

Hybrid Optimization Schemes for Global Optimization: Wing Modeling of Micro-Aerial Vehicles

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Abstract

In this paper, we present a parallel hybrid algorithm for solving global optimization problems that is based on the coupling of a stochastic global (Simultaneous Perturbation Stochastic Approximation, Simulated Annealing, Genetic Algorithms) and a local method (Newton-Krylov Interior-Point) via a surrogate model. There exist several algorithms for finding approximate global solutions, but our technique will further guarantee that such solutions satisfy physical bounds of the problem. First, the Simultaneous Perturbation Stochastic Approximation (SPSA) algorithm conjectures regions where a global solution may exist. Next, some data points from the regions are selected to generate a continuously differentiable surrogate model that approximates the original function. Finally, the Newton-Krylov Interior-Point (NKIP) algorithm is applied to the surrogate model subject to bound constraints for obtaining a feasible approximate global solution. The hybrid optimization code is being applied to Stanford's UFLO Computational Fluid Dynamics (CFD) code. This code is used by the Army High Performance Computing Research Center (AHPARC) to develop a flapping and twisting wing models for Micro-Aerial Vehicles (MAV), hummingbirds-sized airborne vehicles that can be used for sensing and surveillance. We present some preliminary numerical results of the large scale HPC hybrid optimization C code that is being run in the Department of Defense MANA machine from Maui, Hawaii.

1. Introduction

A One major challenge in computational science and engineering is finding an optimal global solution for large-scale nonlinear global optimization problems. Such problems are difficult to solve due to large dense ill-conditioned operators and multiple non-optimal minima solutions among others. Global optimization is the task of finding the best set of parameters that optimize a given objective function. In general, there can be solutions that are locally optimal but not globally optimal. Consequently, global optimization problems are typically classified NP-hard in the context of combinatorial problems.

In the past ten years, global optimization has received a lot of attention due to the success of new algorithms developed for solving large classes of problems from diverse areas such as computational chemistry and biology, structural optimization, computer sciences, operations research, economics, engineering design and control, among others. Such algorithms can be classified as deterministic or stochastic. Hybrid algorithms combine the advantages of more than one computing method to optimize the solution of problems with large numbers of parameters and many solutions that are optimal only for limited ranges of a particular parameter or set of parameters (local minima). We are developing mathematical and computational tools that facilitate the implementation of problem-solving applications on highly parallel systems. We are also demonstrating a practical migration path from current programming approaches to a transaction-based model.

The mathematical side focuses on a hybrid algorithmic approach for solving general optimization problems, including automated parameter estimation problems. In particular, efforts are focused on global optimization problems having many local minima—that is, finding a set of parameters that works best over the entire region of interest from a large group of locally viable candidates.

We consider the global optimization problem in the form:

$$\underset{x}{\text{minimize}} \quad f(x),$$

where the objective function $f: \mathbb{R}^n \rightarrow \mathbb{R}$, $x \in \mathbb{R}^n$ and the global solution x^* is such that $f(x^*) \leq f(x)$, for all x .

2. Hybrid Scheme

Argaez et al. (2007, 2010) have developed an algorithm that couples a stochastic and deterministic method via a surrogate model. The hybrid scheme begins with a global stochastic technique such as the Simultaneous Perturbation Stochastic Approximation (SPSA), Simulated Annealing (SA), Genetic Algorithms (GA), and Global Levenberg-Marquadt (GLM) developed by Velazquez et al. (2001). These techniques are sampling methods that perform a global search of the parameter space. This search may start from multiple initial guesses (parallel multi-start). In many real applications, it is difficult or impossible to compute derivatives of the function being optimized. Most of these global methods do not use derivatives, and thus do not require this information in order to work. Our interest is to combine global and local

strategies for solving the global optimization problem. We now describe the main three components of our hybrid scheme as shown in Figure 1.

2.1 Global Method

First, we apply SPSA as a global method to explore the domain of the function by starting with different initial guesses. This increases the chances for finding regions where a global optimal solution may exist, and allows a rich sampling of the parameter space. SPSA is a stochastic steepest descent direction algorithm introduced by Spall (2003), where a current point is improved by moving randomly in the direction of the negative gradient of the objective function. The algorithm does not depend explicitly on derivative information. A major advantage of SPSA is that it uses only two objective function evaluations in each iteration to obtain the update parameters. Most problems require experiments and/or simulations to evaluate objective and derivate functions that involve expensive computations. The disadvantage is that the algorithm has a slow rate of convergence.

2.2 Surrogate Models

Second, we select the regions given by SPSA with the lowest function values and filter the data by eliminating points outside such regions by using a predefined radius. Then we create a surrogate model $f_s(x)$ using the selected data points $(x_i, f(x_i))$ for $i = 1, \dots, m$ that approximates the original function in this neighborhood. In our particular case, we are interested in finding a quadratic or cubic surrogate model that provides us with a smooth approximation of the objective function within the region of the most promising optimal regions explored by SPSA. The computational cost of evaluating the function and computing derivatives of the surrogate function instead of the original function is less expensive. Particularly, we approximate $f(x)$ by using a Radial Basis Function approach by Orr (1999) using the set of points computed by the SPSA algorithm.

2.3 Local Method

Finally, once the surrogate model is constructed, we can apply our local strategy Newton-Krylov Interior-Point (NKIP). The derivatives necessary to apply this strategy are available from the analytical

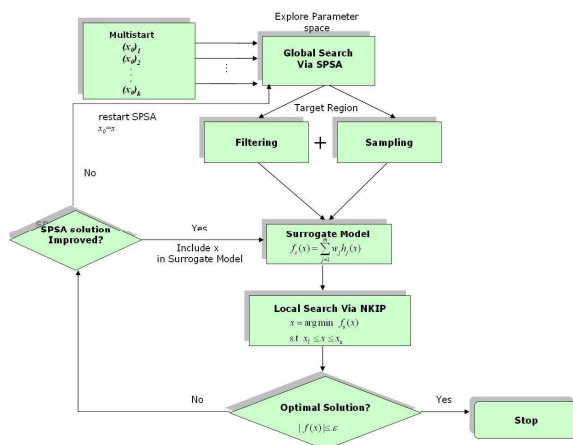


Figure 1. Hybrid algorithm scheme

representation of the surrogate model. The NKIP approach allows for further refinement of the solution yield by SPSA. Moreover, the NKIP strategy incorporates constraints associated to the problem. The solution points obtained by NKIP algorithm are validated against the original model. The NKIP algorithm is a linesearch Newton-Krylov method for solving general nonlinear programming problems. This algorithm was developed for obtaining an optimal solution for large scale and/or degenerate problems by Argáez and Tapia (2002). In this part, we want to solve:

$$\begin{aligned} &\text{minimize } f_S(x) \\ &\text{subject to } a \leq x \leq b \end{aligned}$$

3. Numerical Results and Discussion

Implementing these algorithms for a parallel computing environment requires several novel approaches. For example, introducing individual small operations as the key abstraction for expressing parallelism facilitates maintaining a computer system in a known, consistent state by ensuring that interdependent operations are either all completed successfully or all canceled successfully. We are developing a simple distributed-memory programming model that can scale to systems with thousands of processors. The C version software framework (also available in Matlab by Argáez et al. (2002)) is being tested in the Mana (Maui High Performance Computing Center, Air Force Research Laboratory) and Lonestar TACC (a Dell Linux cluster at the Texas Advanced Computing Center, University of Texas) high performance machines.

3.1 Micro-Aerial Vehicles

Hybrid optimization codes developed as a result of this work are being applied to Stanford's UFLO 2-dimensional CFD code provided and developed by Allaneau and Jameson (2010). This code is used by the AHPCRC to simulate the flapping and twisting wing models for Micro-Aerial Vehicles (MAV), hummingbird-sized airborne vehicles that can be used for sensing and surveillance. The four forces of a MAV as illustrated in Figure 2 are:

- Lift: the upward force created by the wings moving through the air that sustains the airplane in flight (can be increased by increasing the

forward speed of the aircraft or by increasing the angle of attack).

- Drag: the resistance of the airplane to forward motion. It is directly opposed to thrust and is caused by the resistance of air.
- Thrust: the force exerted by the engine. An aircraft is in a state of equilibrium when the thrust and drag are equal and opposite.
- Gravity: the weight of the aircraft.

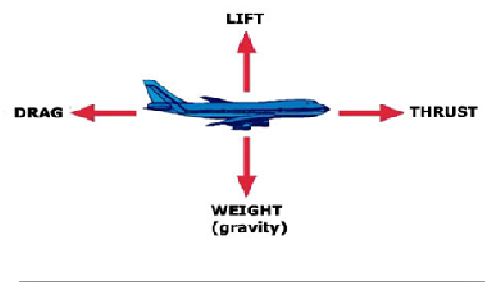


Figure 1. The four forces¹

MAV scale aircraft suffer roughly an order of magnitude loss of aerodynamic efficiency in terms of the lift-to-drag ratio compared with larger aircraft. Traditional aircraft design approaches that assume steady-state aerodynamics and rigid structures do not appear to be adequate for MAV design. However there are numerous examples of efficient flight at these scales among birds and insects. The apparent key to their success is exploitation of unsteady flow phenomenon and flexibility of the aerodynamic surfaces. Compared with the traditional approach of designing aircraft with rigid structures and for steady aerodynamics, however, designing MAV to exploit flexibility and unsteady aerodynamics will be very difficult. First, very few of the efficient computational design tools used for large aircraft design can be used in the unsteady, low Reynolds number regime, requiring costly unsteady numerical flow simulations and experiments as the primary design tools. Addressing this issue requires the development of efficient and physically accurate massively parallel

¹ Available from http://www.centennialofflight.gov/essay/Dictionary/four_forces/DI24.htm; Internet.

CFD tools that incorporate aeroelastic effects and large mesh motions associated with flapping wings.

As vehicle size gets smaller traditional fixed-wing aircraft designs become increasingly less efficient and the design process become increasingly more difficult as shown in Figures 3 and 4. The difficulty comes both from the fundamental physics of the problem and from the lack of appropriate design tools. Accurate simulation of flapping flight to obtain lift, thrust and power requires time-accurate solutions to the 2 or 3 Dimensional Navier-Stokes equations. The cost of these solutions is on the order of hundreds or thousands of CPU hours. Parameterizations of the motions and deformations of a flapping wing require on the order of tens or hundreds of design variables. Even the most efficient optimization algorithms will require hundreds or thousands of these expensive flow solutions to converge to an optimal set of parameters.

The goal is to provide the Stanford's AHPCRC group with optimal motions and deformations for a wing in periodic motion for forward and hovering flight based on averages of lift, thrust and power.

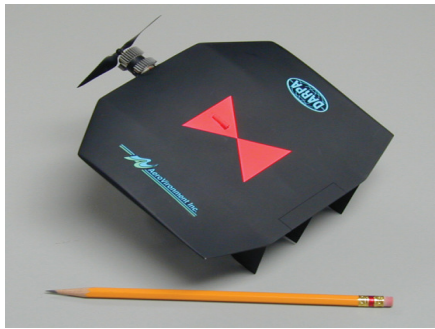


Figure 2. Current MAV²

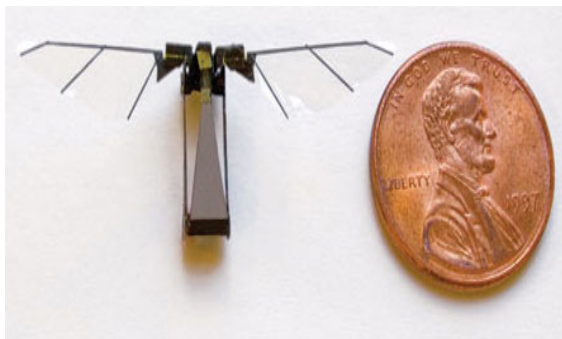


Figure 3. Future MAVs³

² Culbreth, Matt, "Flapping Wing Optimization", Princeton University, November 21, 2009.

³ Culbreth, Matt, "Flapping Wing Optimization", Princeton University, November 21, 2009.

Simulation tools such as UFLO have already gone a long way towards providing these capabilities. The second issue is that the design degrees of freedom increase significantly when considering a flexible wing in a generalized periodic flapping motion. While much has been learned from observing the flight of birds and insects, it is still far from clear how to couple wing flexibility and flapping motion in an optimal way for a given flight performance metric. This motivates the use of numerical optimization techniques coupled with unsteady flow simulations to obtain the periodic wing motions and deformations that best suit different types of flight regimes such as hovering and forward flight. Flapping wing optimization will require thousands or tens of thousands of these flow solutions, making the task essentially infeasible without massively parallel algorithms and hardware. In addition, appropriate objective functions for flapping flight are not as clear as for the steady case.

3.1 Global Optimization Problem

The flapping wing design of MAV is posed as a constrained nonlinear optimization problem:

$$\begin{aligned} & \text{minimize } f(x) \\ & \text{subject to } a \leq g(x) \leq b, \end{aligned}$$

where the design variables x parameterize the flapping motion of the wing, and the objective function f and constraints function g are time averages of integrated force metrics of lift, thrust, and power. Our goal is to maximize thrust and propulsive efficiency:

$$\eta_p = \frac{\bar{P}_{output}}{\bar{P}_{input}},$$

where P represents the power without constraints. The bar indicates that these quantities are time averages of the instantaneous power calculated at each time step. The quantities depend on the lift, thrust, frequency, and amplitude.

Essentially power output represents the power generated in the x direction, which is the direction of forward flight, and is equal to the force in the x direction times the forward speed. The force in the x direction, which we get from UFLO can either be positive or negative depending on the frequency and amplitude of motion. When the force is negative the wing is producing drag, and when it is positive it is producing thrust. The power input is the product of

the velocity in the vertical direction and force in the vertical direction. In the vertical case the velocity and the force can be either positive or negative depending on whether the wing is moving upwards or downwards, but they will always have the same sign, and so the power input will always be positive. By maximizing the ratio of the average power out over power in, we are saying that we want the most power towards propulsion and forward flight for the least power input, which would have to come from motor, batteries, as an example.

Some barriers occur when we obtain a negative efficiency that means you are producing drag and a positive efficiency means that you are producing thrust. In addition, the function can have very large negative values, but can only have positive values less than one. In fact, the function can be negative infinity for the case where amplitude is zero because power input will be zero but power output will still be the product of the drag and the velocity.

3.2 Two-Dimensional Plunging Test Problem

Plunging is the simplest thrust-producing motion and has been widely studied both experimentally and numerically.

Computations were done for a NACA 0012 airfoil oscillating in a uniform flow. Our goal is to use this testcase to maximize the propulsive efficiency of a plunging airfoil as a function of frequency and amplitude. We use the 2-dimensional UFLO Solver of Allaneau and Jameson (2010) to compute the flow around a plunging airfoil. The Stanford's group has applied the well-known software optimization tool SNOPT written by Philip Gill, Walter Murray and Michael Saunders. SNOPT is a software package for solving large-scale linear/nonlinear optimization problems. In particular, it has proven to be effective for nonlinear problems whose functions and gradients are expensive to evaluate. The functions should be smooth but need not be convex. This software has been used successfully in problems with many degrees

of freedom from the areas of engineering, economics, finance, optimal control, among others

In Table 1 we report the numerical results reported by the Stanford's group using SNOPT algorithm, and the hybrid algorithm being developed at UTEP. The first column indicates the methods being used, and the second column shows the value of the objective function obtained by each method. The last four columns indicate the values of the unknown parameters. The reduced frequency and plunging amplitude are bounded between (0,10) and (0,1), respectively. We notice numerically that the reduced frequency can be bounded above by 4 instead of 10. The zero number indicates that such variable was not involved in the optimization case (plunging motion). The call of the UFLO simulator (one function evaluation) requires 32 processors. The SNOPT was run first by optimizing two flapping cycles, and then use this optimum to restart the optimization using 10 cycles. In the case of the hybrid scheme, the solution reported is the one obtained after 10, and in particular the hybrid code uses only two function calls instead of the usual $2n$ function evaluation used to calculate a finite difference approximation of the derivatives.

The results obtained by the hybrid algorithm definitely seem feasible. The propulsive efficiency can be compared between the different parameterizations, and the fact that this case performed similarly but slightly better than the frequency/plunging amplitude case obtained by SNOPT is encouraging. When considering all the four variables, SNOPT cannot report competitive results at this moment. This might indicate that the space is multimodal for the general case. We should mentioned that in the case of the hybrid scheme, the global method SPSA provided good numerical results that were not improved by the construction of a surrogate model due to the flatness of the original function.

Table 1. Numerical comparison between SNOPT and Hybrid Scheme

Method	f = Propulsive Efficiency	x_1 = Reduced Frequency	x_2 = Plunging Amplitude	x_3 =Pitching Amplitude	x_4 =Phase
SNOPT	0.1208	2.6310	0.2520	0	0
SPSA	0.1212	2.5783	0.2783	0	0
HYBRID	0.1211	2.6064	0.2548	0	0
SPSA	0.1471	4.6118	0.1882	0.1241	0.7838

4. Conclusions

The proposed hybrid approach exploits the best of SPSA and NKIP methods for achieving maximum efficiency and robustness in the search of a feasible global solution for the 2-Dimensional Plunging Problem. It is clear that a broad exploration of the parameter space should be done to increase the chance of finding a global optimum when using SPSA. Moreover the solution provided by the hybrid method is guaranteed to be feasible with respect to physical bounds constrained by the problem when using NKIP. Also, the multi-start approach of SPSA will improve the likelihood of obtaining the feasible global solution using less CPU hours in the 2-Dimensional case. Further experiments should be conducted using high-performance computing to exploit the hybrid scheme for solving very large scale 2 and 3-dimensional problems. Finally, the use of parameterization techniques, as shown by Velazquez et al. (2008) and Hernandez IV (2010), will be incorporated in order to work into a lower dimensional space when searching for an optimal solution.

5. Significance to DOD

The hybrid scheme is providing a more feasible solution with less CPU time for generating promising airfoil shapes of future inexpensive MAV that can serve as the soldier's eyes, ears, and nose in situations that are hazardous or that require 24 x 7 attention.

Acknowledgments

This research was performed at the University of Texas at El Paso and Stanford University supported in part by the U.S. Army Research Laboratory, through the Army High Performance Computing Research Center, Cooperative Agreement W911NF-07-0027. The Army High-Performance Computing Research Center supported this project by supplying supercomputer time under the Maui High Performance Computing Center in Hawaii. The University of Hawaii is managing this for the Air Force, as a part of the Department of Defense Supercomputing Center. The authors would like to thank Yves Allaneau and Antony Jameson for allowing us to use the UFLO 2-Dimensional Flow solver.

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