The Future of Fluid Mechanics in Aircraft Design
Re-engineering Engineering
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- Aerodynamic design of aircraft
  - classical theory, wind tunnels, and numerical prediction

- Numerical optimization and automatic design

- Revolutionizing the engineering process in an integrated multidisciplinary environment
BACKGROUND

Why is Fluid Mechanics different?

- Structural analysis is routinely performed by commercial software such as MSC NASTRAN, ANSYS, PATRAN, ELFINI.
- COMPUTATIONAL FLUID DYNAMICS (CFD) remains the subject of intense development in NASA, Los Alamos, Livermore, and universities. Most aerospace companies continue to devote large resources to develop their own in-house software.
  - Boeing Tranair
  - Lockheed TEAM, Splitflow
  - Rockwell USA
Fluid flow is generally more complex than the behaviour of solids.

Impressive structures were built 5,000 years before the first flight of aircraft.

Pyramids of Egypt

Wright Flyer

Fluid mechanics is inherently NONLINEAR, and allows CHAOTIC phenomena such as TURBULENCE.
COMPLEX FEATURES OF FLUID FLOW

- Flow induced by viscosity and compressibility
- Viscous boundary layers and separation (stall)
- Unsteady vortex shedding (Karman vortex street)
- Turbulence
- Shock waves, contact discontinuities and expansion fans
Examples of Complex Flows

from

An Album of Fluid Motion

Milton Van Dyke
Because of the complexity of fluid flow, analytical theories could only be developed by drastic simplification.

CLASSICAL FLUID MECHANICS of “PERFECT FLUIDS” eliminates:
- viscosity
- compressibility

Then, the flow is IRROTATIONAL and one can set:
- \( \mathbf{q} = \nabla \phi \) (vel. vector)
- \( \text{div } \mathbf{q} = \nabla^2 \phi = 0 \) (Laplace's eqn.)
FLUID MECHANICS AND AIRCRAFT DESIGN

- This made possible the development of WING THEORY (Jowkowsky, Lanchester, Prandtl)
- Viscous effects could be estimated by BOUNDARY LAYER THEORY (Prandtl, Schlichting)
- CLASSICAL FLUID MECHANICS provided the INSIGHTS for RATIONAL QUALITATIVE DESIGN.
- This lead to the emergence of aircraft such as the DC3, SPITFIRE, P38, P51, Me262
- QUANTITATIVE ANALYSIS was impossible for complete configurations.
FLUID MECHANICS AND AIRCRAFT DESIGN

- Therefore aerodynamic design of aircraft from the WRIGHT FLYER to the BOEING 747 and F16 relied on WIND TUNNEL TESTING for detailed evaluation (the Wright brothers built a wind tunnel)
- Wind tunnel testing exceeded 25,000 hours for both the B747 and the F16
Wind Tunnel Usage to Develop Major Aircraft

![Diagram showing wind tunnel usage over time with expected trend analysis.](image-url)
Aerodynamic design is performed using:

- Computational Fluid Dynamics (CFD)
- Wind Tunnel testing

as complementary tools
LIMITATIONS OF WIND TUNNEL TESTING

- Non uniform flow
- Interference from slotted walls
- Support interference
- Model imperfections
- Model deflections under load
- Scale effects
- Reynolds no. $\frac{\rho UL}{\mu} = 100$ million (B747), 5 million (wind tunnel)
- Flow can be drastically different (shock location, separation)
Note: One complete airplane development requires about 2.5 million aerodynamic simulations.

Figure 18. Cost and Flowtime Characteristics of Wind Tunnels and CFD
TRANSONIC TESTING REQUIREMENTS

REYNOLDS NUMBER

- Large subsonic transports require 65 million for cruise simulation
Computational methods offer QUANTITATIVE PREDICTIONS of complex flows which previously could only be measured in WIND TUNNELS. They can treat both:

- COMPLEX GEOMETRY
- COMPLEX NONLINEAR EQUATIONS

PANEL METHODS for solving the classical equations for a perfect fluid (Laplace’s equation) on ARBITRARILY COMPLEX GEOMETRY appeared in 1965

Computational methods for solving the NONLINEAR flow models ranging up to the full NAVIER-STOKES equations have become available since 1970

The aircraft designer has to trade-off:

ACCU  RACY vs.  COST
TRUST LEVEL  TURN AROUND TIME

in comparison with wind tunnel data
REQUIREMENTS FOR EFFECTIVE AERODYNAMIC SIMULATION

- Sufficient and known level of accuracy
- Acceptable computational and manpower costs
- Fast turn-around time
ACCURACY REQUIREMENTS

- For aircraft design accuracy is crucial and should be in the range of 0.5%
- The drag coefficient of the B747 is in the range of 0.0275 (depending on the lift). The drag coefficient of current SST designs is in the range of 0.0150
- Thus one needs an accuracy for Cd of 0.0001 (1 count)

- Manufacturers have to guarantee performance
- Errors are very expensive in: redesign, penalty payments, and lost orders
- The MD11 was initially 7% high in fuel consumption (5% due to the engines) and lost orders
MULTIDISCIPLINARY NATURE OF CFD

Mathematics

Computational Aerodynamics

Fluid Mechanics

Computer Science

Aeronautical Engineering
CHOICE OF MATHEMATICAL MODEL

**Must** take into account:
- The suitability of the model for simulation of the expected type of flow (e.g., attached, separated, presence of strong shock waves)
- Available computing power
**NAVIER-STOKES EQUATIONS**

\[ \frac{\partial w}{\partial t} + \frac{\partial F_j}{\partial x_j} = 0 \]

**Mass equation**

\[ w = \rho, \quad F_j = \rho u_j \]

**i-momentum equation**

\[ w_i = \rho u_i, \quad F_{ij} = \rho u_i u_j + p \delta_{ij} - \sigma_{ij} \]

**Energy equation**

\[ w = \rho E, \quad F_j = (\rho E + p) u_j - \sigma_{jk} u_k - \kappa \frac{\partial T}{\partial x_j} \]

**Equation of state**

\[ p = (\gamma - 1) \rho (E - \frac{1}{2} u_i u_i) \]

**Stress tensor**

\[ \sigma_{ij} = \mu \left( \frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right) + \lambda \delta_{ij} \left( \frac{\partial u_k}{\partial x_k} \right) \]
HIERARCHY OF FLUID FLOW MODELS

Decreasing Computational Costs

IV. RANS (1990s)
III. Euler (1980s)
II. Nonlinear Potential (1970s)
I. Linear Potential (1960s)

+ Viscous
+ Rotation
+ Nonlinear

Inviscid, Irrotational
Linear

More Accurate Flow Physics
Increasing Complexity
RANGE OF APPLICABILITY OF DIFFERENT FLOW MODELS

- Navier-Stokes
- Euler
- Linear Potential
- Nonlinear Potential
- Linear Potential
- Subsonic
- Transonic
- Supersonic
- Hypersonic
COMPLEXITY OF FLUID FLOW CALCULATIONS

- At high Reynolds numbers (Re) the smallest scales of turbulence are proportional to $Re^{3/4}$ (Kolmogorov). Complexity to resolve space and time in a full Direct Navier-Stokes (DNS) calculation $\sim Re^3$. For a Boeing 747 $Re \sim 10^8$, complexity $\sim 10^{24}$.

- Inviscid flow past a complete aircraft $\sim 1/2$ million mesh cells.

- Viscous flow simulation with turbulence modeling $\sim 10$ million mesh cells to fully resolve boundary layers.

- Large Eddy Simulation (LES) with subgrid scale (SGS) modeling $\sim 10$ billion mesh cells (W. H. Jou, chief of CFD research at Boeing).
Mesh Requirement for 3D Viscous Calculation

32 cells

32 cells in the boundary layer

512 cells around the wing to limit the mesh aspect ratio (to about 1000)

Surface Mesh

512 cells spanwise

Total: $512 \times 64 \times 512 = 16,777,216$ cells
COMPLEXITY ESTIMATE FOR LES

(Due to W.H. Jou, Boeing)

- Resolve eddies 1/5 of the boundary layer thickness
  - 10 points per eddy = 50 intervals in the b.l.
- Eddies are roughly isotropic
- If b.l. thickness is 0.01 of the chord
  - 5,000 intervals chordwise
- Aspect ratio = 10
  - 50,000 intervals spanwise
- Total ~ 12.5 billion cells in the boundary layer
REVIEW OF ALGORITHMS
NUMERICAL METHODS FOR COMPRESSIBLE FLOW

In order to resolve the complex features of compressible flow such as shock waves, contact discontinuities, and slip lines it has been necessary to develop entirely new numerical methods which reflect the mathematical theory of shock waves due to P.D. Lax
FINITE VOLUME SCHEMES

Subdivide the domain into small hexahedral or tetrahedral cells

Satisfy the conservation laws in integral form

\[ \frac{\partial}{\partial t} \int_{\Omega} w dv + \int_{\partial\Omega} F \cdot dS = 0 \]

for each cell, giving the semi-discrete scheme

\[ \frac{d}{dt} (wV) + \sum_{\text{faces}} F \cdot S = 0 \]

valid in the presence of shock waves and slip lines
STRUCTURED AND UNSTRUCTURED DISCRETIZATIONS

Cell Centered Scheme

Cell-vertex Scheme
Central difference schemes produce spurious oscillations in the solution and overshoots in the vicinity of shock waves.

Oscillations can be eliminated by the use of local extremum diminishing (LED) schemes which prevent maxima from increasing and minima from decreasing. A general semi-discrete scheme in the form

$$\frac{dv_j}{dt} = \sum_{k \neq j} c_{jk} (v_k - v_j)$$

is LED if \( c_{jk} \geq 0 \), and \( c_{jk} = 0 \) if the mesh points \( j \) and \( k \) are not neighbors. This property can be enforced by upwind biasing of the difference formulas.
The **system of equations** for **gas dynamics** allows waves travelling at the speeds $u$, $u+c$, $u-c$ where $u$ is the fluid velocity and $c$ is the speed of sound.

To construct properly **upwind biased** schemes the **fluxes** may be **split** into differences of **characteristic variables** corresponding to the **different waves**. Alternatively the **convective** and **pressure** terms may be separated to produce a Convective Upwind and Split Pressure (CUSP) scheme.

Both **characteristic** and **CUSP** schemes can be formulated to produce **stationary discrete shocks** with a structure containing a **single interior point**.
TIME STEPPING SCHEMES

The semi-discrete scheme which results from the space discretization can be written as a set of ordinary differential equations

\[ \frac{d w}{dt} + R(w) = 0 \]

where w is the vector of the flow variables at the mesh points, and R(w) is the vector of residuals defined by the flux balances.

Implicit schemes of the form

\[ \omega^{n+1} = \omega^n - \Delta t \left\{ (1-\mu) R(\omega^n) + \mu R(\omega^{n+1}) \right\} \]

allow large time steps at the expense of the complexity of solving coupled equations at each time step. The time step of an explicit multistage scheme is limited by a stability restriction, but the computational cost of the time step is reduced, and the scheme is easily parallelized.
MULTIGRID ACCELERATION

Very rapid convergence to a steady state can be achieved by calculating corrections on a sequence of successively coarser meshes which are applied to the fine mesh solution, following a complex cycle.

\[ \text{E} \] = Evaluate the change in the solution

\[ \text{T} \] = Transfer the data without updating the solution

With the aid of this technique equilibrium can be simultaneously established at all scales from local to global.

The multigrid procedure can be used as an efficient method of solving the coupled equations of an implicit time stepping scheme.
REFERENCES

A more detailed discussion of numerical algorithms for CFD can be found in the following:


EXAMPLES OF FLOW CALCULATIONS

- Euler Equations
  - F23
  - SST
  - MD11

- Viscous flows with turbulence modelling
  - RAE 2822 Airfoil
  - F18
  - Transport Wing

- Unsteady Flow
  - Pitching airfoil
Plate 1: Northrop YF-23.

Supplied by R.J. Busch, Jr.
3a: Force Coefficients, Mach 2.1.

3b: Sonic Boom Prediction, Mach 2.5.

Plate 3: Supersonic Transport Calculations.
AIRPLANE code (A. Jameson and T.J. Baker).
CALCULATIONS OF COMPLETE AIRCRAFT USING UNSTRUCTURED TETRAHEDRAL MESHES

Benefits:

- Fast turn-around and reduced cost of mesh generation for arbitrarily complex configurations
- Easy to concentrate additional mesh points where needed for improved resolution
- Computing times for MD11, 350,000 mesh points, 2.1 Million tetrahedra:
  - 4.0 hours on IBM RS6000/590
  - 16 minutes on IBM SP2 (16 Processors)
### Table AIRPLANE Performance on the SP2, Mc Donnell Douglas MD11 Model

<table>
<thead>
<tr>
<th># Nodes</th>
<th>&quot;Wide&quot; Node Performance (seconds/cycle)</th>
<th>Speedup</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>36.03</td>
<td>1.00</td>
</tr>
<tr>
<td>2</td>
<td>18.11</td>
<td>1.99</td>
</tr>
<tr>
<td>4</td>
<td>9.11</td>
<td>3.96</td>
</tr>
<tr>
<td>8</td>
<td>4.66</td>
<td>7.73</td>
</tr>
<tr>
<td>16</td>
<td>2.39</td>
<td>15.08</td>
</tr>
</tbody>
</table>

Note: MD11 (Using local psmooth to reduce message passing):
Total 348,407 nodes, 2,100,466 tetrahedral elements, 75,518 faces, and 2,486,640 edges. Four stages with smoop = 1
Plate 7: Navier-Stokes Predictions for the F-18 Wing-Fuselage at Large Incidence.
Supplied by R.M. Cummings, Y.M. Rizk, L.B. Schiff, and N.M. Chaderjian.
Plate 9: Navier-Stokes Solutions for the ONERA M6 Wing using TLNS3D and CFL3D with Different Turbulence Models.

Supplied by C.L. Rumsey and V.N. Vatsa.
Mach Number Contours. Pitching Airfoil Case. $Re = 1.0 \times 10^6$, $M_\infty = 0.796$, $K_c = 0.202$.

Coefficient of Pressure on a Grid Line Just Outside of the Boundary Layer.
Plate 9: Navier-Stokes Solutions for the ONERA M6 Wing using TLNS3D and CFL3D with Different Turbulence Models.

*Supplied by C.L. Rumsey and V.N. Vatsa.*
ADVANCED CALCULATIONS OF COMPLEX VISCOUS FLOWS

- Currently not accessible tools for industrial use
  - Accuracy is uncertain
    - Turbulence modelling
    - Mesh resolution
  - Too expensive
    - Parallel computing
  - Turn-around too slow
    - Data handling
    - Geometry modelling
    - Mesh generation
Calculations of Ship Hull

Wave and Friction Resistance

Naval Vessels

Sailing Yachts
Solution of the Navier-Stokes Equations with Full Nonlinear Boundary Conditions for the Free Surface
America’s Cup Yacht
2.75 Million Mesh Points

Reynolds Number = 2 Million
Froude Number = 0.32 (9 knots)
Angle of Attack = 5 degrees
KEY ISSUES

- Accuracy - High order schemes
- Treatment of discontinuities and free surfaces - positivity
- Computational efficiency - Multigrid, Implicit schemes
- Problems with multiple scales - Stiff equations, boundary layers, non-computable scales
- Resolution of viscous effects - separation off smooth surfaces, turbulence, sub-grid modelling
- Validation - Interaction with experiments
ADVANCED CFD PAYOFFS

- Faster Design Process
  - Lower Development Cost
- Improved Design Optimization
  - Aerodynamic
    - Multi-Disciplinary
- Better Understanding of Flow Phenomena
  - Insight for Better Design
- Major Targets
  - High Lift
  - Propulsion
  - Flow with Moving Boundaries
The Future of Fluid Mechanics in Aircraft Design

Part II

Re-engineering Engineering

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THE OPPORTUNITY
GOOD SCIENCE AND ENGINEERING

- Good science and engineering requires both:
  - Technical mastery of the details ("God is in the details" - Mies van der Rohe)
  - A clear vision of the big picture

As a specialist it is very easy to become immersed in the technical details and lose the big picture
Traditional Engineering Offices
Grumman Aerodynamics Section in 1968
THE NEW SITUATION

- MODERN WORKSTATIONS such as the SGI Indigo 2 with R8000 processor and the IBM RS6000/590 offer performance near a Cray YMP for $50,000

- These can be DISTRIBUTED and LINKED both to each other and to a powerful CENTRAL SERVER. Small groups of engineers now have the power to accomplish tasks that previously required large teams.
### COMPUTING RESOURCES
**MAE DEPARTMENT - PRINCETON UNIVERSITY**

<table>
<thead>
<tr>
<th>Period</th>
<th>Machine</th>
<th>Speed (mflops)</th>
<th>Memory (mbytes)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1980-85</td>
<td>IBM 4341</td>
<td>.15</td>
<td>2</td>
</tr>
<tr>
<td>1985</td>
<td>Convex C1</td>
<td>3 (x20)</td>
<td>32 (x16)</td>
</tr>
<tr>
<td>1988</td>
<td>Convex C2</td>
<td>15 (x100)</td>
<td>256 (x128)</td>
</tr>
<tr>
<td>1995</td>
<td>IBM SP2</td>
<td>1500 (x10000)</td>
<td>6000 (x30000)</td>
</tr>
</tbody>
</table>

Avarage speed improvement:
- x20 in 5 years
- x1.8 in 1 year
COMPUTATIONAL GEOMETRY

- Computational geometry is an essential tool in modern engineering design
- Fundamental advances in COMPUTATIONAL GEOMETRY provide the MATHEMATICAL FOUNDATIONS underlying CAD systems
- Many of these developments took place in an industrial context including
  - deCasteljou algorithm - Citroen
  - Bezier curves - Renault
  - Nonuniform Rational B-Splines (NURBS) - Boeing
Current CAD systems do not produce geometric definitions with the precision required for CFD.

- In the 777 project, Boeing had to translate CATIA files into the AGPS (Automatic Grid and Panelling System) files for their CFD software. This delayed both the response of aerodynamics to changes introduced by other disciplines, and the response of the other disciplines to aerodynamic modifications.
EARLY IMPACT ON THE DESIGN

The effective exploitation of computational methods requires their use at the stages of conceptual and preliminary design, before the design is frozen.

Because of lengthy turnaround and cost, current computational methods are often only brought into play at a late stage of detailed design. This limits their use to a verification tool.
Early use of computational methods in the design process allows the exploitation of their power to vary the geometric shape and find the optimal solution over a range of design alternatives.
The objective is to develop an advanced computational environment for aerodynamic design to provide fast, cost-effective, computational analysis and optimization capabilities at an early stage in the design cycle. The principal requirements are as follows:

- Analysis turn-around of half an hour for a full aircraft configuration
- Geometry manipulation via a CAD system with access to a central database
- Automated optimization of the design
- Multi-disciplinary analysis
The goal of CFD is captured in the idea of a Numerical Wind Tunnel.

To be properly effective numerical solution methods must be integrated with a central database and a geometry management system.
NUMERICAL WIND TUNNEL CONCEPT

* MDO: Multi-Disciplinary Optimization
NUMERICAL WIND TUNNEL WITH INTEGRATED DESIGN ENVIRONMENT

Initial Design -> Master Definition Central Database

Requirements -> High-Level Redesign -> Monitor Results

Automatic Mesh Generation
- Flow Solution
- Aeroelastic Solution
- Loads

Geometry Modification
- Optimization
- Quantitative Assessment

Human Intensive
Numerically Intensive
OPPORTUNITIES FOR BETTER DESIGN

- Design improvements can be realized via optimization and control theory leading to automatic design of some components
- Disciplines can be united through a shared database allowing concurrent engineering and rational trade-offs
LEVELS OF CFD CAPABILITY FOR DESIGN

- Capability to predict the flow past an airplane in different flow regimes
  - Takeoff
  - Cruise (transonic)
  - Flutter
  - Present turnaround: weeks

- Interactive design calculations to allow immediate improvement

- Automatic design optimization
MULTI-DISCIPLINARY OPTIMIZATION (MDO)

- Ideally the designer needs effective multi-disciplinary optimization (MDO) methods to make the best trade-offs between aerodynamics, structural, and other requirements while meeting constraints such as take-off and landing field length.

- If the disciplines are represented by over-simplified models of insufficient accuracy the results of MDO can be very misleading. The quality of the result is no better than the quality of the least accurate model.
AUTOMATIC SHAPE DESIGN VIA CONTROL THEORY

- Apply the theory of control of partial differential equations (of the flow) by boundary control (the shape)

- Find the Frechet derivative (infinite dimensional gradient) of the drag (or other performance measure) with respect to the shape by solving and adjoint equation in addition to the flow equation

- Modify the shape in the sense defined by the smoothed gradient

- Repeat the iterations until the performance approaches an optimum value
Aerodynamic Shape Optimization of Wing and Wing-Body Configurations Using Control Theory

J. Reuther
RIACS/NASA Ames
A. Jameson
Princeton University
AIAA Paper 95-0123
January 1995
Design Using Control Theory

Let $I$ be the **cost** (or **objective**) function

$$I = I(w, \mathcal{F})$$

where

$$w = \text{flowfield variables}$$

$$\mathcal{F} = \text{grid variables}$$

The **first variation** of the cost function is

$$\delta I = \frac{\partial I^T}{\partial w} \delta w + \frac{\partial I^T}{\partial \mathcal{F}} \delta \mathcal{F}$$

The **flowfield equation** and its **first variation** are

$$R(w, \mathcal{F}) = 0$$

$$\delta R = 0 = \left[ \frac{\partial R}{\partial w} \right] \delta w + \left[ \frac{\partial R}{\partial \mathcal{F}} \right] \delta \mathcal{F}$$
Introducing a **Lagrange Multiplier**, $\psi$, and using the **flowfield equation** as a constraint

$$
\delta I = \frac{\partial I^T}{\partial w} \delta w + \frac{\partial I^T}{\partial F} \delta F - \psi^T \left\{ \frac{\partial R}{\partial w} \delta w + \frac{\partial R}{\partial F} \delta F \right\}
$$

$$
= \left\{ \frac{\partial I^T}{\partial w} - \psi^T \left[ \frac{\partial R}{\partial w} \right] \right\} \delta w + \left\{ \frac{\partial I^T}{\partial F} - \psi^T \left[ \frac{\partial R}{\partial F} \right] \right\} \delta F
$$

By choosing $\psi$ such that it satisfies the **adjoint equation**

$$
\left[ \frac{\partial R}{\partial w} \right]^T \psi = \frac{\partial I}{\partial w},
$$

we have

$$
\delta I = \left\{ \frac{\partial I^T}{\partial F} - \psi^T \left[ \frac{\partial R}{\partial F} \right] \right\} \delta F
$$

This reduces the **gradient** calculation for an arbitrarily large number of design variables to

**One Flow Solution**

+ **One Adjoint Solution**
Application to the 3D Euler Equations

Define the Euler equations as

\[ \frac{\partial w}{\partial t} + \frac{\partial f_i}{\partial x_i} = 0 \]

where

\[ w = \begin{bmatrix} \rho \\ \rho u_1 \\ \rho u_2 \\ \rho u_3 \\ \rho E \end{bmatrix}, \quad f_i = \begin{bmatrix} \rho u_i \\ \rho u_i u_1 + p \delta_{i1} \\ \rho u_i u_2 + p \delta_{i2} \\ \rho u_i u_3 + p \delta_{i3} \\ \rho u_i H \end{bmatrix}. \]

Making a transformation to general coordinates, they become

\[ \frac{\partial W}{\partial t} + \frac{\partial F_i}{\partial \xi_i} = 0 \]

\[ W = J \begin{bmatrix} \rho \\ \rho u_1 \\ \rho u_2 \\ \rho u_3 \\ \rho E \end{bmatrix}, \quad F_i = J \begin{bmatrix} \rho U_i \\ \rho U_i u_1 + p \frac{\partial \xi_i}{\partial x_1} \\ \rho U_i u_2 + p \frac{\partial \xi_i}{\partial x_2} \\ \rho U_i u_3 + p \frac{\partial \xi_i}{\partial x_3} \\ \rho U_i H \end{bmatrix}. \]
The cost function for the inverse problem can be defined on the wing

\[ I = \frac{1}{2} \int_{B_W} (p - p_d)^2 \, d\xi_1 d\xi_3, \]

Taking the first variation of the cost function

\[ \delta I = \int_{B_W} (p - p_d) \, \delta p \, d\xi_1 d\xi_3. \]

Taking the first variation of the governing equations

\[ \frac{\partial}{\partial \xi_i} (\delta F_i) = 0, \]

where

\[ \delta F_i = \left[ \frac{\partial f_j}{\partial w} \right] J \frac{\partial \xi_i}{\partial x_j} \delta w + \delta \left( J \frac{\partial \xi_i}{\partial x_j} \right) f_j. \]

Set

\[ C_i = \left[ \frac{\partial f_j}{\partial w} \right] J \frac{\partial \xi_i}{\partial x_j}, \quad Q_i = \delta \left( J \frac{\partial \xi_i}{\partial x_j} \right) f_j. \]

Multiplying by a Lagrange Multiplier \( \psi \) and integrating by parts gives

\[ \int_D \frac{\partial \psi^T}{\partial \xi_i} C_i \delta w d\xi_i + \int_D \frac{\partial \psi^T}{\partial \xi_i} Q_i d\xi_i + \int_B (\bar{n}_i \psi^T \delta F_i) d\xi_j = 0 \]
On the boundary

\[
\delta F_2 = J \begin{bmatrix}
0 \\
\frac{\partial \xi_2}{\partial x_1} \delta p \\
\frac{\partial \xi_2}{\partial x_2} \delta p \\
\frac{\partial \xi_2}{\partial x_3} \delta p \\
0
\end{bmatrix} + p \begin{bmatrix}
0 \\
\delta (J \frac{\partial \xi_2}{\partial x_1}) \\
\delta (J \frac{\partial \xi_2}{\partial x_2}) \\
\delta (J \frac{\partial \xi_2}{\partial x_3}) \\
0
\end{bmatrix}
\text{ on } B_W
\]

Choose \( \psi \) to satisfy the adjoint equation

\[
\frac{\partial \psi}{\partial t} - C_i^T \frac{\partial \psi}{\partial \xi_i} = 0 \quad \text{in } D,
\]

with the boundary condition

\[
J \left( \psi_2 \frac{\partial \xi_2}{\partial x_1} + \psi_3 \frac{\partial \xi_2}{\partial x_2} + \psi_4 \frac{\partial \xi_2}{\partial x_3} \right) = (p - p_d) \quad \text{on } B_W.
\]

The cost function thus becomes

\[
\delta I = -\int_D \frac{\partial \psi^T}{\partial \xi_i} \delta \left( J \frac{\partial \xi_i}{\partial x_j} \right) f_i d\xi_k
\]

\[
-\int_{B_W} \left\{ \psi_2 \delta \left( J \frac{\partial \xi_2}{\partial x_1} \right) + \psi_3 \delta \left( J \frac{\partial \xi_2}{\partial x_2} \right) + \psi_4 \delta \left( J \frac{\partial \xi_2}{\partial x_3} \right) \right\} p d\xi_1 d\xi_3
\]
J WING 5585 DESIGN

Cl 0.550 Mach 0.850
10 Cycles
Initial Cd 0.0236
Final Cd 0.0119

192 x 32 x 48 = 294,912 Grid Cells
33 x 128 = 4224 Design Variables

Computational Cost
IBM 530 Workstation 15.0 hours
IBM 590 Workstation 2.5 hours
Convex C220 (1 CPU) 7.5 Hours
Figure 9: FLO67 check on initial wing.
$M = 0.85, C_L = 0.5506, C_D = 0.0236, \alpha = -1.260^\circ$. 
Figure 10: FLO67 check on redesigned wing.

$M = 0.85$, $C_L = 0.5500$, $C_D = 0.0119$, $\alpha = 0.210^\circ$. 
For a long range transport aircraft:

- Wing $C_d$ 0.0109
- Vortex
- Shock wave 0.0010
- Friction 0.0045
- Fuselage 0.0050
- Tail 0.0020
- Nacelles 0.0015
- Excreences 0.0010
- Total $C_d$ 0.0259
- $C_l$ 0.5500
- $L/D$ 21.2
Pressure Maps

Upper Surface, $M = 0.81$, Alpha Sweep

5 deg.
4 deg.
3 deg.
alpha = 2 deg.
6 deg.
7 deg.
8 deg.
9 deg.
flow
FACTORS WHICH MAKE RE-ENGINEERING OF THE DESIGN PROCESS POSSIBLE AND NECESSARY

- Advances in CAD
- Advances in numerical and mathematical solution methods
  - Increased computing power and networking
  - Shared database
- Numerically controlled machine tools and computer control of manufacturing
- Fast prototyping
Computational engineering is emerging as the new controlling discipline across a variety of industries:

- Aerospace
- Automotive
- Shipbuilding
The engineering process needs to be radically reformulated to take full advantage of advances in:

- Distributed high performance computing power and techniques
- Information technology
- Smart manufacturing

Equipped with workstations of Cray YMP power a group of 3 or 4 engineers can carry out design analyses that previously might have required a team of 50.

- We can now coordinate the disciplines through a shared database
- Automatic optimization techniques and fast prototyping can drastically accelerate the design cycle
MULTI-DISCIPLINARY AERODYNAMIC DESIGN ENVIRONMENT

AOM  Aerodynamic Optimization Module
CAD  Computer Aided Design
CFD  Computational Fluid Dynamics
GUI  Graphic User Interface
IPCP  Inter-Process Communication Protocol
MDO  Multi-Disciplinary Optimization
MSD  Master Surface Definition
Companies which re-engineer the engineering process for integrated multidisciplinary design will gain a decisive market advantage.

Companies which fail to recognize the new situation may be at a fatal disadvantage.

Universities which fail to respond to these fundamental changes taking place in the engineering process will lose their prominence.
Computational geometry is an essential tool in modern engineering design. Fundamental advances in computational geometry provide the mathematical foundations underlying CAD systems. Many of these developments took place in an industrial context including de Casteljau algorithm, Bezier curves, and Nonuniform Rational B-Splines (NURBS).
BOTTLENECK IN CURRENT USE OF CFD FOR DESIGN

- Current CAD systems do not produce geometric definitions with the precision required for CFD.
  - In the 777 project, Boeing had to translate CATIA files into the AGPS (Automatic Grid and Panelling System) files for their CFD software. This delayed both the response of aerodynamics to changes introduced by other disciplines, and the response of the other disciplines to aerodynamic modifications.
EARLY IMPACT ON THE DESIGN

- The effective exploitation of computational methods requires their use at the stages of **conceptual** and **preliminary design**, before the design is **frozen**.
- Because of lengthy **turnaround** and **cost**, current computational methods are often only brought into play at a **late stage** of detailed design. This limits their use to a verification tool.
Early use of computational methods in the design process allows the exploitation of their power to vary the geometric shape and find the optimal solution over a range of design alternatives.