AERO-STRUCTURAL WING DESIGN OPTIMIZATION USING HIGH-FIDELITY SENSITIVITY ANALYSIS

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Abstract

This paper develops and implements a framework for the computation of coupled aero-structural sensitivities which are required for the design of aircraft where aeroelastic interactions are significant. All aero-structural sensitivities are computed using high-fidelity models of both the aerodynamics and the structure of the wing with a coupled-adjoint approach that uses single discipline sensitivity information to calculate the sensitivities of the coupled system. The sensitivities of drag with respect to a set of shape design variables are computed using the aero-structural adjoint method and compared with sensitivities given by the complex-step derivative approximation and finite-differences. The aero-structural adjoint is shown to be both accurate and efficient, and to have a significant cost advantage when the gradient of a small number of functions with respect to a large number of design variables is needed. To demonstrate the usefulness of computing aero-structural sensitivities with the proposed method, results of two drag minimization problems with 190 shape design variables are presented. These results emphasize the importance of aero-structural coupling even in a conventional swept-wing design.

Introduction

A considerable amount of research has been conducted on MultiDisciplinary Optimization (MDO) and its application to aircraft design. The survey paper by Sobieski [16] provides a comprehensive discussion of much of the work in this area. The efforts described therein range from the development of techniques for inter-disciplinary coupling to applications in real-world design problems. In most cases, sound coupling and optimization methods were shown to be extremely important since some techniques, such as sequential discipline optimization, were unable to converge to the true optimum of a coupled system. Wakayama [17], for example, showed that in order to obtain realistic wing planform shapes with aircraft design optimization, it is necessary to include multiple disciplines in conjunction with a complete set of real-world constraints.

Aero-structural analysis has traditionally been carried out in a cut-and-try basis. Aircraft designers have a pre-conceived idea of the shape of an "optimal" load distribution and then tailor the structure's jig shape so that the deflected wing shape under a 1-g load gives the desired distribution. While this approach is typically sufficient for traditional swept-back wing designs, the complexity of aero-structural interactions can be such that, in more advanced designs where little experience has been accumulated or where multiple design points are part of the mission, it can result in sub-optimal designs. This is the case in the design of both small and large supersonic transports, where simple beam theory models of the wing cannot be used to accurately describe the behavior of the wing's structure. In some cases, these aircraft must also cruise for significant portions of their flight at different Mach numbers. In addition, a variety of studies show that this type of aircraft configuration exhibits a range of undesirable aeroelastic phenomena that can only be suppressed when aero-structural interactions are taken into account at the preliminary design stage [2].

Unfortunately, the modeling of the various disciplines in most of the work that has appeared so far has remained at a relatively low level. While useful at the conceptual design stage, lower-order models cannot accurately represent a variety of nonlinear phenomena such as wave drag, which can play an important role in the search for the optimum design. An exception to this low-fidelity modeling is the recent work by Giunta [4] and by Maute et al. [10] where aero-structural sensitivities are calculated using higher-fidelity models.

The objective of this work is to develop techniques for computing high-fidelity coupled sensitivities of aero-structural problems inexpensively so that the resulting information may be used by a gradient-based optimizer to perform realistic design studies. Typical cost functions include different aerodynamic surface pressure integrals (e.g. drag), structural weight, and principal stresses. The aero-structural design problem is usually parameterized using shape design variables that modify the Outer-Mold Line (OML) and the sizes of the structural elements. The MDO framework is built upon prior work by the authors on aero-structural high-fidelity sensitivity analysis [13, 9].

The following sections begin with an introduction to analytic sensitivity analysis where the adjoint method is derived. We then generalize this theory for coupled systems and derive the aero-structural sensitivity equations we want to solve. We then solve these equations to obtain the vector of sensitivities of the wing drag coefficient with respect to wing-shape variables and show that, when using the coupled-adjoint method, these sensitivities can be obtained accurately and efficiently. Finally, we present results of the application of this sensitivity computation method to the aero-structural optimization of a transonic wing.

Analytic Sensitivity Analysis

When solving an optimization problem the goal is usually to minimize an *objective function*, I, by carefully choosing the values of a set of *design variables*. In general, the objective function depends not only on the design variables, but also on the physical state of the problem that is being modeled, thus we can write

$$I = I(x_j, y_k), \tag{1}$$

where x_j represents the design variables and y_k the state variables.

For a given vector x_j, the solution of the governing equations of the system yields a vector y_k, thus establishing the dependence of the state of the system on the design variables. We will denote the governing equations as

$$R_{k'}\left(x_j, y_k\left(x_j\right)\right) = 0. \tag{2}$$

The first instance of x_j in the above equation signals the fact that the residual of the governing equations may depend *explicitly* on x_j. In the case of a structural solver, for example, changing the size of an element has a direct effect on the stiffness matrix. By solving the governing equations we determine the state, y_k, which depends *implicitly* on the design variables through the solution of the system.

Since the number of equations must equal the number of state variables for a solvable system, the ranges of the indices k and k' are the same, i.e., $k, k' = 1, \ldots, n_R$. For a structural solver, for example, n_R is the number of free degrees of freedom; while for a Computational Fluid Dynamics (CFD) solver, n_R is the number of mesh points multiplied by the number of state variables at each point (four in the two-dimensional case and five in three-dimensions.) For a coupled system, R_k' represents *all* the governing equations of the different disciplines.

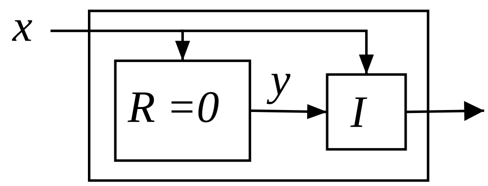

Figure 1: Schematic representation of the governing equations ($R = 0$), design variables (x), state variables (y), and objective function (I).

A graphical representation of the system of governing equations is shown in Figure 1, with the design variables x_j as the inputs and I as the output. The two arrows leading to I illustrate the fact that the objective function may depend on the design variables not only explicitly, but also through the state variables that satisfy the governing equations.

When solving the optimization problem using a gradient-based optimizer, the total variation of the objective function with respect to the design variables, $\frac{dI}{dx_j}$, must be calculated. As a first step towards obtaining this total variation, we use the chain rule to write the total variation of I as

$$\delta I = \frac{\partial I}{\partial x_j}\delta x_j + \frac{\partial I}{\partial y_k}\delta y_k, \quad \text{for} \quad k = 1, \ldots, n_R, \quad j = 1, \ldots, n_x, \tag{3}$$

where we use index notation to denote the vector products. If we were to use this equation directly, the vector of δy_k's would have to be calculated for each δx_j by solving the governing equations n_x times. If there are many design variables and the solution of the governing equations is costly, using equation (3) directly can be impractical.

We now observe that the variations δx_j and δy_k in the total variation of the objective function (3) are not independent of each other if we restrict ourselves to the solution of the governing equations (2). A relationship between these two sets of variations can be obtained by realizing that the variation of the residuals (2) must be zero, i.e.

$$\delta R_{k'} = \frac{\partial R_{k'}}{\partial x_j}\delta x_j + \frac{\partial R_{k'}}{\partial y_k}\delta y_k = 0, \tag{4}$$

for all $k = 1, \dots, n_R$ and $j = 1, \dots, n_x$.

Since this residual variation (4) is zero we can add it to the objective function variation (3) without modifying the latter, i.e.

$$\delta I = \frac{\partial I}{\partial x_j}\delta x_j + \frac{\partial I}{\partial y_k}\delta y_k + \psi_{k'}\left(\frac{\partial R_{k'}}{\partial x_j}\delta x_j + \frac{\partial R_{k'}}{\partial y_k}\delta y_k\right), \tag{5}$$

where $\psi_{k'}$ are arbitrary scalars called *adjoint variables*. This approach is identical to the one used in non-linear constrained optimization, where equality constraints are added to the objective function, and the arbitrary scalars are known as *Lagrange multipliers*.

We can now group the terms in equation (5) that contribute to the same variation and write

$$\delta I = \left(\frac{\partial I}{\partial x_j} + \psi_{k'}\frac{\partial R_{k'}}{\partial x_j}\right)\delta x_j + \left(\frac{\partial I}{\partial y_k} + \psi_{k'}\frac{\partial R_{k'}}{\partial y_k}\right)\delta y_k. \tag{6}$$

If we set the term multiplying δy_k to zero, we are left with the total variation of I as a function of the design variables and the adjoint variables, removing the dependence of the variation on the state variables. Since the adjoint variables are arbitrary, we can accomplish this by solving the *adjoint equations*

$$\frac{\partial R_{k'}}{\partial y_k}\psi_{k'} = -\frac{\partial I}{\partial y_k}. \tag{7}$$

These equations depend only on the partial derivatives of both the objective function and the residuals of the governing equations with respect to the state variables. Since these partial derivatives can be calculated directly without solving the governing equations, the adjoint equations (7) only need to be solved for each I and their solution is valid for all the design variables.

When adjoint variables are found in this manner, we can use them to calculate the total sensitivity of I with the first term of equation (6), i.e.

$$\frac{dI}{dx_j} = \frac{\partial I}{\partial x_j} + \psi_{k'}\frac{\partial R_{k'}}{\partial x_j}. \tag{8}$$

The cost involved in calculating sensitivities using the adjoint method is practically independent of the number of design variables. After having solved the governing equations, the adjoint equations are solved only once for each I, independently of the number of design variables. The terms in the adjoint equations are inexpensive to calculate, and the cost of solving the adjoint equations is similar to that involved in the solution of the governing equations.

The adjoint method has been widely used for single discipline sensitivity analysis and examples of its application include structural sensitivity analysis [1] and aerodynamic shape optimization [6].

Aero-Structural Sensitivity Analysis

The analysis presented in the previous section for single discipline systems can be generalized for multiple, coupled systems. The same total sensitivity equations (7, 8) apply, provided that the governing equations and state variables corresponding to all the disciplines are included in $R_{k'}$ and y_k respectively.

In the case of an aero-structural system we have aerodynamic (A) and structural (S) analyses, and two sets of state variables: the flow state, w, and the structural displacements, u.

We now use a notation that is specific to the aero-structural system, rather than the general notation we have used in the previous section. We no longer use index notation and we split the vectors of residuals, states and adjoints into two smaller vectors corresponding to the aerodynamic and structural systems, i.e.

$$R_{k'} = \begin{bmatrix} R_A \\ R_S \end{bmatrix}, \quad y_k = \begin{bmatrix} w \\ u \end{bmatrix} \quad \text{and} \quad \psi_{k'} = \begin{bmatrix} \psi_A \\ \psi_S \end{bmatrix}. \tag{9}$$

Figure 2 shows a diagram representing the coupling in this system.

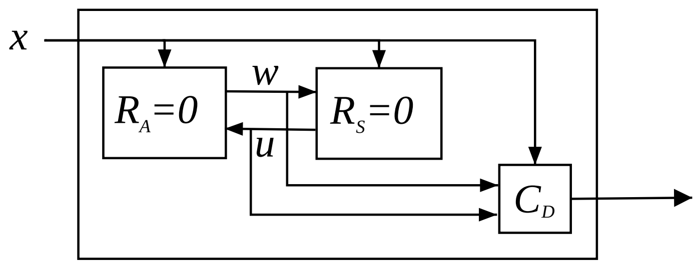

Figure 2: Schematic representation of the aero-structural governing equations.

Since for the problem at hand we are interested in the sensitivities of the drag coefficient, C_D takes the place of I. We continue to denote the wing-shape variables by the same vector, x.

Using this new notation, the adjoint equation (7) for an aero-structural system can be written as

$$\begin{bmatrix} \partial R_A/\partial w & \partial R_A/\partial u \\ \partial R_S/\partial w & \partial R_S/\partial u \end{bmatrix}^T \begin{bmatrix} \psi_A \\ \psi_S \end{bmatrix} = \begin{bmatrix} \partial C_D/\partial w \\ \partial C_D/\partial u \end{bmatrix}. \tag{10}$$

In addition to the diagonal terms of the matrix that appear when we solve the single discipline adjoint equations, we also have cross terms expressing the sensitivity of one discipline to the other's state variables. The residual sensitivity matrix in this equation is identical to that of the Global Sensitivity Equations (GSE) introduced by Sobieski [15]. Considerable detail is hidden in the terms of this equation and therefore we will describe each of the terms in more detail.

- $\partial R_A/\partial w$: This term represents the variation of the residuals of the CFD equations due to changes in the the flow variables. When a flow variable at a given cell center is perturbed, the sum of the fluxes on that cell is altered. Only that cell and the neighboring ones are affected. Therefore, even though $\partial R_A/\partial w$ is a large square matrix, it is also extremely sparse and its non-zero terms can be calculated with finite-differences.

- $\partial R_A/\partial u$: This represents the effect that the structural displacements have on the residuals of the CFD solution through the perturbation of the CFD mesh. When the wing deflects, the mesh must be warped, resulting in a change in the geometry of a subset of grid cells. Even though the flow variables are kept constant, the change in the geometry has an influence on the sum of the fluxes, whose variation can be easily obtained by re-calculating the residuals for the warped cells.

- $\partial R_S/\partial w$: The linear structural equations can be written as $Ku - f = 0$, where K is the stiffness matrix and f is the vector of applied forces. The only term that the flow variables affect directly is the applied force, and therefore this term is equal to $-\partial f/\partial w$, which can be found by examining the procedure that integrates the pressures to obtain the applied forces.
- $\partial R_S/\partial u$: Since the forces do not depend directly on the displacements, this term is simply the stiffness matrix, K.
- $\partial C_D/\partial w$: The direct sensitivity of the drag coefficient with respect to the flow variables can be obtained analytically by examining the numerical integration of the surface pressures.
- $\partial C_D/\partial u$: This term represents the change in the drag coefficient due to the wing's displacements while keeping the pressure distribution constant. The structural displacements affect the drag directly, since they change the wing surface geometry over which the pressure distribution is integrated.

The computation of all of these terms is inexpensive when compared to the cost of an aero-structural solution because it does not require an iterative procedure.

Since the factorization of the full matrix in the system of equations (10) would be extremely costly, our approach uses an iterative solver, much like the one used for the aero-structural solution, where the adjoint vectors are *lagged* and the two different sets of equations are solved separately. For the calculation of the adjoint vector of one discipline, we use the adjoint vector of the other discipline from the previous iteration, i.e., we solve

$$\left[\frac{\partial R_A}{\partial w}\right]^T \psi_A = \frac{\partial C_D}{\partial w} - \left[\frac{\partial R_S}{\partial w}\right]^T \tilde{\psi}_S, \tag{11}$$

$$\left[\frac{\partial R_S}{\partial u}\right]^T \psi_S = \frac{\partial C_D}{\partial u} - \left[\frac{\partial R_A}{\partial u}\right]^T \tilde{\psi}_A. \tag{12}$$

The final result given by this system, is the same as that of the original coupled-adjoint equations (10). We will call this the *Lagged-Coupled Adjoint* (LCA) method for computing sensitivities of coupled systems. Note that these equations look like the single discipline adjoint equations for the aerodynamic and the structural solvers, with the addition of forcing terms in the right-hand-side that contain the cross terms of the residual sensitivity matrix.

As noted previously, $\partial R_S/\partial u = K$ for a linear structural solver. Since the stiffness matrix is symmetric, $K^T = K$, and the structural adjoint equations (12) have the same "stiffness matrix" as the structural governing equations. Therefore, the structural solver can be used to solve for the structural adjoint vector, ψ_S, by using the *pseudo-load* vector given by the right–hand side of equation (12).

Once both adjoint vectors have converged, we can compute the final sensitivities of the objective function by using

$$\frac{dC_D}{dx} = \frac{\partial C_D}{\partial x} - \psi_A^T \frac{\partial R_A}{\partial x} - \psi_S^T \frac{\partial R_S}{\partial x}, \tag{13}$$

which is the coupled version of the total sensitivity equation (8). We will now describe the partial derivatives in the above equation.

- $\partial C_D/\partial x$: This is the change in the drag coefficient due to wing-shape perturbations, while keeping the pressure distribution constant. This sensitivity is analogous to the partial derivative $\frac{\partial C_D}{\partial u}$ that we described above and can be easily calculated using finite-differences.

- $\partial R_A/\partial x$: The direct effect of shape perturbations on the CFD residuals is similar to that of the displacements on the same residuals, $\partial R_A/\partial u$, that we mentioned previously.

- $\partial R_S/\partial x$: The only term of the structural equations that the shape perturbations affect directly is the stiffness matrix, and thus this partial derivative is equal to $\partial K/\partial x \cdot u$.

As in the case of the partial derivatives of equations 10, all the terms can be computed without incurring a large computational cost.

Results

The focus of this section is on the validation of the coupled aero-structural sensitivities obtained using an implementation of the lagged coupled-adjoint method. The computed sensitivities are primarily intended to be used in conjunction with a gradient-based optimizer to guide the evolution of the wing design. To illustrate this purpose, we also include two design examples that are not meant to represent a true wing design problem, where more suitable choices of objective function, design variables, and constraints would be necessary.

Aero-Structural Analysis

The coupled-adjoint procedure was implemented as a module that was added to the aero-structural design framework previously developed by the authors [13]. The framework consists of an aerodynamic analysis and design module (which includes a geometry engine and a mesh perturbation algorithm), a linear finite-element structural solver, an aero-structural coupling procedure, and various pre-processing tools that are used to setup aero-structural design problems.

The aerodynamic analysis and design module, SYN107-MB [12], is a multiblock parallel flow solver for both the Euler and the Reynolds Averaged Navier-Stokes equations that has been shown to be accurate and efficient for the computation of the flow around full aircraft configurations [14]. An aerodynamic adjoint solver is also included in this package in order to perform aerodynamic shape optimization in the absence of aero-structural interaction.

The structural analysis package is FESMEH, a finite element solver developed by Holden [5]. The package is a linear finite-element solver that incorporates two element types and computes the structural displacements and stresses of wing structures. Although this solver is not as general as some commercially-available packages, it is still representative of the challenge involved in using large models with tens of thousands of degrees of freedom. High-fidelity coupling between the aerodynamic and the structural analysis programs is achieved using a linearly consistent and conservative scheme [13, 3].

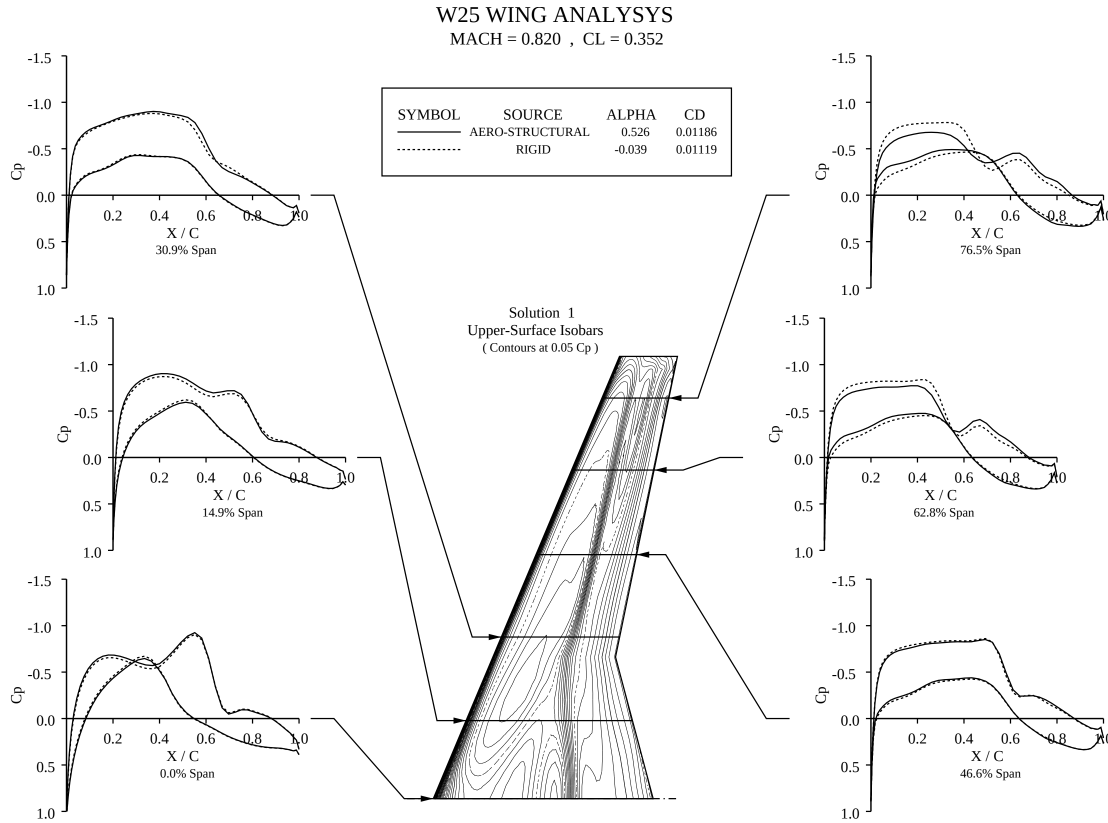

Figure 3: Wing pressure distributions for rigid and aero-structural configurations.

The subsequent results have been obtained for the isolated wing of a small transonic business jet flying at a free-stream Mach number of 0.82. The wing planform and pressure distributions are shown in Figure 3 both with and without the aeroelastic deflections, for the same lift coefficient of 0.352.

The structural model of the wing is constructed using a wing box with six spars evenly distributed from 5% to 50% of the chord at the root and from 5% to 80% of the chord at the tip section. Ribs are distributed along the span at every tenth of the semispan. A total of 640 finite elements were used in the construction of this model. Appropriate thicknesses of the spar caps, shear webs, and skins were chosen to model the wing's real structure. The production airplane has a very stiff wing that results in rather small deflections. In order to investigate the effect of larger wing tip deflections, the Young's modulus of the material was reduced so that a wing tip deflection of approximately 4% of the semispan was achieved for the cruise condition. Figure 4 shows a view of the deflected and rigid geometries of the wing box. The nodes of the structural models are shown, but the finite elements are not drawn for clarity.

The displacement and rotation of the wing tip is shown in Figure 5 and it is clear that the incidence of the tip section is decreased. This is expected, since for swept-back wings, the aeroelastic axis is located ahead of the quarter-chord line. The lower incidence of the outboard sections of the deflected wing explains the increase in the angle of attack (from

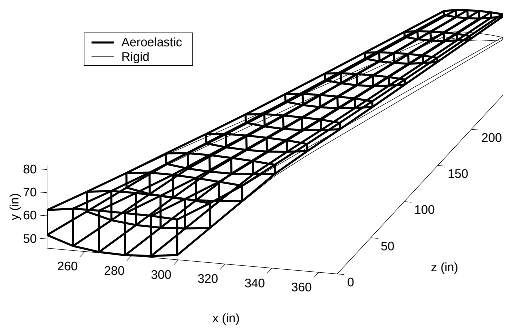

Figure 4: Model of the wing structure showing the jig shape and the deflected shape.

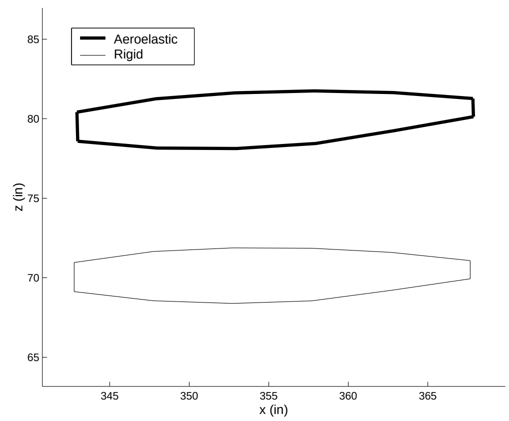

Figure 5: Deflection of the rib located at the wing tip.

−0.039 to 0.526 degrees) that is necessary to maintain the same wing lift coefficient.

Aero-Structural Sensitivities

The sensitivity results of the coupled-adjoint method are validated by comparison with finite-differences and complex-step derivative approximations. The complex-step approximation is a relatively new method that computes sensitivities (e.g. dC_D/dx_j) using the formula

$$\frac{dC_D}{dx_j} \approx \frac{\operatorname{Im}\left[C_D\left(x_j + ih\right)\right]}{h}. \tag{14}$$

Details of this simple yet powerful approximation can be found in work published by the authors [8, 9, 7]. As in the case of finite-differences, the cost of a full gradient calculation scales linearly with the total number of design variables. Unlike finite-differences, however, the accuracy of the complex-step method is extremely insensitive to the step size, h, making it much more robust.

The design variables are wing-shape perturbations in the form of Hicks-Henne *bump* functions [12], two sets of which we used to obtain the results presented herein. The first set consists of shape perturbations that we evenly distributed spanwise along the quarter-chord of the wing, both on the top and on the bottom surfaces.

Figure 6 shows the results of the sensitivity calculations of the coefficient of drag, C_D, to the amplitude of this set of shape functions. The upper plot represents the effect of 9 of these design variables placed along the upper surface of the wing, while the lower plot shows the same results for the 9 bumps placed along the lower surface. In each of the plots we present the sensitivities with and without the presence of aero-structural coupling.

When only the aerodynamics is taken into account, the shape of the upper surface of the wing has a large effect on the drag coefficient, specially for the sections near the root. The influence of the lower surface shape, on the other hand, is much smaller, being one order of magnitude smaller. The finite-difference and complex-step results are almost identical, the $\mathcal{L}_2$ norm of the difference between the two being 5×10^{-5}. The aerodynamic

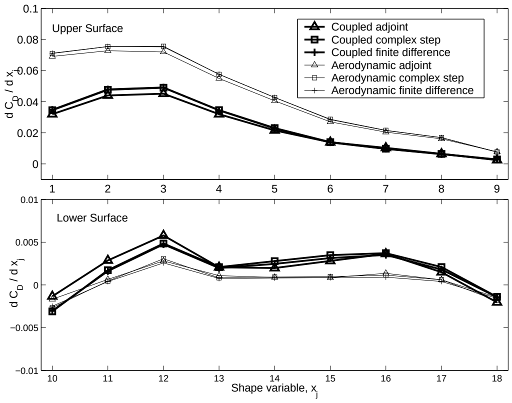

Figure 6: Sensitivities of the drag coefficient with respect to shape perturbations distributed spanwise.

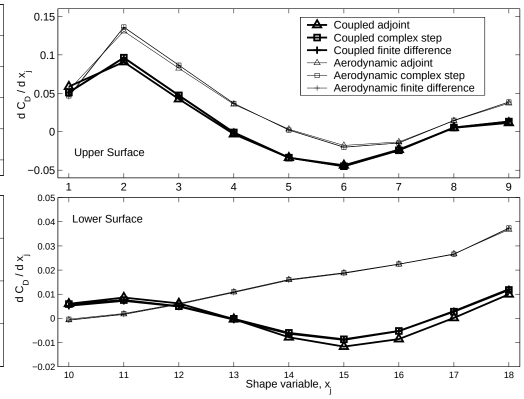

Figure 7: Sensitivities of the drag coefficient with respect to shape perturbations distributed chordwise.

adjoint follows closely, with $\mathcal{L}_2 = 4 \times 10^{-4}$; the greater difference is attributable to the fact that we use a discretization of the continuous adjoint equations that is only consistent with the values given by finite-differences in the limit of very fine meshes. This fact has been demonstrated in previous work within our research group [11].

When aero-structural deflections are taken into account, there is a significant change in the sensitivities. The values of the aero-structural sensitivities with respect to the upper surface shape modifications shown in Figure 6 have magnitudes that are between 50% and 100% smaller than their rigid counterparts. This is an expected result since the wing exhibits aeroelastic twist relief and therefore, at a fixed angle of attack, the total C_L is lower than for the rigid case.

On both surfaces, the sensitivity values given by the complex-step and finite-difference methods agree extremely well. The aero-structural adjoint sensitivities agree with the reference results thus validating the implementation of the method. The magnitude of the $\mathcal{L}_2$ norms of the differences between the finite-difference and adjoint results are of the same order as those noted for the rigid wing case.

For the second set of sensitivities we chose to distribute the Hicks-Henne bump functions evenly in the chordwise direction of an airfoil section located midspan. The resulting sensitivities are shown in Figure 7. Again, 9 bumps are evenly distributed along the top and 9 on the bottom of the airfoil. The agreement between the adjoint results and the reference sensitivities given by finite differences and the complex step is very similar to what we described for the first set of sensitivities.

Table 1 summarizes the computational times required for the calculation of the complete gradient of the drag coefficient with respect to the design variables of the problem. The times are normalized with respect to the computational time required for a single aero-structural solution. Using our framework, this computational time is only 25% higher than the time required for a rigid aerodynamic solution converged to the same level.

Note that the cost of the sensitivity calculation using either the finite-difference or complex-step methods is linearly dependent on the number of design variables in the problem, whereas the cost of the coupled-adjoint procedure is essentially independent of

Aero-structural	Solution	1.0
	Finite difference	14.2
	Complex step	34.4
	Coupled adjoint	7.5
Aerodynamic	Solution	0.8
	Finite difference	13.3
	Complex step	32.1
	Adjoint	3.3

Table 1: Computation time comparison.

this number. In more realistic design situations where the number of design variables chosen to parameterize the surface is much larger than 18, the computational efficiency of the adjoint sensitivity calculation would become even more obvious.

The cost of a finite-difference gradient evaluation for the 18 design variables is about 14 times the cost of a single aero-structural solution for computations that have converged to six orders of magnitude. Notice that one would expect this method to incur a computational cost equivalent to 19 aero-structural solutions (the solution of the baseline configuration plus one flow solution for each design variable perturbation.) The cost is lower than this because the additional calculations do not start from a uniform flow-field initial condition, but from the previously converged solution.

The cost of the complex-step procedure is more than twice of that of the finite-difference procedure since the function evaluations require complex arithmetic. We feel, however, that the complex-step calculations are worth this cost penalty since there is no need to find an acceptable step size a priori, as in the case of the finite-difference approximations.

Finally, the coupled-adjoint method requires the equivalent of 7.5 aero-structural solutions to compute the whole gradient. As mentioned previously, this computational cost is practically independent of the total number of design variables in the problem and would therefore remain at the same order of magnitude, even in the more realistic case of 200 or more design variables. In contrast, the finite-difference method would require a computational effort of around $\frac{200}{18} \times 14.2 = 157.8$ times the cost of an aero-structural solution to compute the same gradient.

Wing Optimization

In this section, the sensitivity analysis procedure we presented is used for the drag minimization of the same swept-back transonic wing flying at a Mach number of 0.82, and at a wing lift coefficient of 0.352. Two separate design calculations are presented: the first one is carried out under the assumption that the wing structure is infinitely rigid, while the second one includes the effects of aero-structural coupling. In both calculations there is a total of 190 shape design variables, 10 of which are wing section twists and the rest are bump functions distributed throughout the wing's surface. In addition, four linear constraints are imposed at 11 evenly-spaced spanwise defining stations: two spar thickness constraints at the 40% and 80% chord locations, a trailing edge angle constraint, and a leading edge radius constraint.

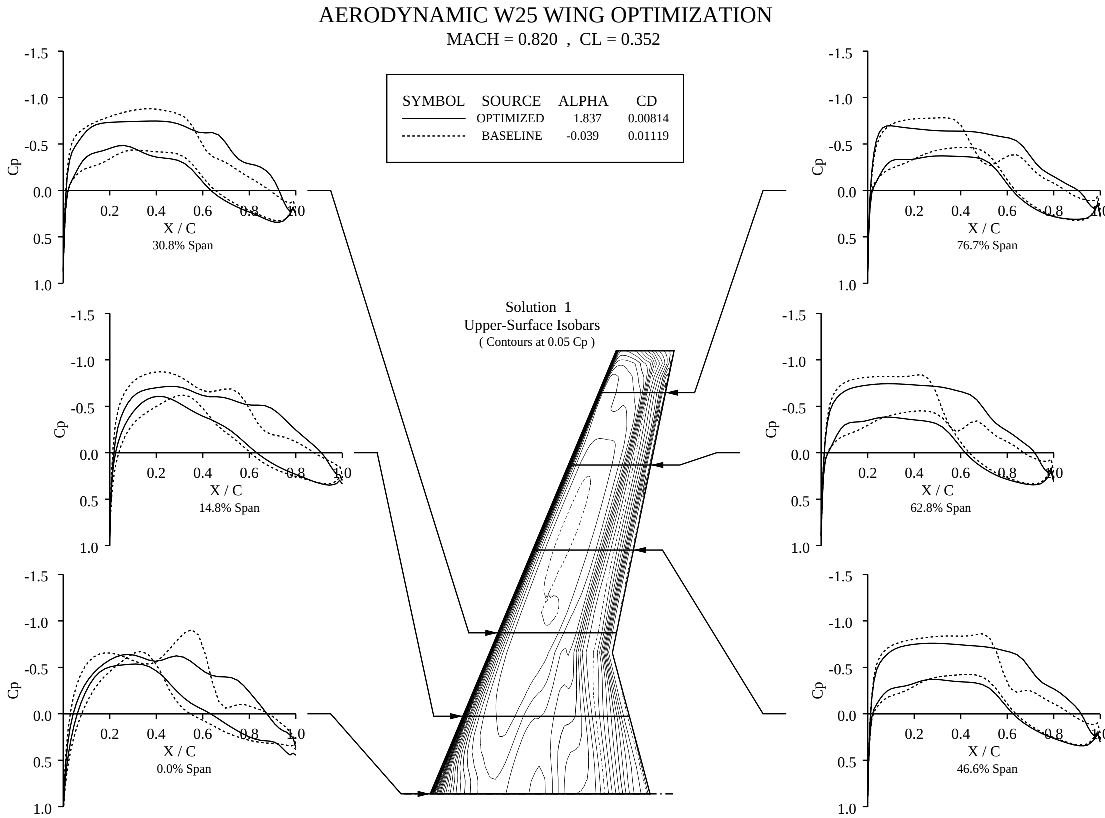

Figure 8: Wing pressure distributions for baseline and optimized rigid designs.

Rigid Wing-Shape Optimization

The results of the drag minimization problem for the rigid wing case are shown in Figure 8. These result are representative of our earlier work and are provided for comparison with the aero-structural optimization results of the following section. Chordwise pressure distributions for six different spanwise stations are shown. The solid lines represent the outcome of the design optimization, while the dashed lines correspond to the rigid analysis of the baseline configuration. Through the modifications in the shape of the airfoil sections on the wing, the coefficient of drag has decreased from 0.01119 to 0.00814 in 12 design iterations. As seen in Figure 8, the strong shock wave that existed along the span has been eliminated, and the strong recompression in the root section of the wing has been smoothed. The decrease in drag coefficient is mainly due to the elimination of the shock wave drag, although the spanload has changed slightly resulting in a reduction in induced drag. The resulting rigid shape would usually be passed on to the structural designer so that a jig shape that deforms to this geometry under a 1-g load can be designed.

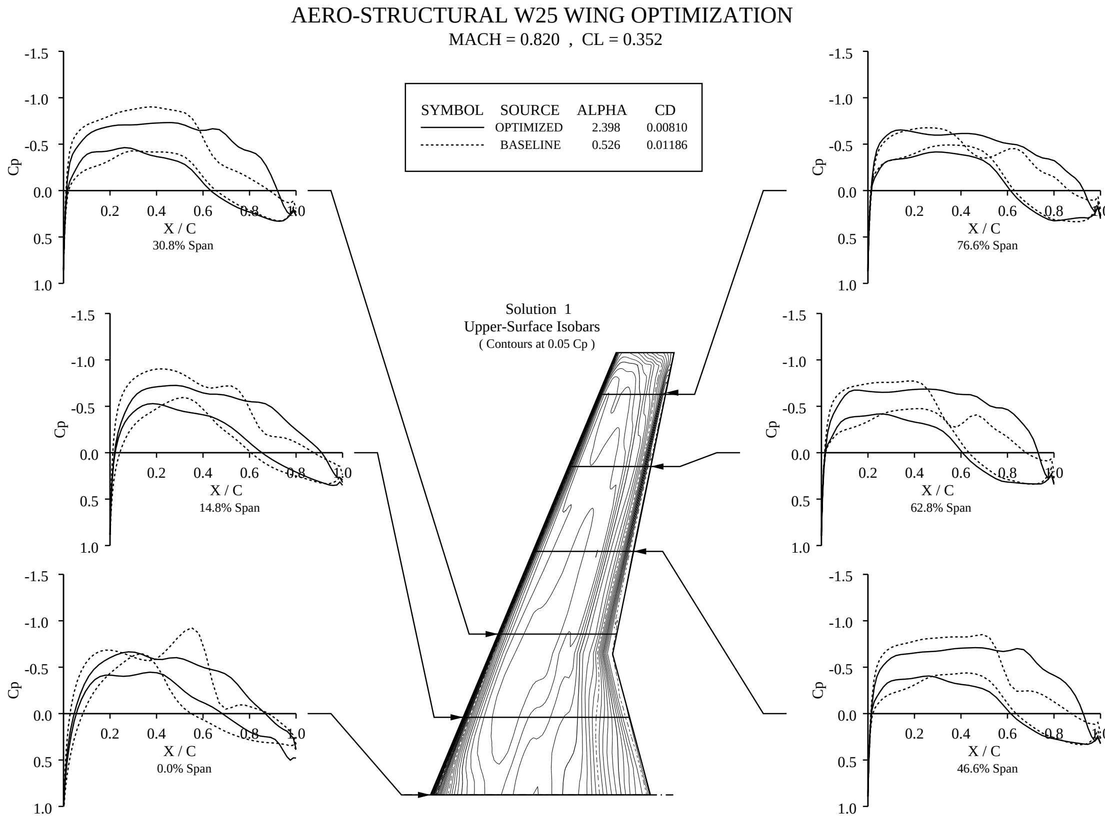

Figure 9: Wing pressure distributions for baseline and optimized aero-structural designs.

Aero-Structural Wing-Shape Optimization

Figure 9 shows the result of minimizing drag after 12 design iterations for the same configuration, design variables, and constraints mentioned above. The only difference is that aero-structural effects are included and the sensitivity information used to guide the progress of the design uses the aero-structural adjoint method. The result of this optimization is interesting in two ways. Firstly, even with the inclusion of aero-structural effects, the optimization procedure has managed to completely eliminate the strong shock wave that existed all along the upper surface of the wing. The importance of this result lies in the fact that in order to completely eliminate the shock wave drag, the correct sensitivities of drag coefficient to shape deformations in the presence of structural deflections are needed. This serves as independent validation of the sensitivities computed through the coupled-adjoint procedure. The second point of interest is the character of the optimized pressure distributions shown in Figure 9. Since the optimizer is only able to change the shape of the wing surface, it only has an indirect effect on the resulting twist distribution. This leads to pressure distributions with a high load in the aft portion of most airfoils sections. The aero-structural analysis of the baseline wing yields a drag coefficient of 0.01186, a higher value than the original rigid baseline drag coefficient of 0.01119. The drag coefficient decreased to 0.00810, a value which is not significantly lower than the result of the rigid optimization. The comments made for the rigid case

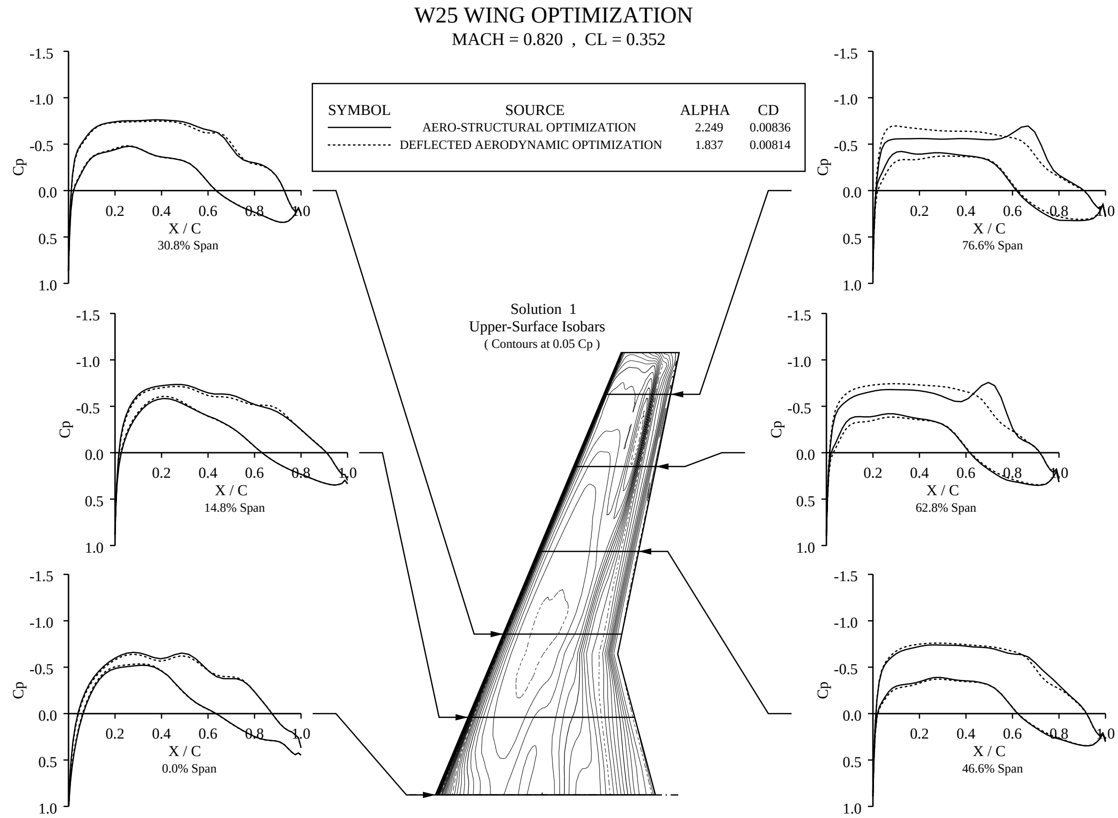

Figure 10: Wing pressure distributions for the optimized aero-structural design and the deflected optimized rigid design.

regarding the importance of the decrease in shock wave and induced drag also apply to these optimization results.

Comparison of Results

In Figure 10 we show a comparison of the pressure distributions resulting from the aero-structural optimization shown in Figure 9 and an aero-structural analysis of the optimum rigid design of Figure 8. The goal of this comparison is to emphasize that the two optimization runs will yield different results, particularly in the outboard section of the wing where the twist and bending deflections are the largest. The value of performing aero-structural optimization is especially important in unconventional structural configurations and in situations where additional design variables affecting the weight and stiffness of the structure are considered.

Conclusions

A new methodology for coupled sensitivity analysis of high-fidelity aero-structural analysis was presented. The underlying theory in this methodology is a generalization of the adjoint method for sensitivity analysis that can be applied to any multidisciplinary system. The lagged-coupled adjoint approach was proposed, that makes viable the solution of the adjoint equations for the case of high-fidelity aero-structural analysis with a large number of state and coupling variables.

The lagged aero-structural adjoint was implemented, adding sensitivity computation capability to the existing wing optimization framework. The current implementation computes the sensitivities of the wing drag coefficient with respect to a series of bump functions distributed throughout the surface of the wing.

The accuracy and cost of computation of these sensitivities using the lagged aero-structural adjoint method was compared to finite-differences and the complex-step derivative approximation. The lagged-coupled adjoint was shown to produce accurate sensitivities with a considerable reduction in computational cost. The reason for this drastic reduction is the fact that the cost of computing sensitivities using an adjoint method is not strongly dependent on the number of design variables.

Typically, hundreds of variables are used in this type of wing-shape optimization, making the coupled-adjoint approach even more attractive. The sensitivities obtained by this method were used to solve a wing optimization problem with a large number of design variables and the method was shown to be suitable for use in design.

Future work in this area of research is expected to add further capability to the multidisciplinary wing optimization framework. The sensitivity of other aerodynamic quantities will be computed and the sizes of structural elements will be added to the set of design variables. The computation of sensitivities of structural displacement and stress with respect to the design variables will also be implemented in order to add realistic constraints to the wing optimization problem.

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References

[1] H. M. Adelman and R. T. Haftka. Sensitivity Analysis of Discrete Structural Systems. *AIAA Journal*, 24(5):823–832, May 1986.

[2] K. G. Bhatia and J. Wertheimer. Aeroelastic Challenges for a High Speed Civil Transport. AIAA Paper 93-1478, Feb. 1993.

[3] S. A. Brown. Displacement Extrapolation for CFD+CSM Aeroelastic Analysis. AIAA Paper 97-1090, Jan. 1997.

[4] A. A. Giunta. A Novel Sensitivity Analysis Method for High Fidelity Multidisciplinary Optimization of Aero-Structural Systems. AIAA Paper 2000-0683, Jan. 2000.

[5] M. E. Holden. *Aeroelastic Optimization Using the Collocation Method.* PhD thesis, Stanford University, Stanford, CA 94305, 1999.

[6] A. Jameson. Aerodynamic Design via Control Theory. *Journal of Scientific Computing*, 3:233–260, 1988.

[7] J. R. R. A. Martins, 2001. http://aero-comlab.stanford.edu/jmartins.

[8] J. R. R. A. Martins, I. M. Kroo, and J. J. Alonso. An Automated Method for Sensitivity Analysis Using Complex Variables. AIAA Paper 2000-0689, Jan. 2000.

[9] J. R. R. A. Martins, P. Sturdza, and J. J. Alonso. The Connection Between the Complex-Step Derivative Approximation and Algorithmic Differentiation. AIAA Paper 2001-0921, Jan. 2001.

[10] K. Maute, M. Lesoinne, and C. Farhat. Optimization of Aeroelastic Systems Using Coupled Analytical Sensitivities. AIAA Paper 2000-0560, Jan. 2000.

[11] S. Nadarajah and A. Jameson. A Comparison of the Continuous and Discrete Adjoint Approach to Automatic Aerodynamic Optimization. AIAA Paper 2000-0667, Jan. 2000.

[12] J. Reuther, J. J. Alonso, A. Jameson, M. Rimlinger, and D. Saunders. Constrained Multipoint Aerodynamic Shape Optimization Using an Adjoint Formulation and Parallel Computers: Part I. *Journal of Aircraft*, 36(1):51–60, 1999.

[13] J. Reuther, J. J. Alonso, J. R. R. A. Martins, and S. C. Smith. A Coupled Aero-Structural Optimization Method for Complete Aircraft Configurations. AIAA Paper 99-0187, 1999.

[14] J. Reuther, J. J. Alonso, J. C. Vassberg, A. Jameson, and L. Martinelli. An Efficient Multiblock Method for Aerodynamic Analysis and Design on Distributed Memory Systems. AIAA paper 97-1893, June 1997.

[15] J. Sobieszczanski-Sobieski. Sensitivity of Complex, Internally Coupled Systems. *AIAA Journal*, 28(1):153–160, Jan. 1990.

[16] J. Sobieszczanski-Sobieski and R. T. Haftka. Multidisciplinary Aerospace Design Optimization: Survey of Recent Developments. AIAA Paper 96-0711, Jan. 1996.

[17] S. R. Wakayama. *Lifting Surface Design using Multidisciplinary Optimization*. PhD thesis, Stanford University, Stanford, CA 94305, Dec. 1994.