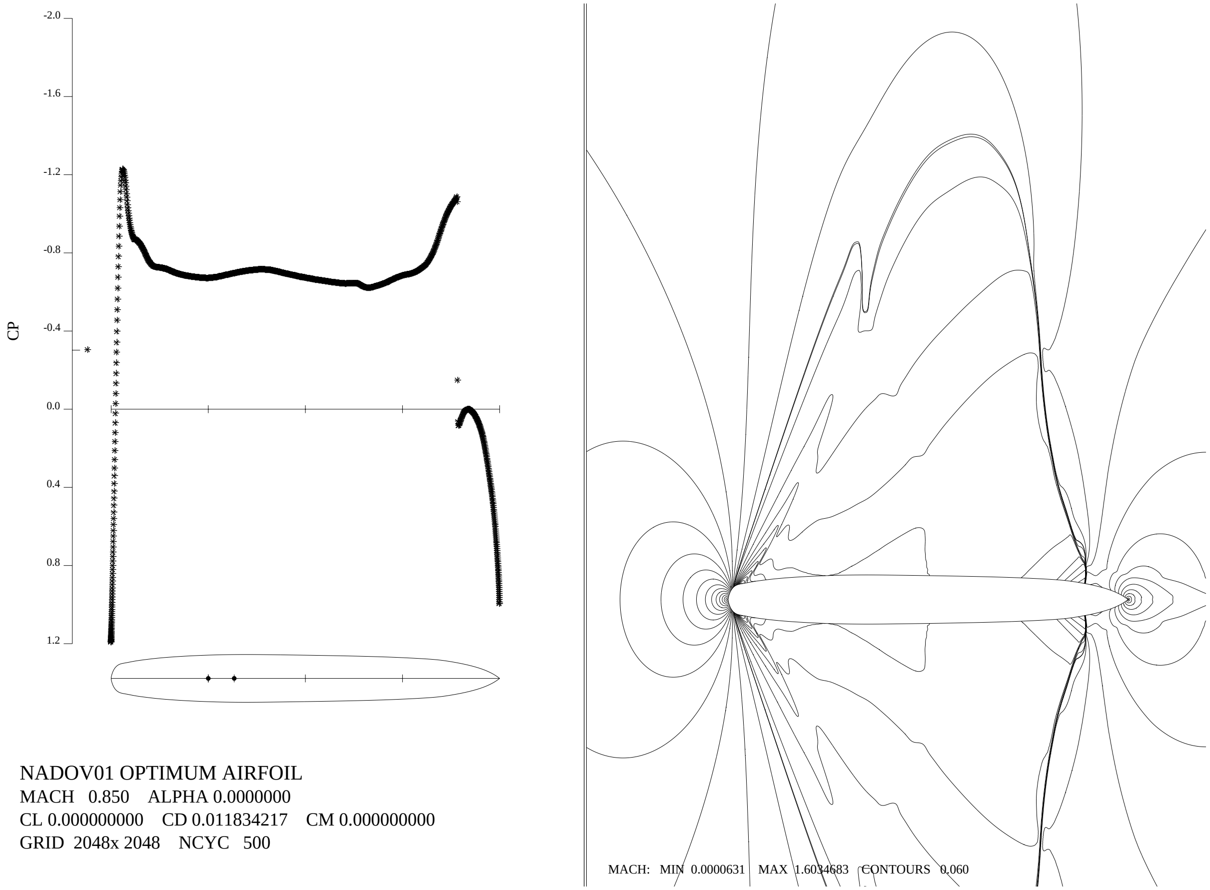
FLO82 Solution, CMAES-N09 Airfoil, $M = 0.85$, $\alpha = 0^\circ$.

SYN83 RESULTS

Table VI:
SYN83 Results (C_d in $counts$).

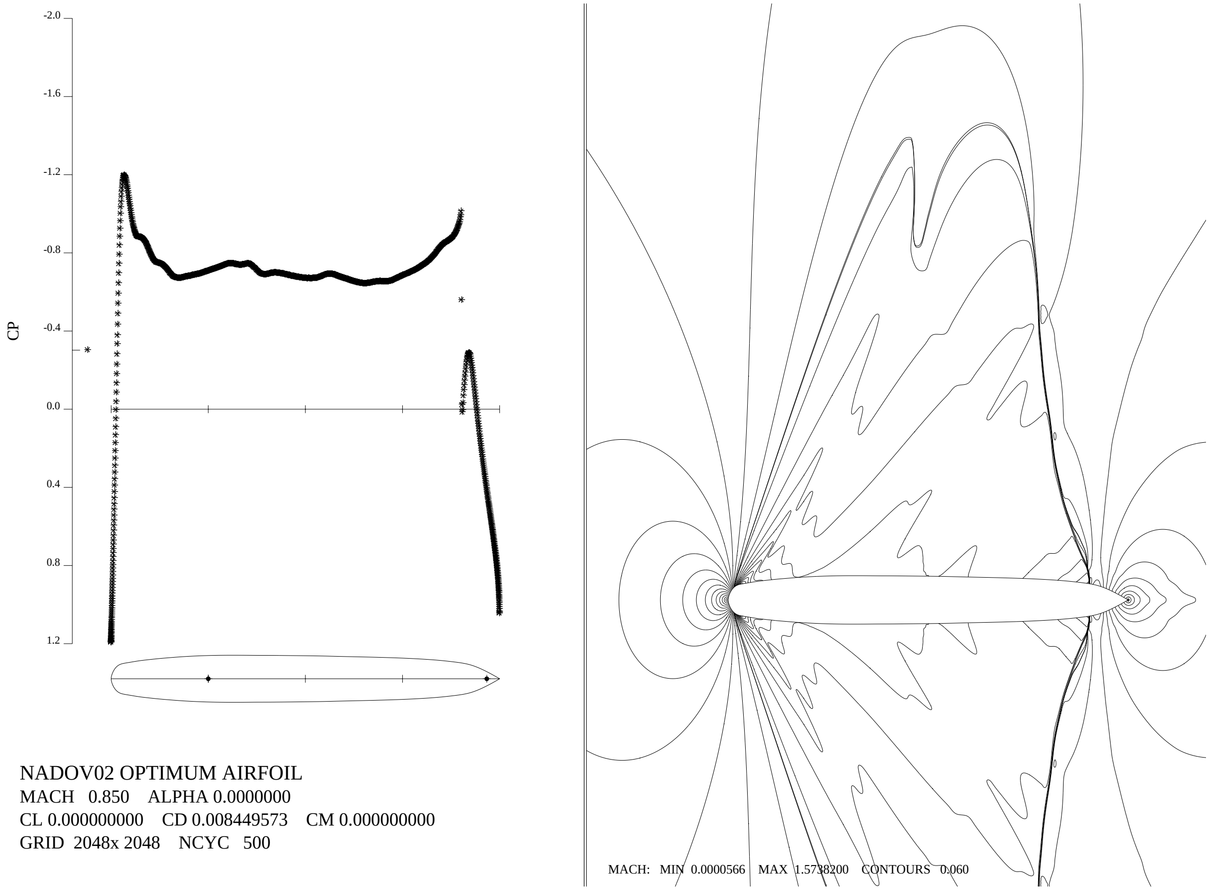
Airfoil		C_d	ΔC_d
Seed	NACA0012-ADO	456.34	-
Design	SYN-NADOV01	103.71	-352.63
Seed	SEED02	101.79	-354.55
Design	SYN-NADOV02	79.31	-377.03

SYN83 RESULTS



FLO82 Solution, SYN83-NADOV01 Airfoil, $M = 0.85$, $\alpha = 0^\circ$.

SYN83 RESULTS



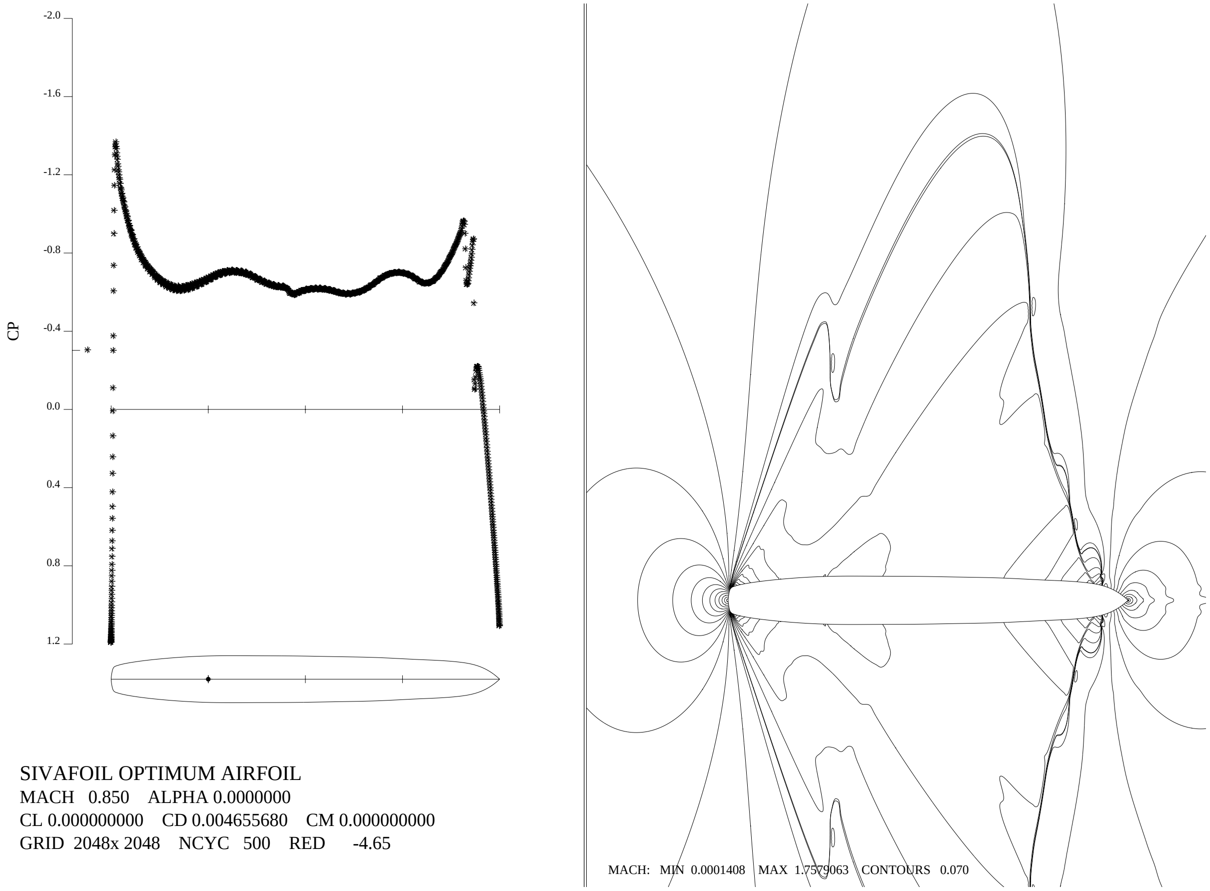
FLO82 Solution, SYN83-NADOV02 Airfoil, $M = 0.85$, $\alpha = 0^\circ$.

FLO82 CROSS ANALYSIS

Table VII:
FLO82 Drag Assessment (C_d in $counts$).

Airfoil	N256	N512	N1024	N2048	Continuum	ΔC_d
NACA0012-ADO	470.19	470.09	471.13	471.23	471.27	-
NADOT101	487.50	488.20	488.48	488.58	488.62	+17.35
CMAES03	296.90	294.30	294.66	294.79	294.88	-176.39
CMAES06	194.24	189.00	189.48	189.68	189.82	-281.45
CMAES09	116.85	103.95	101.78	101.77	101.77	-369.50
SYN-NADOV01	153.78	123.24	119.03	118.34	118.21	-353.06
SYN-NADOV02	109.25	86.87	84.68	84.50	84.48	-386.79
SYNT101	122.22	99.20	96.75	96.64	96.63	-374.64
SIVAFOIL	118.60	72.95	51.56	46.56	45.03	-426.24

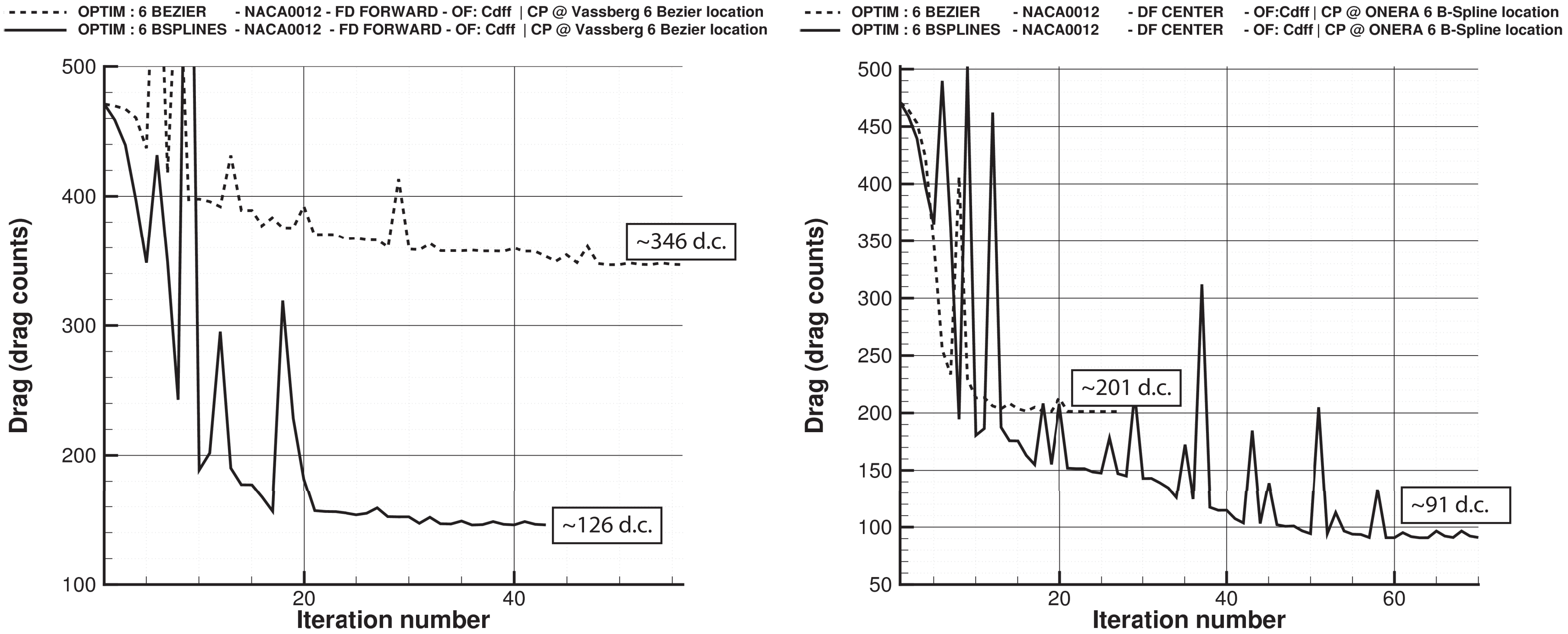
BISSON-NADARAJAH RESULTS



SIVAFOIL OPTIMUM AIRFOIL
MACH 0.850 ALPHA 0.0000000
CL 0.000000000 CD 0.004655680 CM 0.000000000
GRID 2048x 2048 NCYC 500 RED -4.65

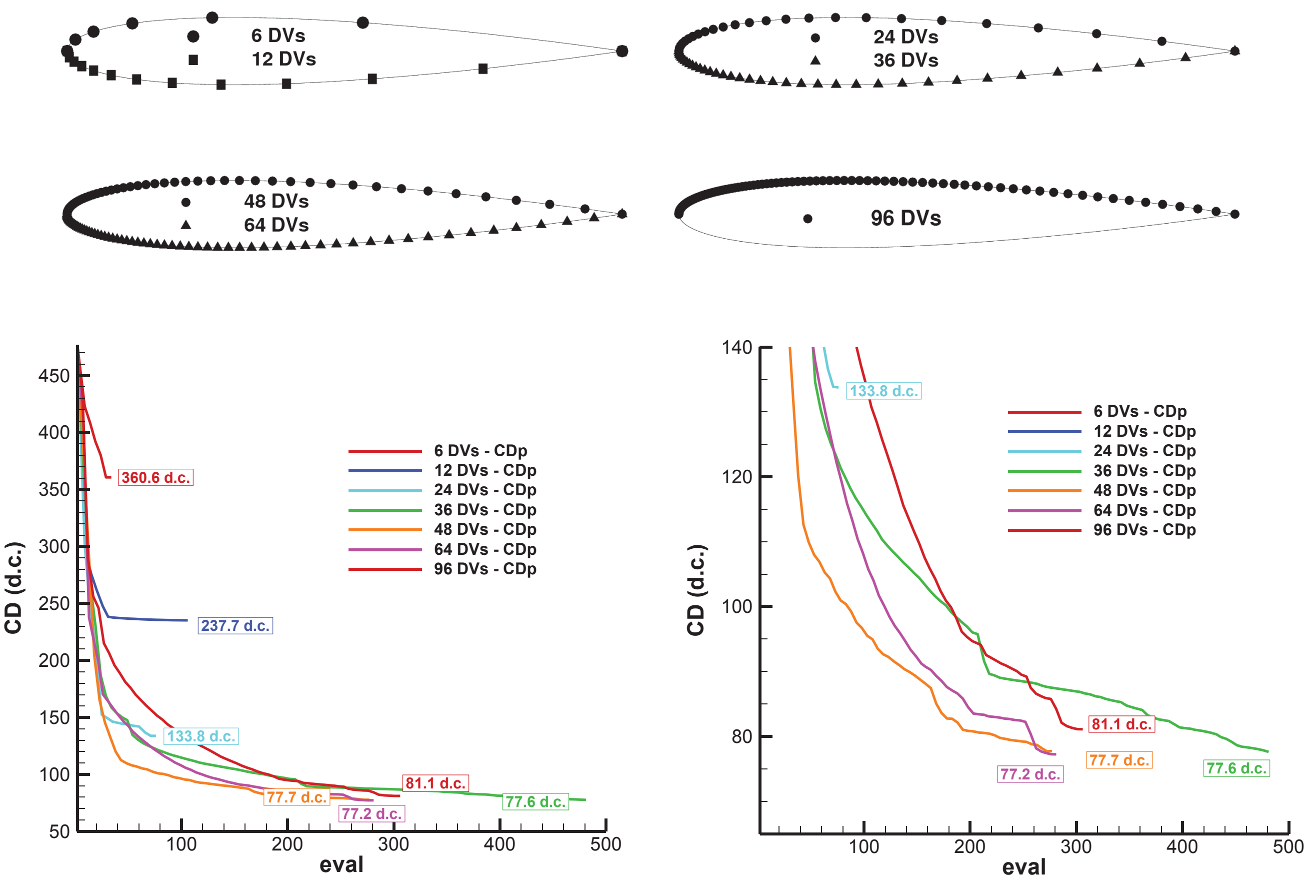
FLO82 Solution, Bisson-Nadarajah Airfoil, $M = 0.85$, $\alpha = 0°$.

CARRIER, ET.AL. RESULTS



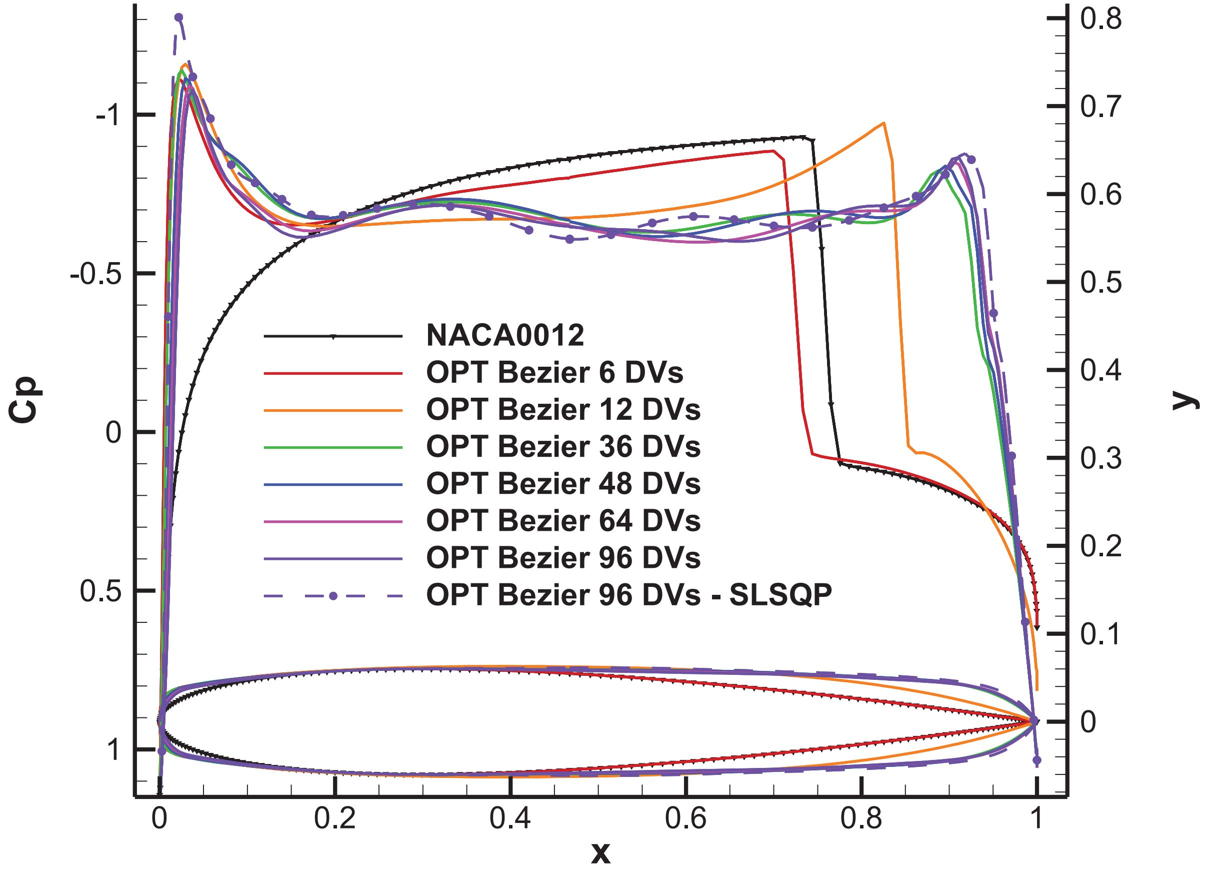
Carrier, *et.al.*: Bezier & B-Spline Comparisons.

CARRIER, ET.AL. RESULTS



Carrier, *et.al.*: Systematic Study, Drag-Convergence.

CARRIER, ET.AL. RESULTS



Carrier, *et.al.*: Geometry & Pressure Comparisons

NACA0012-ADO SUMMARY

- **CHALLENGING PROBLEM**
  - OPTIMUMS EXHIBIT SINGULAR BEHAVIOR
- **DIMENSION MATTERS**
  - DESIGNS IMPROVE (TO A POINT)
  - OPTIMIZATION COSTS CAN SCALE
  - OPTIMIZATIONS CAN HANG
- **LOCAL VS. GLOBAL CONTROL**
  - BETTER DESIGNS WITH LOCAL CONTROL
  - CODER VS. CARRIER
- **DISTRIBUTION OF DVs MATTER**

ONERA-M6 OUTLINE

- **PROBLEM STATEMENT**
  - THREE VARIATIONS
- **SYN107 RESULTS**
  - DRAG CONVERGENCE HISTORIES
  - COMPARISON OF PRESSURES
  - COMPARISON OF GEOMETRIES
- **DISCUSSIONS**
  - DRAG LOOPS
  - DESIGN SPACE
  - AREA RULING

ONERA-M6 PROBLEM STATEMENT

The second example case we present is based on the benchmark ONERA-M6 wing at flow conditions:

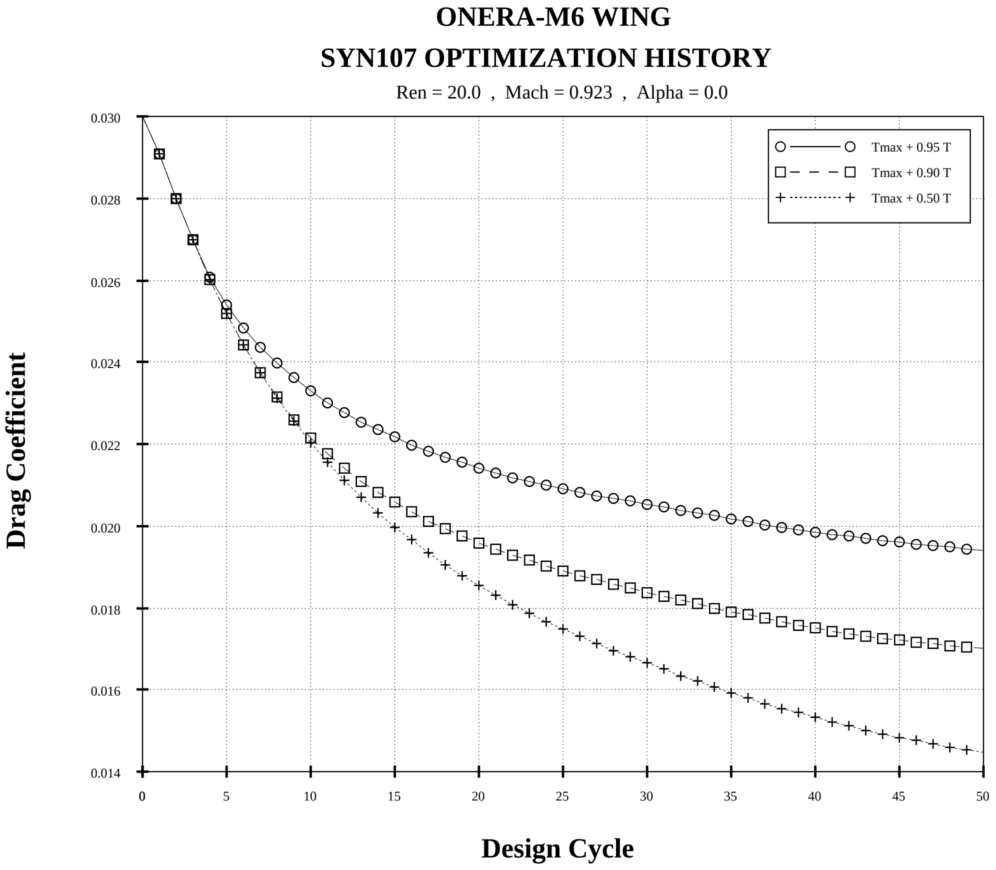
$$M = 0.923,\ \alpha = 0^\circ,\ Ren = 20x10^6.$$

We conduct 3 optimizations with varying thickness constraints. The objective is to minimize total drag, subject to the following geometric constraints on each airfoil section.

$$\begin{aligned} Tmax_{opt} &\geq Tmax_{M6}, \\ Tdist_{opt} &\geq [0.95,\ 0.90,\ 0.50] * Tdist_{M6}. \end{aligned}$$

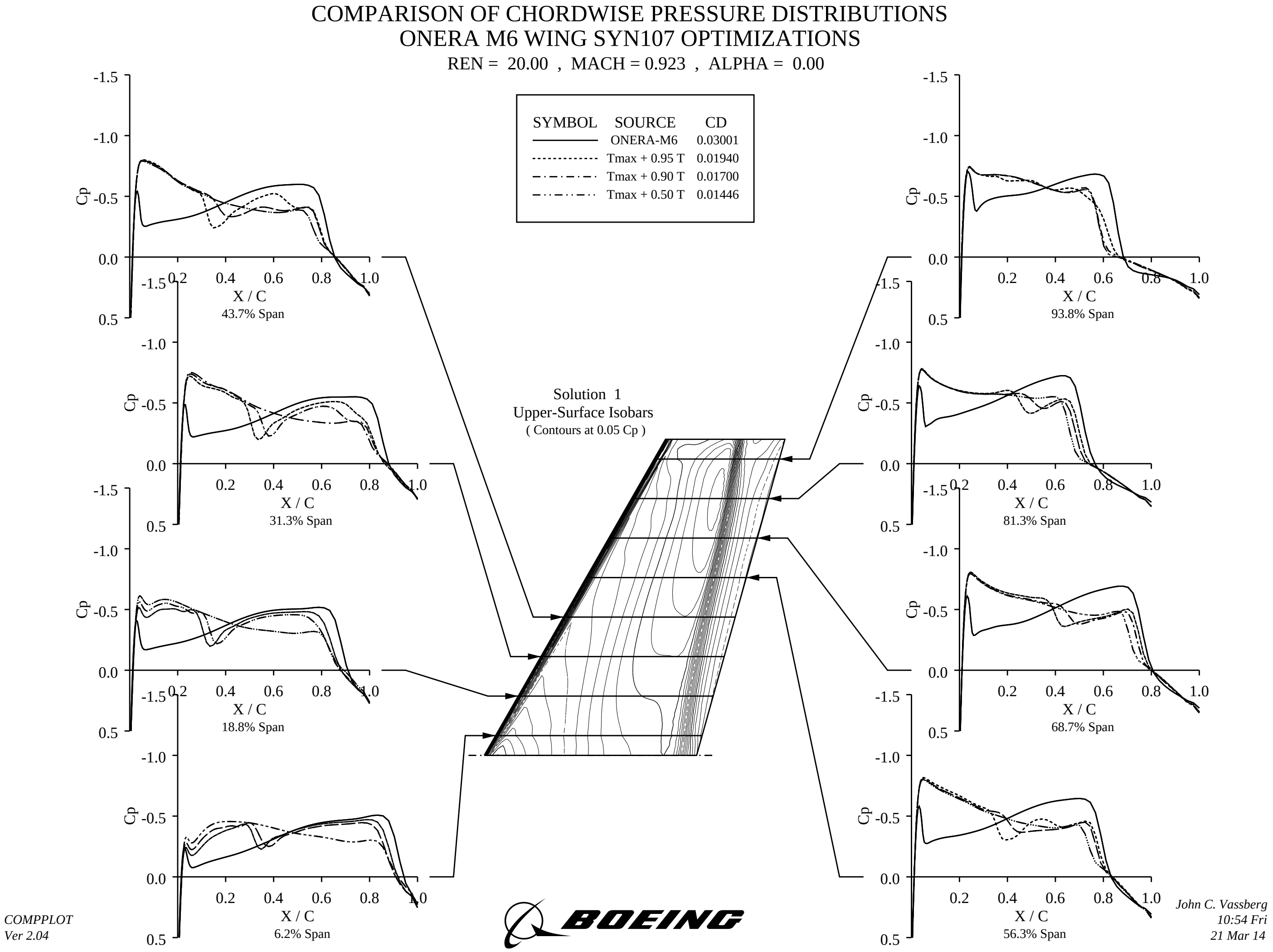
SYN107 is run with its B-Spline design space and is arbitrarily terminated after 50 design cycles.

ONERA-M6 CASE

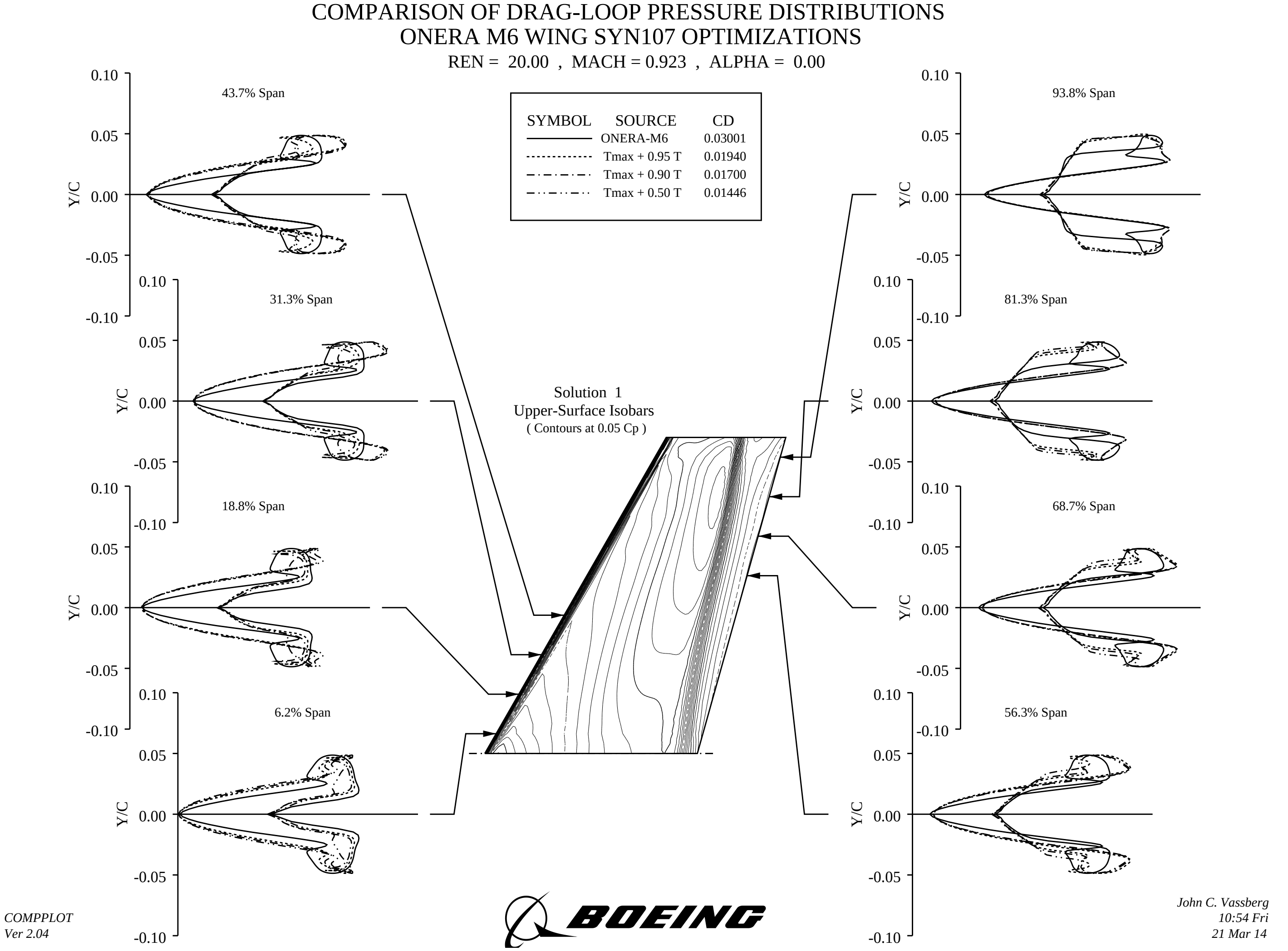


JC Vassberg
21 Mar 2014

SYN107 Design History of Drag, ONERA-M6 Wing.

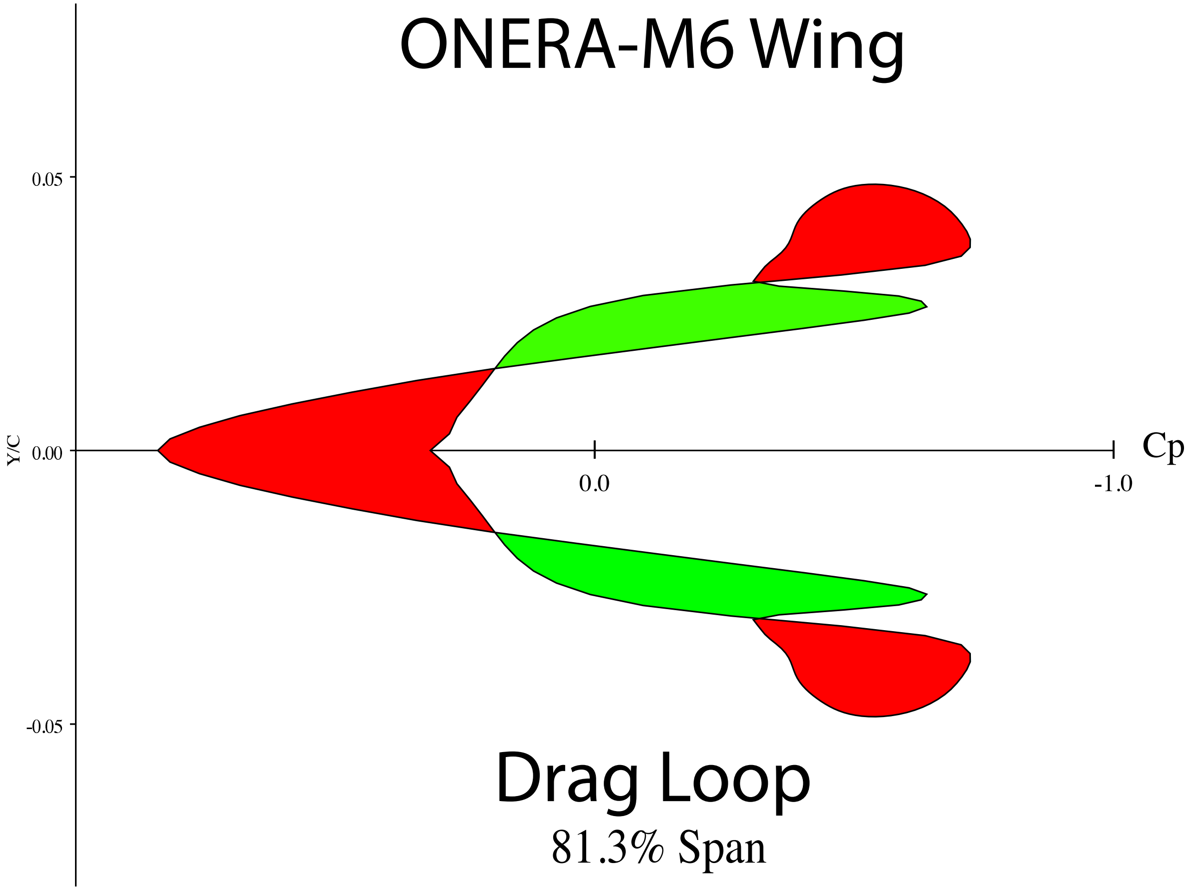


ONERA-M6 Baseline & Optimized Wings Surface Pressures.



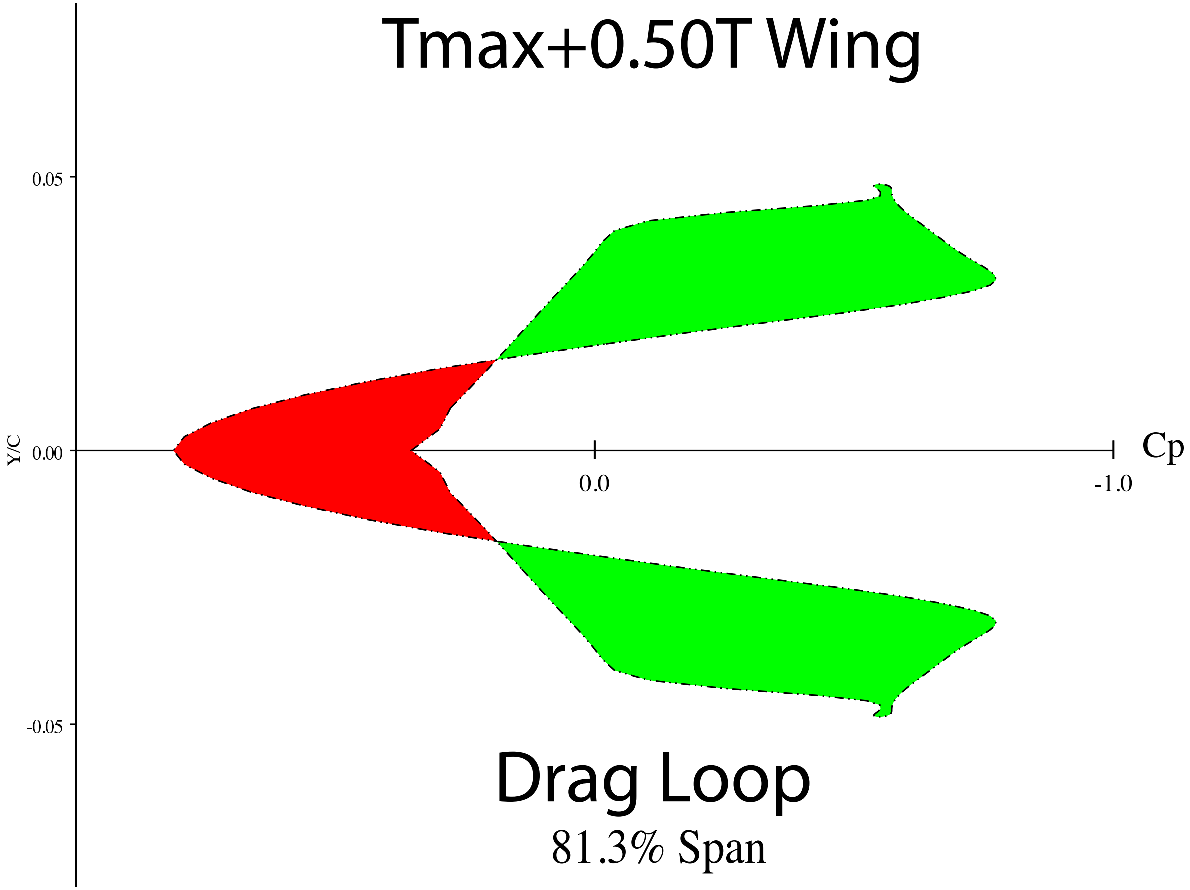
ONERA-M6 Baseline & Optimized Wings Drag Loops.

ONERA-M6 CASE



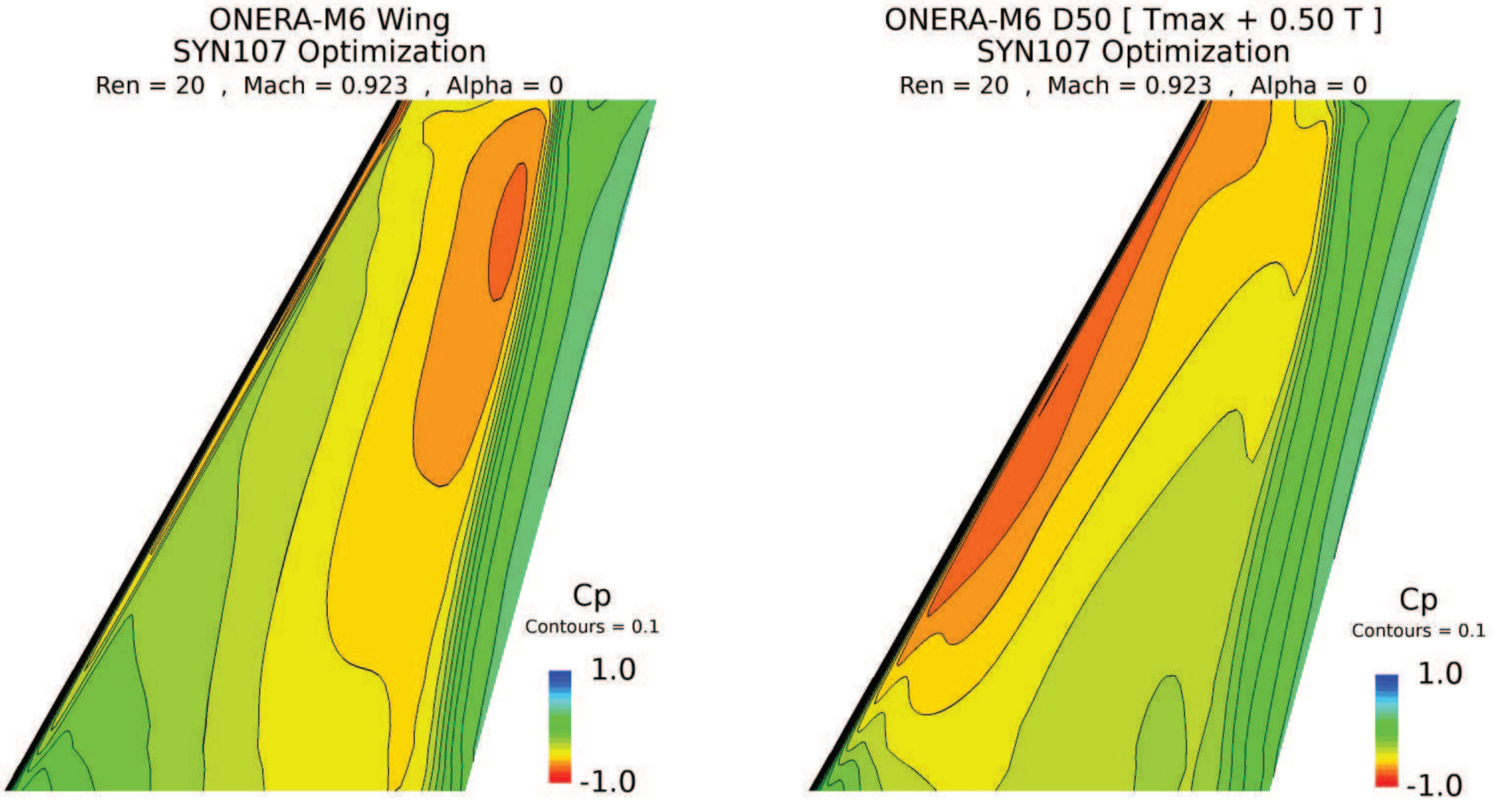
ONERA-M6 Baseline Sectional Drag Loop, $\eta = 0.813$.

ONERA-M6 CASE



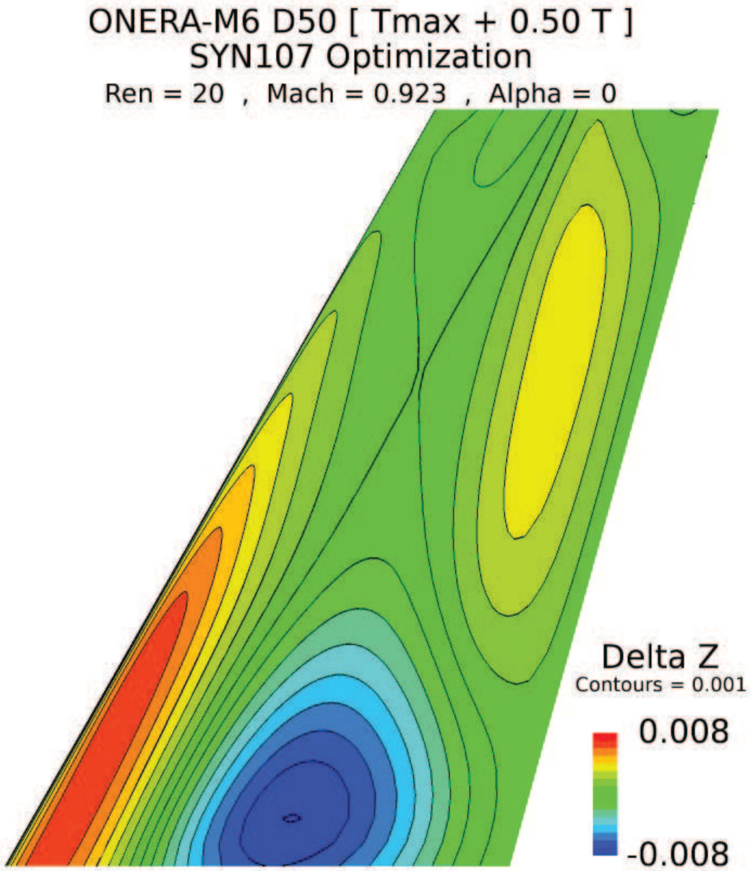
ONERA-M6 Optimum Sectional Drag Loop, $\eta = 0.813$.

ONERA-M6 CASE



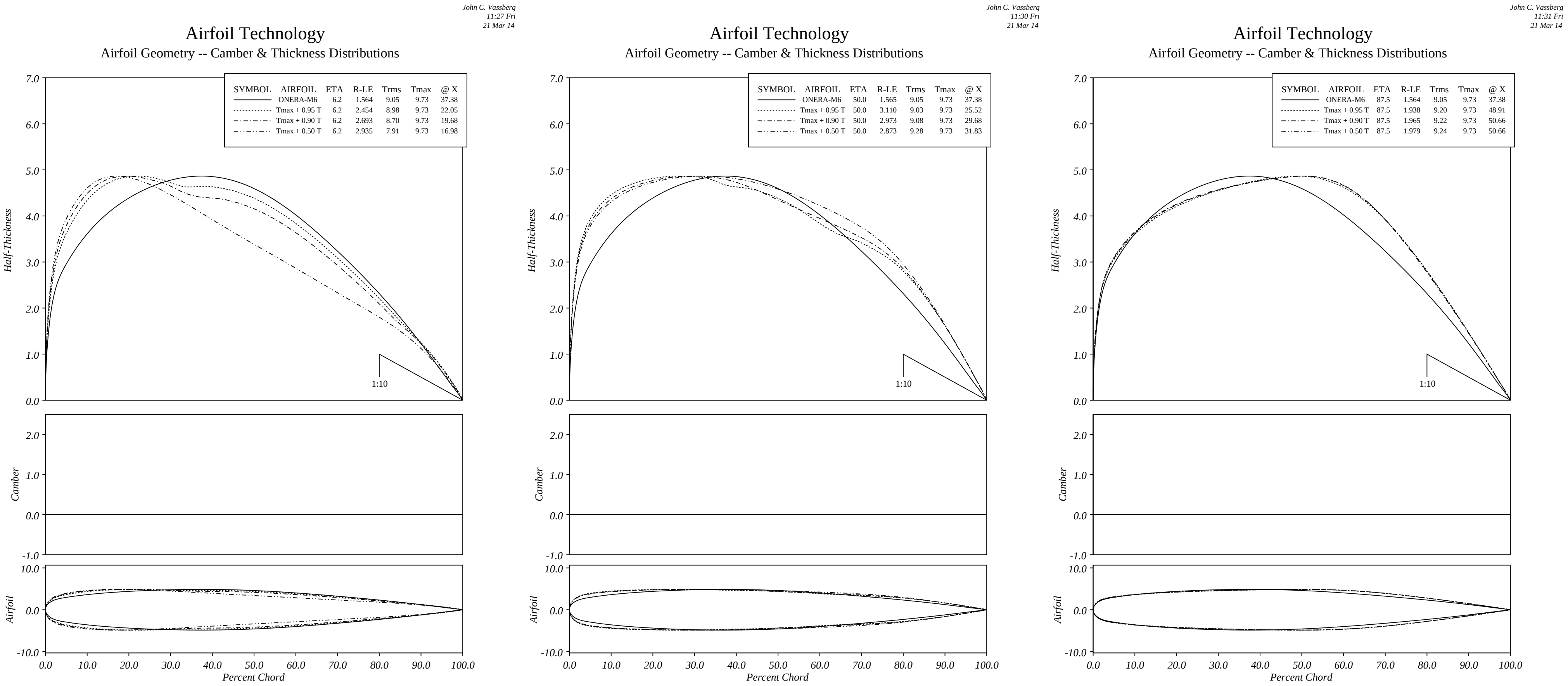
ONERA-M6 Baseline & Optimized Wings Isobars.

ONERA-M6 CASE



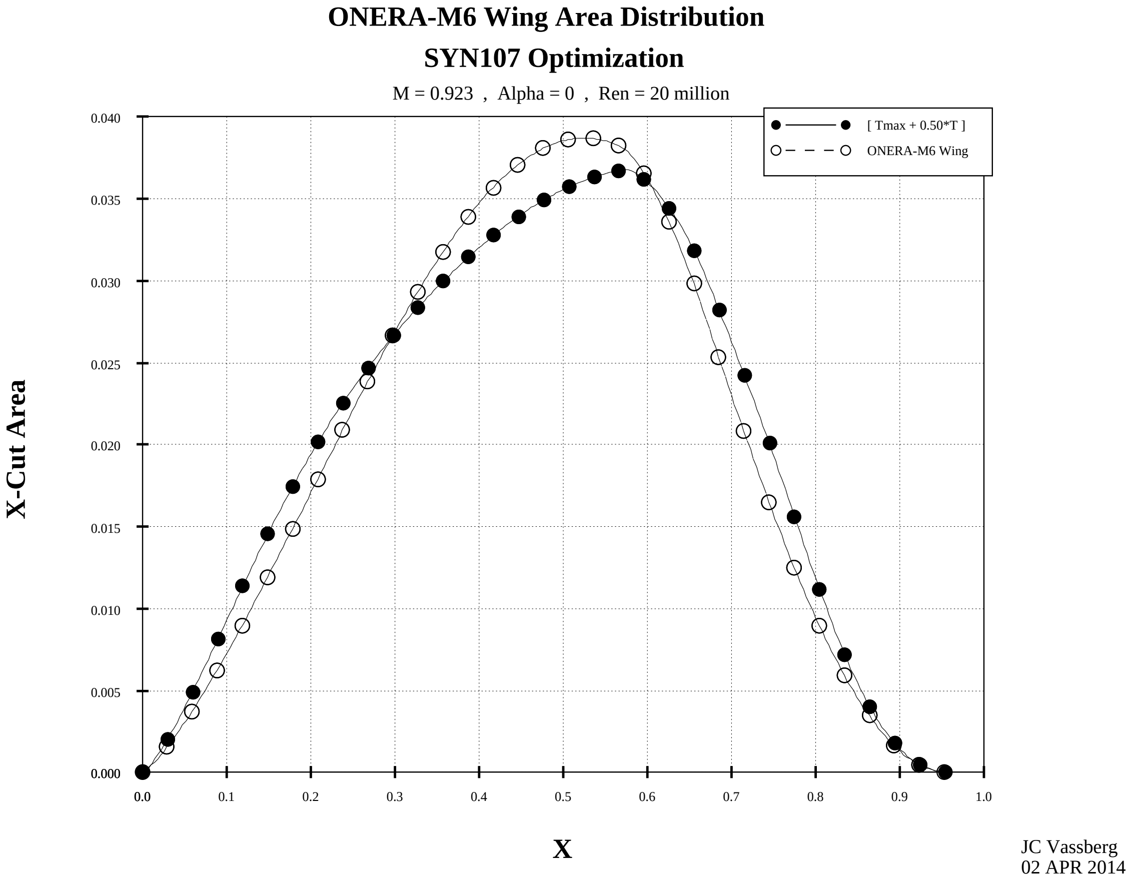
Surface ΔZ of ONERA-M6 Wing (Optimized - Baseline).

ONERA-M6 CASE



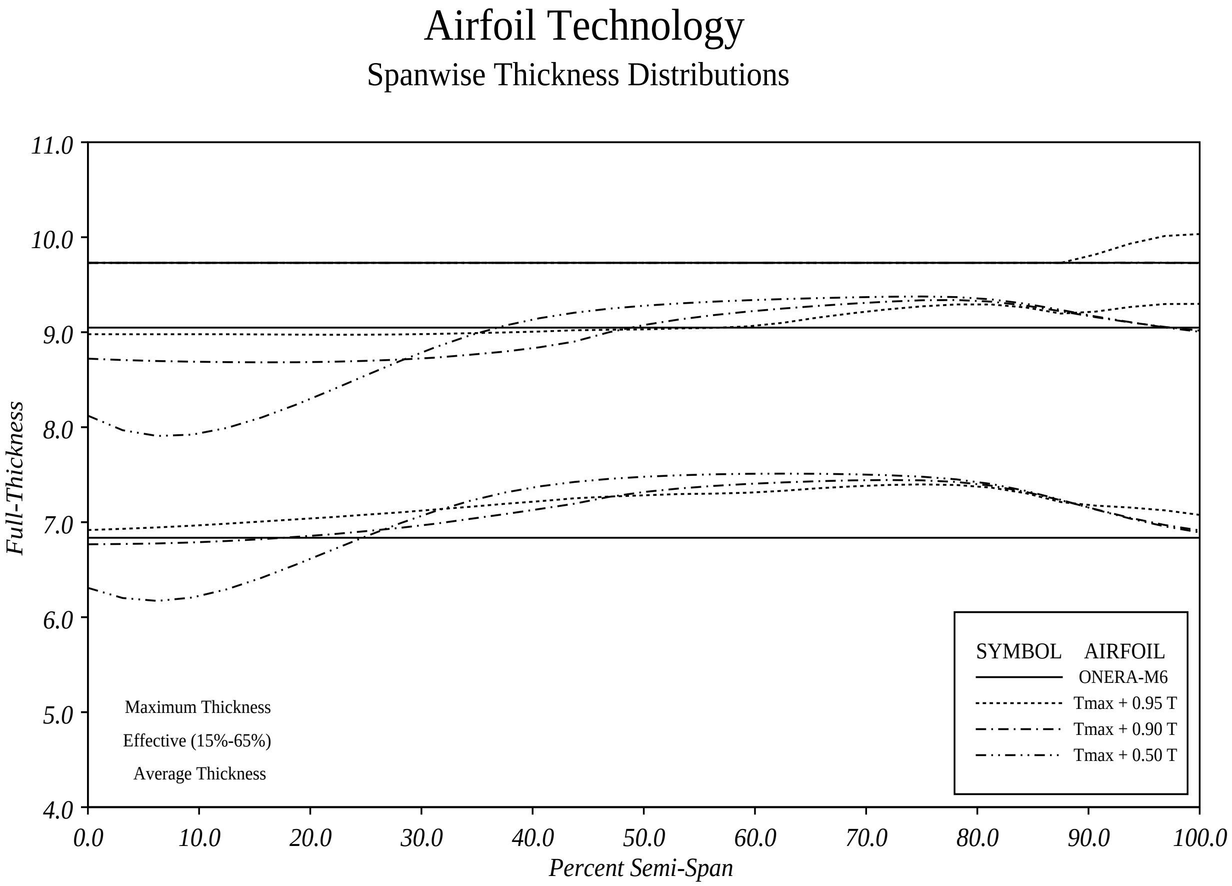
ONERA-M6 Airfoil Sections, $\eta = [0.06,\ 0.50,\ 0.87]$.

ONERA-M6 CASE



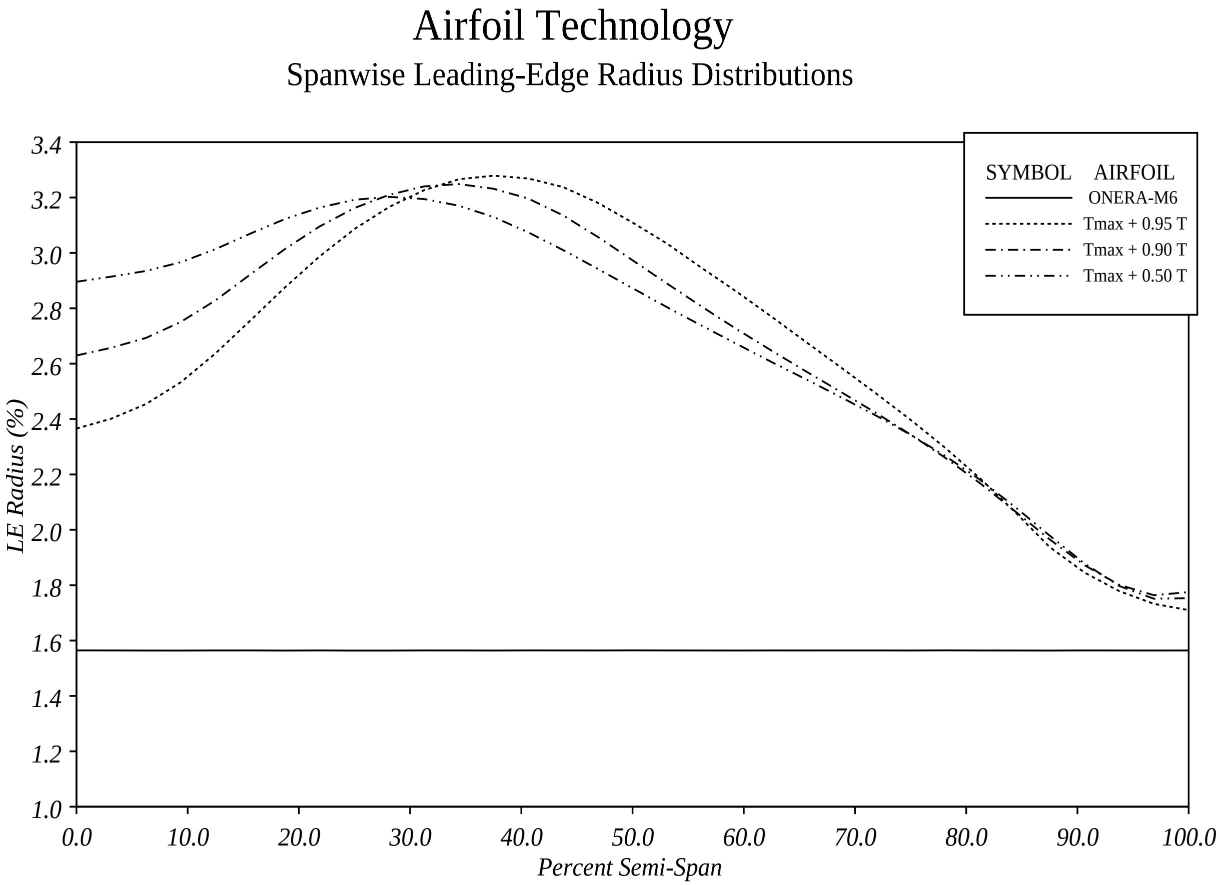
X-Cut Area Distributions - Area Ruling.

ONERA-M6 CASE



ONERA-M6 Baseline & Optimums Thickness Distributions.

ONERA-M6 CASE



ONERA-M6 Baseline & Optimums R_{LE} Distributions.

ONERA-M6 SUMMARY

- **OPTIMIZATIONS HOLDING TMAX**
  - LARGE DRAG REDUCTIONS OBTAINED
- **DRAG LOOPS EXPLAINED**
- **DISTRIBUTION OF SHAPE CHANGES**
  - MODERATE ΔZs ALIGNED WITH SHOCK
  - LARGEST ΔZs IN UNEXPECTED REGIONS
  - IN RETROSPECT - AREA RULING
- **USER DEFINED DESIGN SPACE,** $N << 100$
  - LIKELY INADEQUATE PLACEMENT OF DVs
  - LIKELY NOT RECOGNIZED AS SUCH

ADO-CRM-WING OUTLINE

- **PROBLEM STATEMENT**
  - NASA COMMON RESEARCH MODEL
  - WING EXTRACTED & TRANSFORMED
- **SYN107 RESULTS**
  - SINGLE- & MULTI-POINT DESIGNS
  - PRESSURE DISTRIBUTIONS
  - DRAG CONVERGENCE HISTORIES
  - DRAG POLARS, LIFT & PITCH CURVES
- **DISCUSSIONS**
  - VOLUME & PITCH CONSTRAINTS

ADO-CRM-WING MODEL PROBLEM

The model problem for our third test case is based on the NASA Common Research Model (CRM) developed by Vassberg. Wing extracted and transformed by Osusky.

The objective is to minimize the drag of the ADO-CRM-Wing at the flow condition of $M = 0.85$ and $Re = 5 \times 10^6$, considering a fully-turbulent flow, and subject to the following constraints.

$$\begin{aligned} C_L &= 0.50 + [0.55,\ 0.45] \\ C_M &\geq -0.17 \\ Volume &\geq Volume_{initial} \end{aligned}$$

where, $Volume$ refers to the internal volume of the wing.

ADO-CRM-WING MODEL PROBLEM

Reference Quantities:

$$\begin{aligned} Cref &= 1.0, \\ Sref/2 &= 3.407014, \\ b/2 &= 3.75820, \\ AR &= 8.29117, \\ (X,Y,Z)_{ref} &= (1.2077,\ 0.0,\ 0.007669). \end{aligned}$$

Note:
These reference quantities are different than those of the CRM configuration, in that they have been scaled down by $Cref$. Also, the wing has been shifted inward to place the side-of-body station at the symmetry plane. Hence, $Sref$ is further reduced.

ADO-CRM WING CASE

Table VIII: SYN107 Optimizations

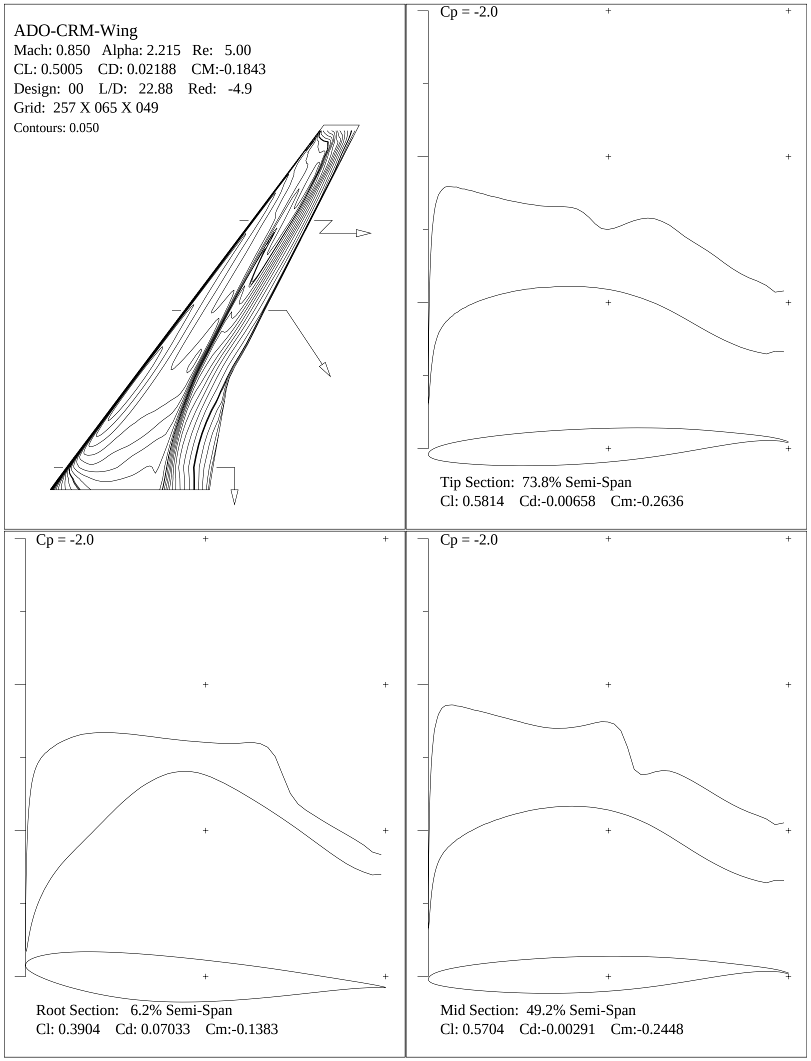
$C_L = 0.5$, $M = 0.85$, $Re = 5 \times 10^6$

$WING$	C_L	C_D	C_M
BASELINE	0.5005	0.02188	-0.1843
CRMADOV09	0.4989	0.02074	-0.1696
CRMADOV10	0.4993	0.02089	-0.1702

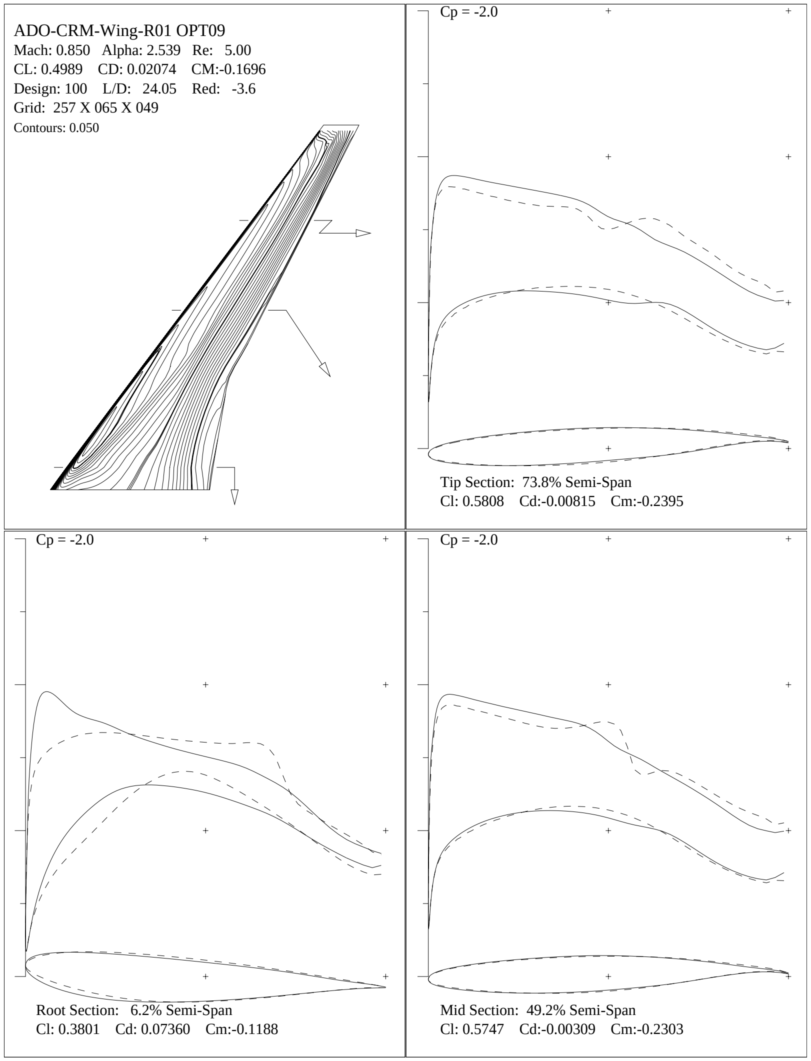
$WING$	C_L	dC_D/dC_L	dC_M/dC_L
BASELINE	0.5005	0.05355	-0.34786
CRMADOV09	0.4989	0.04375	-0.33373
CRMADOV10	0.4993	0.04364	-0.30294

$WING$	C_Lcor	C_Dcor	C_Mcor
BASELINE	0.5	0.02185	-0.18416
CRMADOV09	0.5	0.02079	-0.16993
CRMADOV10	0.5	0.02092	-0.17043

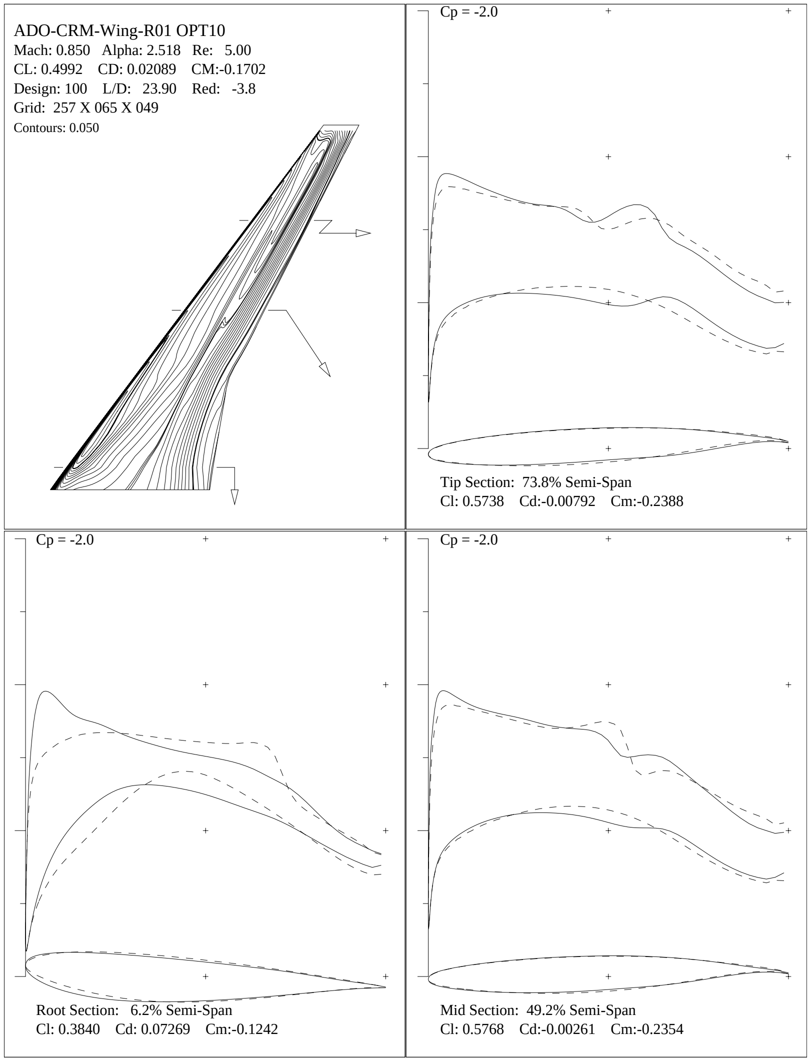
Note: OVERFLOW Cross-Analysis of CRMADOV09 shows a 10.0-count Improvement.



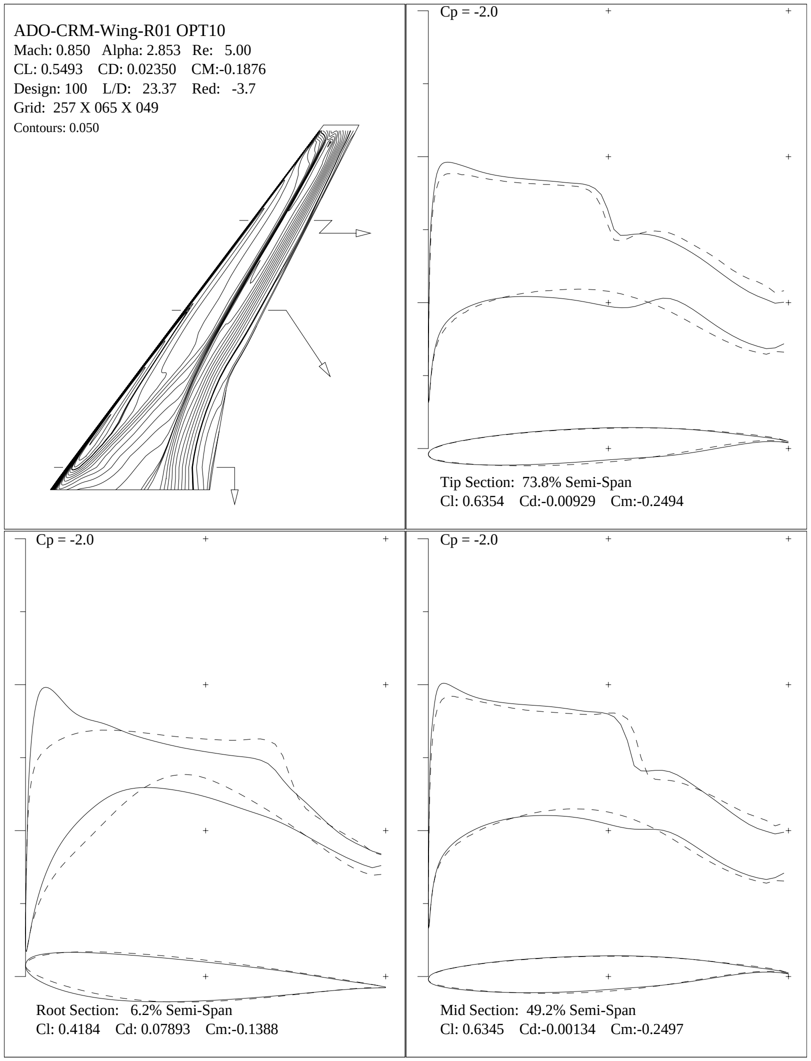
Baseline ADO-CRM-Wing Solution.



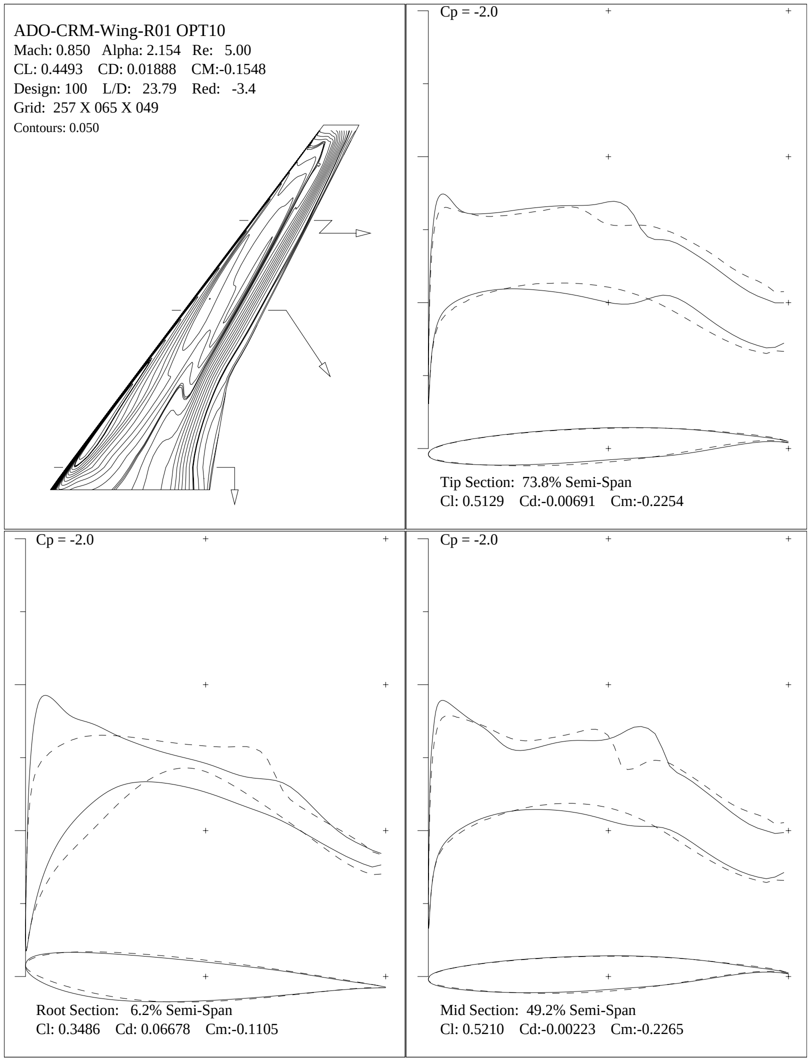
CRMADOV09 Wing, $M = 0.85$, $C_L = 0.50$, $Re = 5 \times 10^6$.



CRMADOV10 Wing, $M = 0.85$, $C_L = 0.50$, $Re = 5 \times 10^6$.

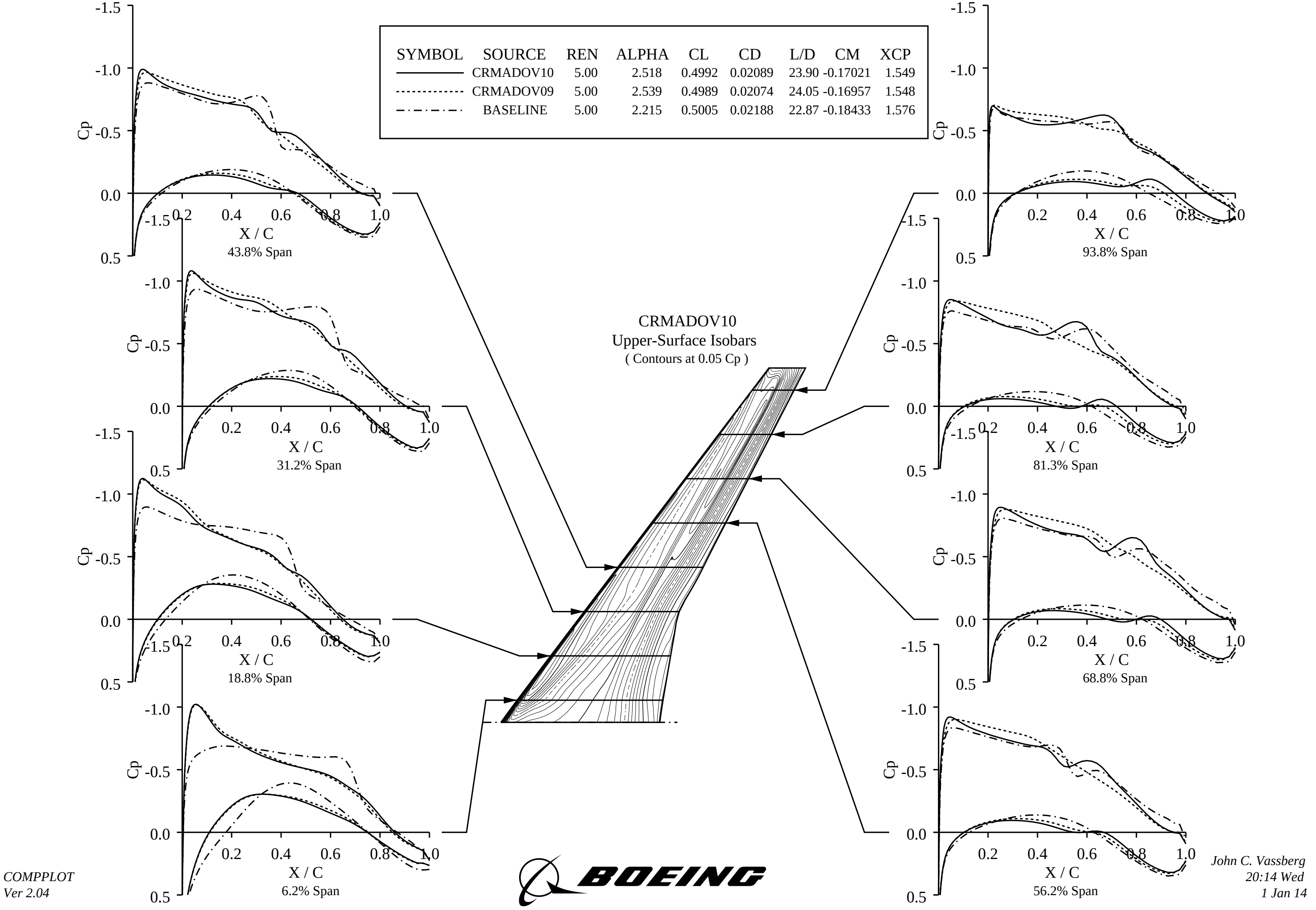


CRMADOV10 Wing, $M = 0.85$, $C_L = 0.55$, $Re = 5 \times 10^6$.

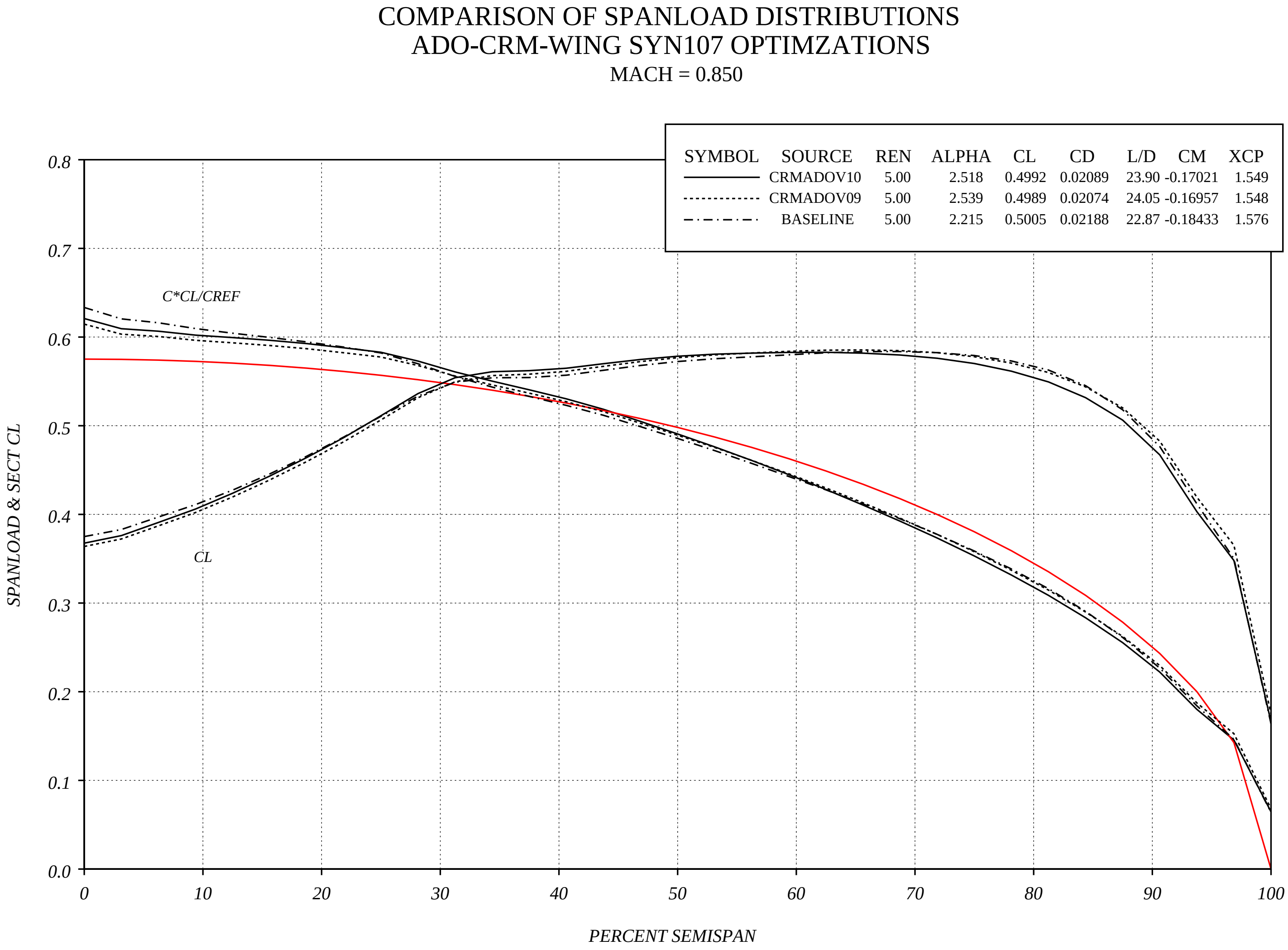


CRMADOV10 Wing, $M = 0.85$, $C_L = 0.45$, $Re = 5 \times 10^6$.

COMPARISON OF CHORDWISE PRESSURE DISTRIBUTIONS
ADO-CRM-WING SYN107 OPTIMIZATIONS
MACH = 0.850

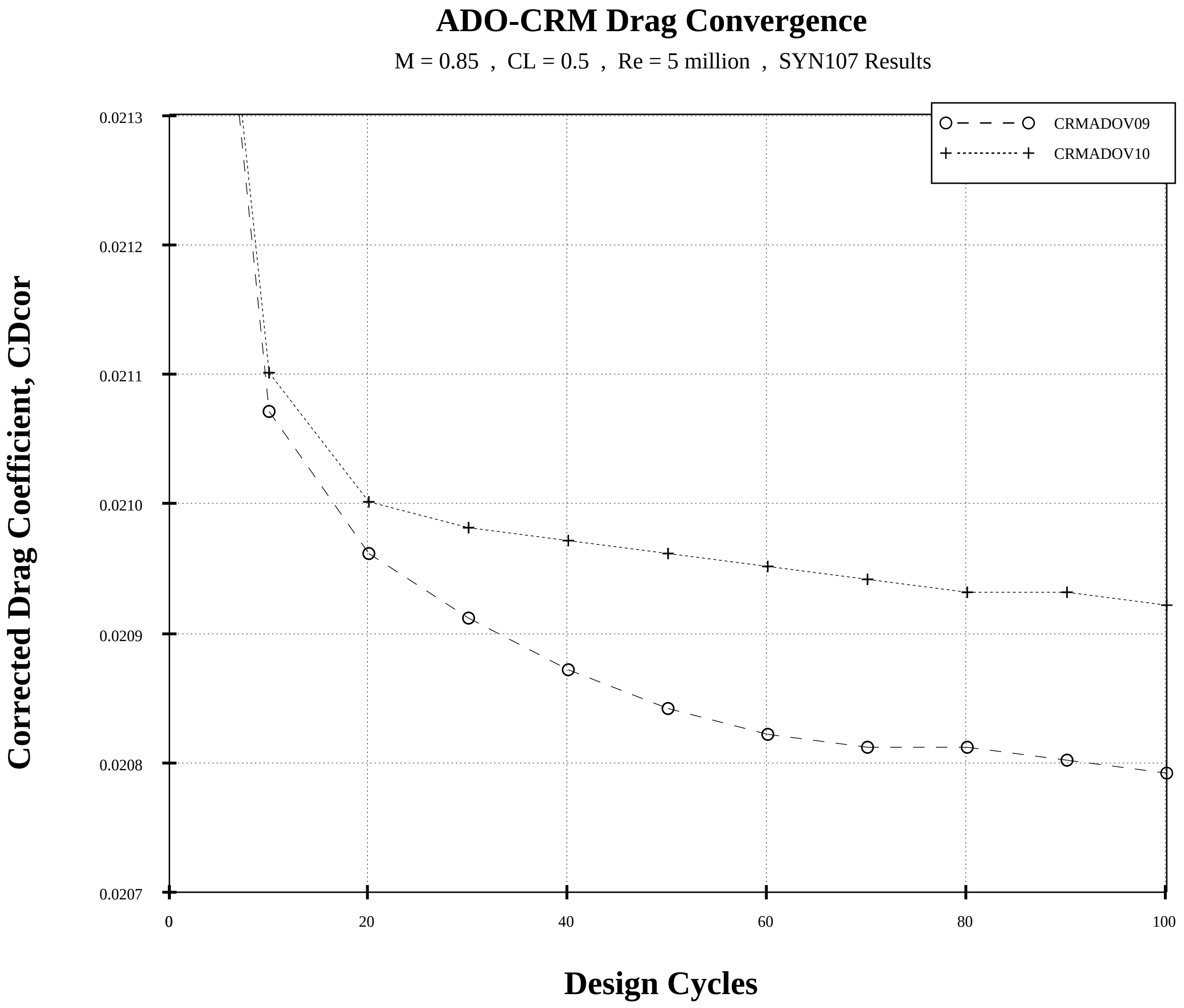


SYMBOL	SOURCE	REN	ALPHA	CL	CD	L/D	CM	XCP
———	CRMADOV10	5.00	2.518	0.4992	0.02089	23.90	-0.17021	1.549
········	CRMADOV09	5.00	2.539	0.4989	0.02074	24.05	-0.16957	1.548
-·-·-·	BASELINE	5.00	2.215	0.5005	0.02188	22.87	-0.18433	1.576

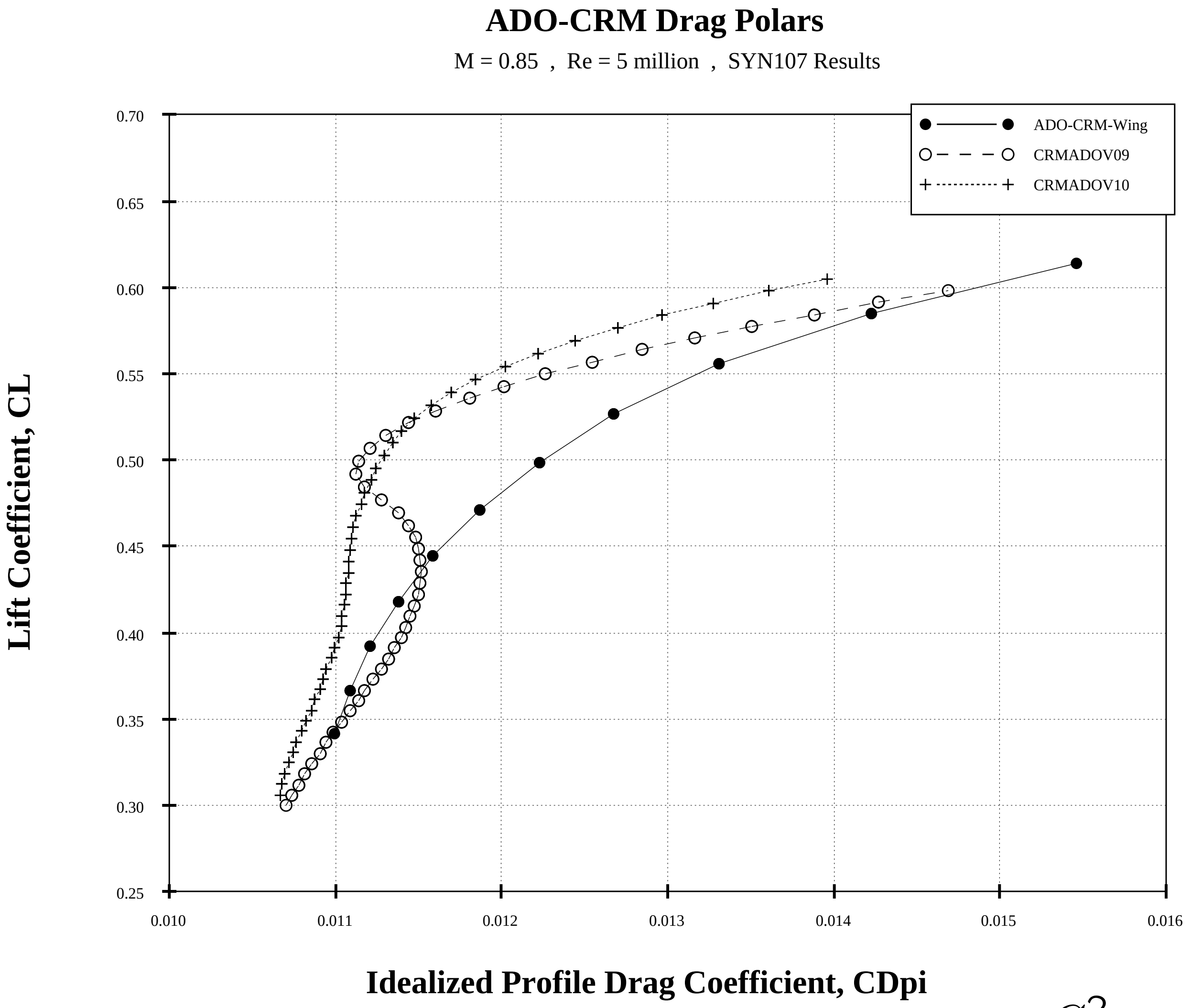


SYMBOL	SOURCE	REN	ALPHA	CL	CD	L/D	CM	XCP
———	CRMADOV10	5.00	2.518	0.4992	0.02089	23.90	-0.17021	1.549
·········	CRMADOV09	5.00	2.539	0.4989	0.02074	24.05	-0.16957	1.548
—·—·—	BASELINE	5.00	2.215	0.5005	0.02188	22.87	-0.18433	1.576

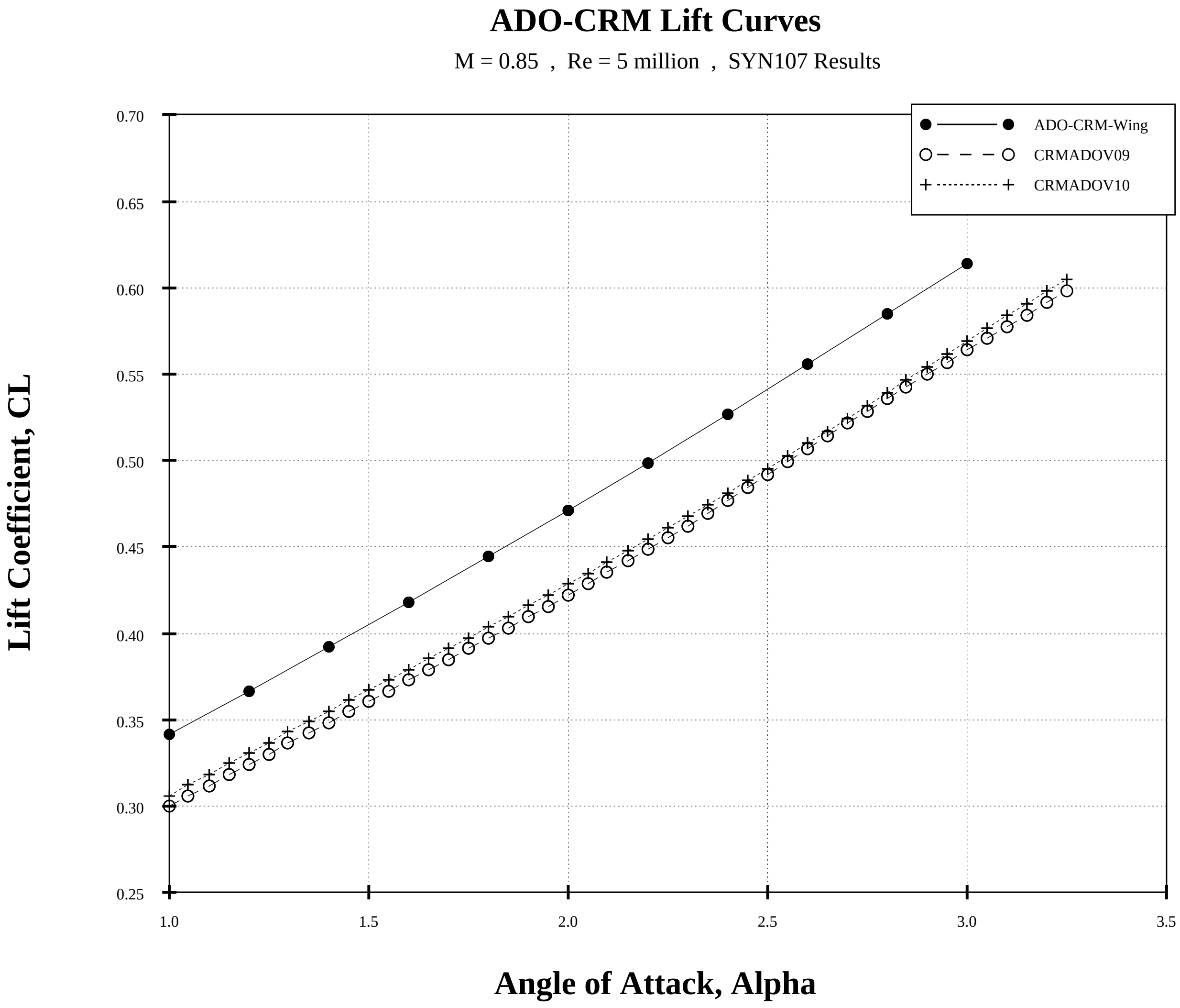
ADO-CRM Spanloads, $M = 0.85$, $C_L = 0.50$, $Re = 5 \times 10^6$.



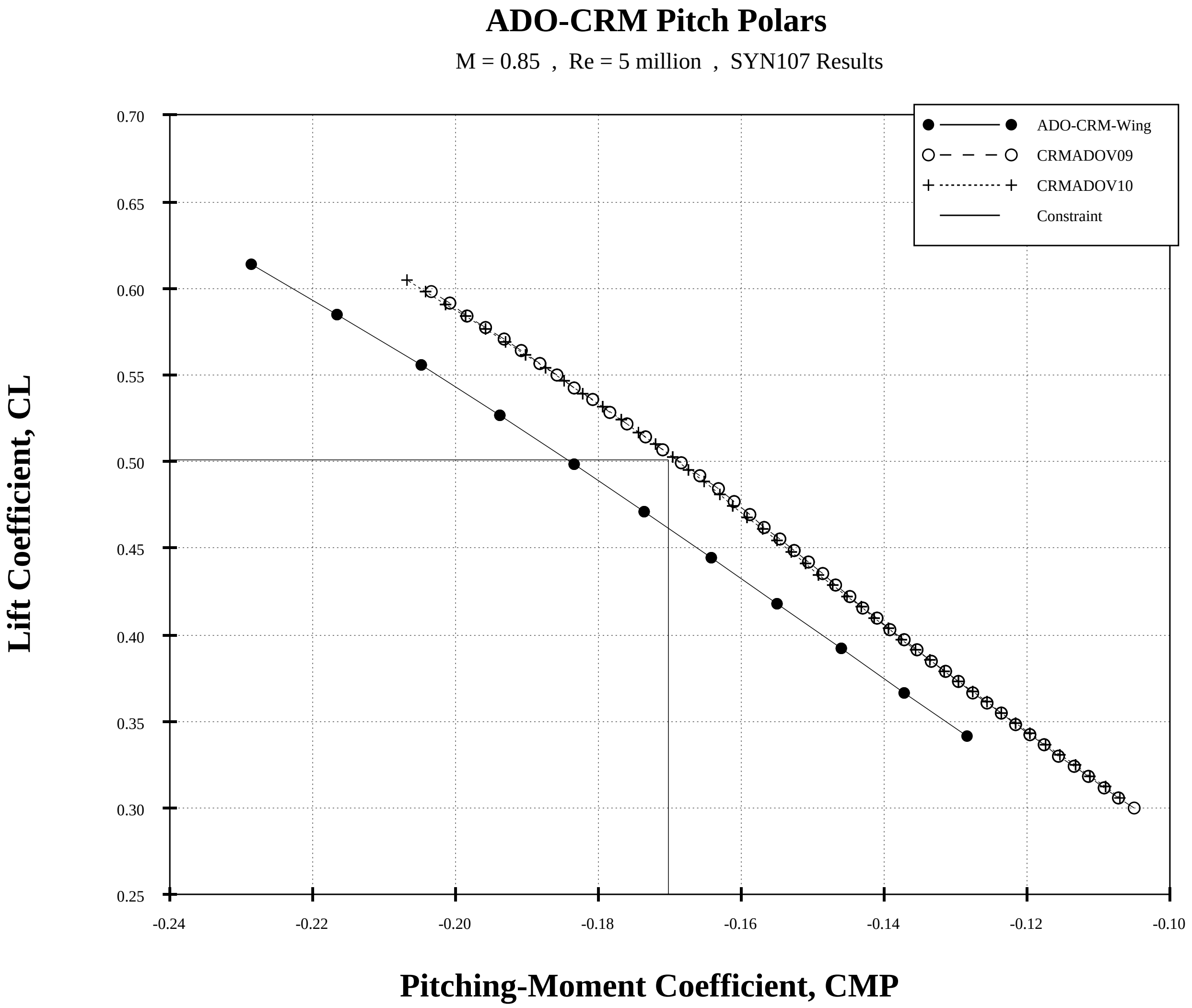
ADO-CRM Drag Histories, $M = 0.85$, $C_L = 0.50$, $Re = 5 \times 10^6$.



ADO-CRM Drag Polars, $C_{Dpi} = C_D - \frac{C_L^2}{\pi AR}$.



ADO-CRM Lift Curves, $M = 0.85$, $Re = 5 \times 10^6$.



ADO-CRM Pitch Polars, $M = 0.85$, $Re = 5 \times 10^6$.

ADO-CRM-WING SUMMARY

- **SYN107 OPTIMIZATIONS**
  - $C_L = 0.5$, $C_M \leq -0.17$ & WING VOLUME
  - CRMADOV09: $\Delta C_D = 10.6\ counts$
  - CRMADOV10: $\Delta C_D = 9.3\ counts$
- **CRMADOV09 $\simeq$ THEORETICAL LIMIT**
  - ESSENTIALLY NO SHOCK DRAG
  - CLOSE TO ELLIPTIC SPANLOAD
- **OVERFLOW CROSS-ANALYSIS**
  - CRMADOV09: $\Delta C_D = 10.0\ counts$

Influence of Shape Parameterization on Aerodynamic Shape Optimization

John C. Vassberg

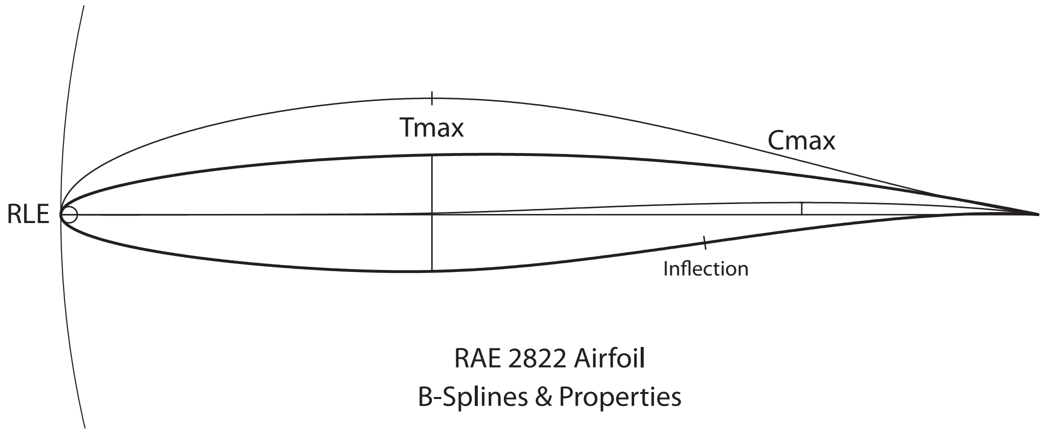
Boeing Technical Fellow
Advanced Concepts Design Center
Boeing Commercial Airplanes
Long Beach, CA 90846, USA

Antony Jameson

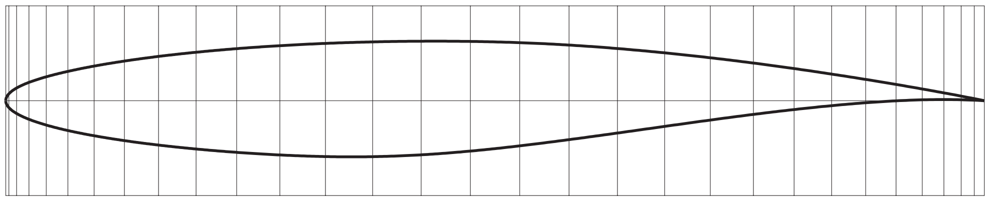
T. V. Jones Professor of Engineering
Dept. of Aeronautics & Astronautics
Stanford University
Stanford, CA 94305-3030, USA

Von Karman Institute
Brussels, Belgium
9 April, 2014

QUESTIONS



RAE 2822 Airfoil
B-Splines & Properties



RAE 2822 Airfoil
Curve Segments