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APPLICATION OF OPTIMIZATION TECHNIQUES TO THE LATERAL CONTROL OF THE E-2A

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SUMMARY*

A hypothetical redesign of the lateral directional stability augmentation system for the Grumman E-2A (Hawkeye) aircraft is described. The major importance of this work is the mathematical formulation and computational solution of a general problem. Due to safety of flight considerations, one of the basic tradeoffs is between structural weight and control authority. The measure of a limited authority controller is defined in terms of reducing unfavorable aircraft reactions without degrading response. In these terms, the best controller can be found as the solution of the optimal regulator problem. The results presented here demonstrate the utility of optimal control theory as a design aid. However, classical theory can only indirectly handle such real world constraints as:

1) Fixed feedback control structure

2) Constant feedback gains for short time intervals

3) Best compromise feedback gains for several flight conditions

In conclusion, the stage is set for dealing directly in the future with the constrained problem.

INTRODUCTION

The objective is to find the most efficient way of determining a practical automatic flight control system which will provide:

1) Rapid and precise response to the pilot's commands

2) Acceptable stability in the Dutch roll, spiral and roll subsidence modes

It is assumed that a desirable trajectory of the aircraft is known for a representative maneuver. The problem can be given an explicit mathematical form, which will open it to an organized attack, by using some measure of the deviation between the actual and desired trajectories as an index of performance. The system can then be optimized against this measure. This amounts to a simultaneous attack on the problems both of stabilization and control, because the input which causes the maneuver should also excite any unstable modes, and the resulting oscillations will lead to greater deviations from the desired trajectory, and a larger value of the performance index. The particular system which is found to be optimal will depend on the choice of the performance index, and optimization is here introduced not with the aim of finding a unique 'best' system, but rather as a means of guiding the calculation. The question is whether a performance index can be found for which the corresponding optimal system satisfies all the criteria embodied in objectives 1 and 2.

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In formulating the problem it is necessary to consider the amount of complexity to be allowed in the configuration of the control system. If, for example, the criterion of merit is taken to be speed of response, then it can be shown that the optimal system is a bang-bang system, which would require on board computation of switching times. In general, one can either try to choose the performance index in such a way that the optimal system is within the desired class of configurations, or one can attach specific constraints on the type of configuration. It is assumed here at the outset that the desired configuration is a linear control system in which the pilot's commands are modified by feedback signals from measurements of the aircraft's motion. It is assumed also that it is desired to use constant feedback gains in the short period, though the gains may be altered with the flight condition. Rather than trying to mechanize a particular optimal system, one may, of course, prefer to use the insight gained from knowledge of optimal systems to design a simpler system.

CONCLUSIONS AND RECOMMENDATIONS

A good method of finding an acceptable system, with performance as good as or better than that obtainable by traditional methods, is to calculate the optimal system for each flight condition, and then to try to simplify it by eliminating some feedbacks and fixing other feedbacks. This method has the advantages that:

1) It is very rapid

2) It is possible to trade different qualities, such as suppression of sideslip and speed of response, by altering the coefficients in the performance index used in the determination of the optimal system

A natural extension is to try to optimize the parameters of the simplified system. This is unfortunately more difficult than the calculation of the free optimal system; the elimination of feedbacks amounts to a constraint on the problem when it is formulated mathematically. Current efforts are concentrating on solving this problem by computerized search. The recently developed accelerated gradient methods give useful approximations to the exact solution, Reference 7.

DISCUSSION

Kalman Method

The most convenient optimization technique is the Kalman method for linear systems with an integral quadratic performance index. The method treats multi-input multi-output and single-input single-output systems in a uniform manner, providing a solution to the regulator problem; that is, the transfer of the system from an arbitrary initial state to equilibrium. The solution is easy to compute, requiring the integration of the matrix Riccati equation, which is well behaved; the equations are the dual of the equations for determining an optimal filter for noisy measurements. The system is assumed to be described by a set of first order equations for 'state variables', in matrix notation:

$$\dot{x} = Ax + Bu$$

where x is a vector representing the state of the system and u is the input or control vector. If it is desired to control an output vector

$$y = Cx$$

the performance index is taken to be a quadratic measure of the output, with a penalty added for the amount of control effort:

$$J = \int_0^T (y^T Q y + u^T R u)\,dt$$

where Q and R are positive definite weighting matrices.

Provided that the system is:

a) Completely controllable by the input

b) Completely observable by the output

the principal properties of the method are:

1) The solution is a closed loop system using feedbacks from every state variable (though it may also be interpreted as giving the optimal history of the controls of an open loop system).

2) In general the optimal gains vary with time, but if the system dynamics are constant and the performance index is measured over an infinite time interval, they are constant and independent of the trajectory. These constant gains are gains determined as the asymptotic values approached in the integration of the matrix Riccati equation.

3) The resulting system is stable.

4) The solution for the control signal is a global optimum.

5) Judged by a suitable quadratic measure, the closed loop system is less sensitive to variations of the system parameters than the equivalent open loop system (this is a generalization of the classical Bode sensitivity criterion for a single-input single-output system).

A system is defined to be completely 'controllable' if it can be brought into an arbitrary state by the use of the controls in a finite time interval. If the system equations are transformed to normal form, with a separate equation for each mode, then the system is completely controllable only if each mode is controllable. If there are some uncontrollable modes, then only the controllable part of the system can be stabilized. A system is defined to be completely 'observable' if its state can be determined by measuring only the output variables for a finite time. If the equations are in normal form, the system is completely observable only if every mode affects at least one output variable. If some modes are not represented in the output, then instability of one of these modes would not be reflected by an increase in the performance index. Properties 1-4 were proved by Kalman who introduced the concepts of controllability and observability, Reference 1. The definition of controllability and observability in terms of normal modes is due to Gilbert, Reference 3. Property 5 has been proved by Kreindler, Reference 4. It has also been proved that if the closed loop system is less sensitive than the equivalent open loop system, then it must be optimal for some quadratic performance index, References 2 and 5.

Limits of control authority cannot be met directly. The magnitudes of the gains, and hence of the actual control signals for a particular trajectory, depend on the penalty on the control signals which is included in the performance index. A penalty which results in signals within given limits has to be found by trial and error. Thus, although the feedbacks are independent of the trajectory for a particular performance index (property 3), the appropriate performance index may in fact depend on the trajectory. Without some penalty on the control signals the solution is singular, indicating that an arbitrary trajectory can be obtained by applying infinite control force.

If the system is to be optimized by reference to a desirable model, supplementary equations can be used to introduce the model as an uncontrollable part of the system in parallel with the basic plant, and the output variables can be formed as differences between the plant variables and the model variables. In this way the problem is reduced to the regulator problem treated by Kalman.

It is to be expected that an optimal system would require feedbacks from every state variable (property 1), because this is the minimum amount of information needed to predict the trajectory in the absence of further control.

Application of the Kalman method to the lateral control problem

The present problem can be brought within the scope of the Kalman method provided that the linear perturbational equations give a sufficiently accurate description of the aircraft. Written as a set of first order equations, these are:

$$\begin{bmatrix} \dot{\beta} \\ \dot{p} \\ \dot{\phi} \\ \dot{r} \end{bmatrix} = \begin{bmatrix} \frac{Y_\beta}{V} & \frac{Y_p}{V} & \frac{g}{V} & \frac{Y_r}{V} - 1 \\ \frac{\ell_\beta + An_\beta}{1-AB} & \frac{\ell_p + An_p}{1-AB} & 0 & \frac{\ell_r + An_r}{1-AB} \\ 0 & 1 & 0 & 0 \\ \frac{n_\beta + B\ell_\beta}{1-AB} & \frac{n_p + B\ell_p}{1-AB} & 0 & \frac{n_r + B\ell_r}{1-AB} \end{bmatrix} \begin{bmatrix} \beta \\ p \\ \phi \\ r \end{bmatrix} + \begin{bmatrix} \frac{Y_{\delta r}}{V} & \frac{Y_{\delta a}}{V} \\ \frac{\ell_{\delta r} + An_{\delta r}}{1-AB} & \frac{\ell_{\delta a} + An_{\delta a}}{1-AB} \\ 0 & 0 \\ \frac{n_{\delta r} + B\ell_{\delta r}}{1-AB} & \frac{n_{\delta a} + B\ell_{\delta a}}{1-AB} \end{bmatrix} \begin{bmatrix} \delta r \\ \delta a \end{bmatrix} \tag{1}$$

where β, p, ϕ, and r are the sideslip angle, roll rate, roll angle and yaw rate, δr and δa are the rudder and aileron angles, and Y_β, ℓ_β, n_β ....are the side acceleration, rolling acceleration and yawing acceleration due to β, p, r, δr, and δa. Also A and B are ratios of the moments of inertia with respect to stability axes:

$$A = \frac{I_{xz}}{I_{xx}}, \quad B = \frac{I_{xz}}{I_{zz}}$$

(See Appendix)

The rudder and aileron angles are here treated as state variables, and the control variables are taken to be the signals to the actuators. The actuator equations can be approximated by first order lags from the control signals $u_{\delta r}$ and $u_{\delta a}$:

$$\dot{\delta r} = -\omega_{\delta r}\delta r + \omega_{\delta r}u_{\delta r}$$

$$\dot{\delta a} = -\omega_{\delta a}\delta a + \omega_{\delta r}u_{\delta a} \qquad (2)$$

A more accurate representation of the actuators could be used, but since the optimal control system will modify the actuator characteristics by introducing feedbacks from the actuator variables, the additional information would be difficult to interpret.

Some equations must be added to define the input and desired trajectory. These are needed to formulate the problem as a regulator problem. Their introduction restricts the input to the class of functions which satisfy differential equations with constant coefficients. A representative pilot input can be constructed as the solution of a second order equation, or of two first order equations for the wheel angle δw and its rate rw. The desired roll rate p_D is taken to be directly proportional to the wheel angle; that is,

$$p_D = K\delta w$$

where K is a constant to be selected. In practice the aircraft can respond only after a time lag: this choice amounts to a requirement that its response should both be proportional to the input and as fast as possible. To keep track of the desired roll angle ϕ_D we also introduce it explicitly as the integral of p_D. We thus add the equations

$$\dot{\phi}_D = K\delta w$$

$$\dot{\delta w} = rw$$

$$\dot{rw} = -\omega_1\omega_2\delta w - (\omega_1 + \omega_2)rw$$

The particular input and hence the maneuver are determined by the coefficients ω_1 and ω_2 and the initial conditions for ϕ, ϕ_D, δw and rw. These extra equations amount simply to a command generator: a more elaborate model with, for example, a time lag, could easily be introduced.

Since we wish to execute a roll while preventing the occurrence of yawing oscillations, which would lead to large sideslip angles, we take as output variables β, $p - p_D$, and $\phi - \phi_D$. This leads to the performance index

$$J = \int_0^T \left[Q_1 \beta^2 + Q_2 (\phi - \phi_D)^2 + Q_3 (p - p_D)^2 + R_1 u_{\delta r}{}^2 + R_2 u_{\delta a}{}^2 \right] dt \qquad (4)$$

By varying the weights Q_1, Q_2, Q_3, R_1 and R_2 we can vary the emphasis on different qualities and limit the magnitude of the control signals. The solution calls for feedback signals from β, p, ϕ, r, δr, and δa, combined with feedforward signals formed from the pilot input δw, its rate rw, and its integral ϕ_D. This gives the block diagram shown in fig. 1. It should be noted that the side accelerometer reading a_y and the sideslip are related by:

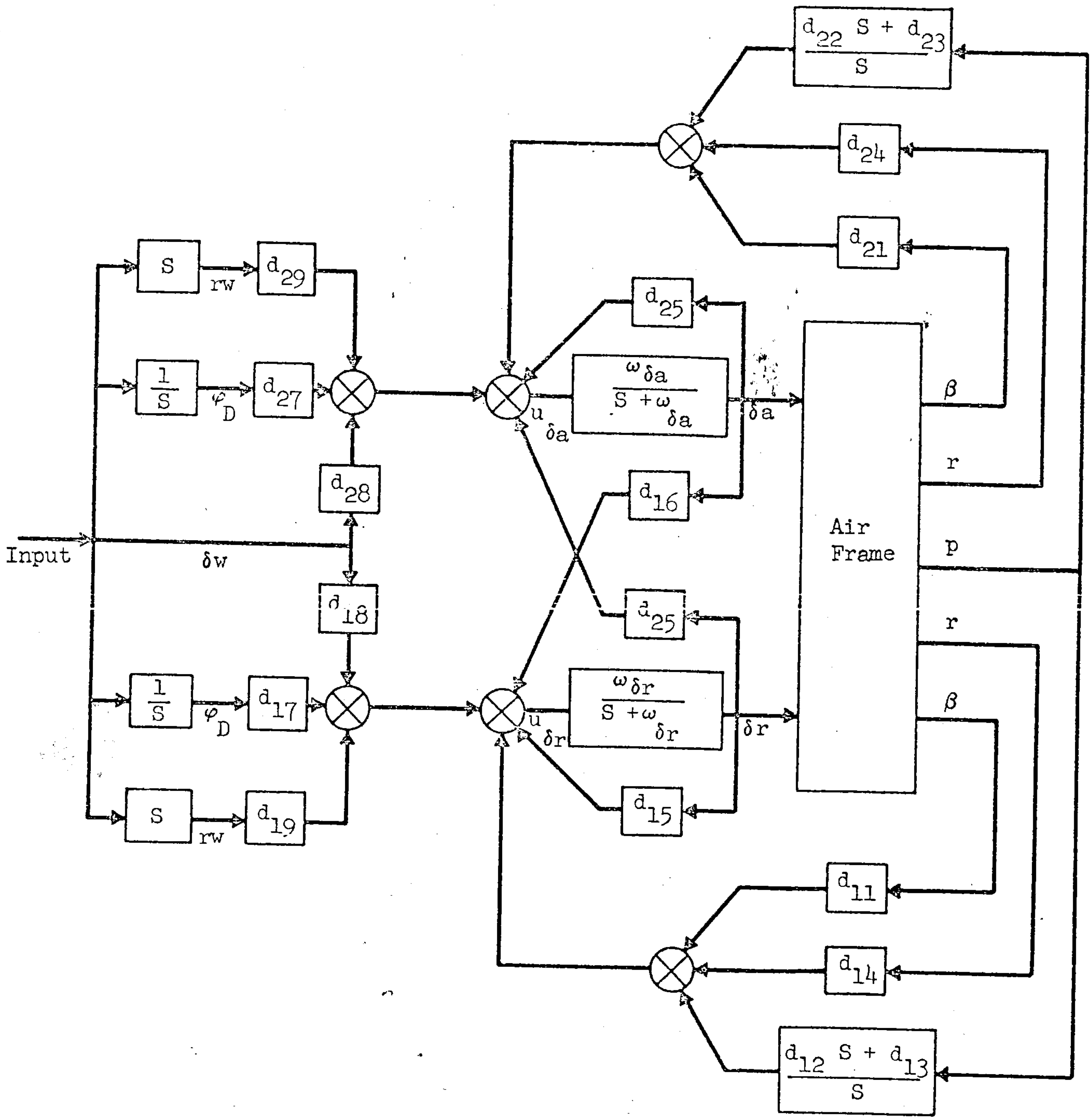

Figure 1 Optimal Control System of Unconstrained Configuration for an Input of the Form $rw\ (s) = \frac{1}{(S + \omega_1)\ (S + \omega_2)}$

$$a_y = \frac{1}{g}\left[\left(Y_\beta \beta + Y_p\ p + Y_r\ r + Y_{\delta r}\ \delta r + Y_{\delta a}\ \delta a \right)\right] \tag{5}$$

Thus feedbacks from β, p and r are equivalent to feedbacks from a_y, p and r, δr and δa, and this configuration could be mechanized by measuring a_y instead of β. The optimal feedforward gains depend on the input. It can, however, be shown by writing the matrix Riccati equation in partitioned form, that the optimal feedback gains are independent of the input, and are influenced only by the choice of the performance index (and hence indirectly by the authority limits).

To compute the solution within this framework we have developed an optimization program which:

(1) Sets up the lateral equations of motion, given a description of the airframe and flight condition

(2) Finds the roots of the open loop system, sets up the equations in normal form, and checks the controllability and observability of the system

(3) Integrates the matrix Riccati equation for a prescribed optimization interval and calculates the corresponding set of constant feedbacks

(4) Finds the roots of the closed loop system

(5) Calculates the trajectory of the system for a given input

(6) If the peak control signals are outside specified limits, alters the performance index and repeats (3) and (5) until a solution is found within these limits.

Using an IBM 360-75, the program takes about 7 seconds to integrate the matrix Riccati equation over a period of 10 seconds in a typical case. We have also developed a program for the analysis of a system with a given set of feedbacks, not necessarily optimal, which:

(1) Sets up the lateral equations of motion

(2) Finds the roots of the closed loop system

(3) Calculates the trajectory for a given input (possibly not continuous)

(4) Calculates the time dependent statistics of the response to a statistical input (if desired).

This program takes about 2 seconds to calculate the trajectory over a period of 10 seconds, and will plot via the high speed on-line printer all the state, output and control variables in another 4 seconds, if this is required. The two programs can be used together, first to determine optimal control systems for different choices of performance index, and then to compare these with other simpler non optimal systems.

Optimal Control Systems for the E-2A

The E-2A aircraft was selected for study because its low speed handling qualities have typical limitations due to design constraints. This is true despite the use of a fairly complex stability augmentation system.

The optimization program was used to determine the optimal feedback and feedforward gains for the E2A in 5 flight conditions:

(1)	Power Approach, 1.4 Vspa	(1 PA)
(2)	Cruise at 10000 feet, 1.4 Vscr	(2 CR 10000)
(3)	Cruise at 30000 feet, 1.4 Vscr	(3 CR 30000)
(4)	Power at 10000 feet, .8 Vh	(4 P 10000)
(5)	Power at 30000 feet, .8 Vh	(5 P 30000)

Data for the E2A in these conditions was taken from the appendix to Reference 6, Table B, and is reproduced in the Appendix. The control actuators were represented by first order lags with bandpass frequencies of 10 radians/sec for the rudder and 20 radians/sec for the ailerons. The input was standardized as a demand for a roll from -45° to $+45^\circ$ with a peak roll rate as near as possible to 45°/second; a rate only, in fact, attainable in flight conditions with a fairly high dynamic pressure. Using a gain constant K=1 between the control wheel angle δw and the desired roll rate p_D, this maneuver is obtained by setting the initial control wheel rate rw(0) to 180°/sec and $\omega_1 = 1, \omega_2 = 2$ in equations (3). The authority of the stability augmentation system over the rudder is limited to $\pm 10^\circ$ during the power approach and up to maximum of 160 knots EAS, $\pm 3^\circ$ up to a maximum of 220 knots EAS, and $\pm 2^\circ$ at higher speeds. Only in condition 4 is the equivalent airspeed greater than 220 knots. The limits of the deflection of each aileron are -25° and $+15^\circ$ during the power approach, and -18° and $+15^\circ$ in the other flight conditions. It is possible to optimize the control of both the rudder and the aileron, or to assume that the pilot's command is fed directly to the aileron, and to optimize the control of the rudder only. Both approaches were tried.

When the control system has authority over both the rudder and the aileron, it was found to be sufficient to integrate the matrix Riccati equation over a period of 10 seconds to obtain convergence to a final answer. Tables 1-3 show the optimal gains for the different flight conditions with three choices of the weighting coefficients in the performance index (4). For all three tables the same coefficients $Q_2 = 1$ and $Q_3 = 1$ were used for the roll rate and roll angle errors $p-p_D$ and $\phi - \phi_D$. Also, it was found that by fixing the coefficients for the penalties on the rudder and aileron signals at $R_1 = 30$ and $R_2 = 10$, the control signals can be kept within the authority limits during the test rolling maneuver from -45° to $+45^\circ$. By raising the coefficient Q_1 for the sideslip angle β from 10 Design 1 (Table 1) to 100 Design 2 (Table 2) to 1000 Design 3 (Table 3) systems can be generated which progressively eliminate sideslip at the expense of a slight reduction in the speed of response. This is illustrated in Figure 2 in which trajectories resulting from these three choices of Q_1 are overplotted for a single flight condition, the cruise at 10,000 feet. The first three plots show the achieved sideslip, roll rate, and bank histories and the corresponding ideal histories. The next three plots illustrate the resulting histories of yaw rate, rudder, and aileron for the three designs. The remaining two plots show the prescribed wheel rate and wheel deflection histories. Note that the desired roll rate history is taken to be the wheel deflection history.

Table 4 contains the relevant peak and final values of the aircraft state variables for all five flight conditions to indicate the trade for increasing values of gain. The system implied by $Q_1 = 100$ was selected because it suppresses unwanted sideslip without gains so high as to result in large signals in any of the individual feedback paths. This is desirable so that certain paths can be safely eliminated either by design or as a result of failures. The curves for the other flight conditions were similar to those of Figure 2.

Table 5 compares the characteristic roots of the free airframe with those of the dual input optimal closed loop system for $Q_1 = 100$. The spiral mode of the free airframe is generally unstable, its roll subsidence mode is stable, and its Dutch roll is lightly damped with a natural frequency roughly proportional to the square root of dynamic pressure. The optimal control system stabilizes the spiral mode for every flight condition, increases the rate of roll subsidence, and yields a well damped Dutch roll with a damping factor .6 and a damped natural frequency from $1\frac{1}{2}$ to 2 times that of the free airframe. The assumed bandpass frequencies of the actuators are much higher than both the aircraft natural frequency and the highest expected frequency component of the pilot's input, thus the actuator characteristics were hardly altered by the introduction of feedbacks.

For the study of optimization of the rudder circuit alone, a connection coefficient from the input to the aileron was chosen so that the aircraft would roll approximately from -45° to $+45^{\circ}$ while all the feedbacks to the aileron were deleted. The coefficient Q_1 in the performance index was fixed at the value of 100 already found to give desirable results. The roll rate and roll angle errors and their corresponding coefficients Q_2 and Q_3 were eliminated from the performance index because the rudder should not be used to control the roll angle. The coefficient for the rudder signal was fixed at $R_1 = 30$. It was found that the matrix Riccati equation now only converged after integration over a much longer period, on the order of 100 seconds. Although the aircraft meets Kalman's requirements of controllability, longer optimization intervals are apparently required because the slow spiral mode is only weakly represented in the performance index when neither the roll angle nor the roll rate is included as an output variable. For an optimization interval of 10 seconds, the initial feedbacks are much the same as the corresponding values computed for the rudder and aileron case. In the latter case, the "ten second" aileron feedbacks stabilize the spiral model. Table 6 compares the feedbacks obtained for an optimization interval of 10 seconds with those obtained for an interval of 100 seconds.

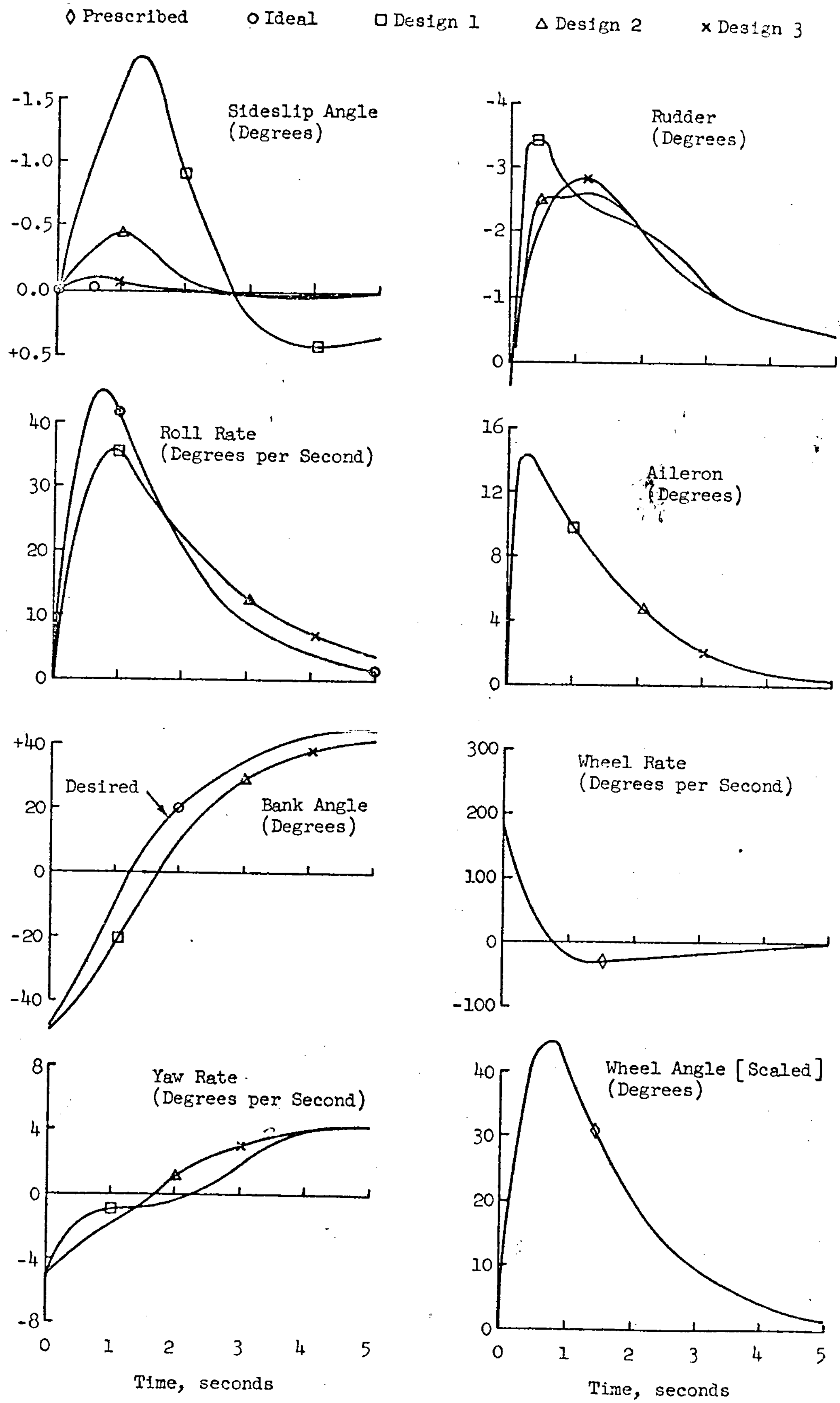

Figure 2 Comparison Time Histories

To discover the effects of optimization interval in this case, comparison trajectories were run using both sets of gains. Table 7 indicates that the "10 second" gains do considerably better than the converged gains in terms of the suppression of peak values of sideslip for a given peak value of rudder over the first 10 seconds of the maneuver. This indicates that for cases in which aircraft behavior over short intervals is more important than stability on the infinite time interval, short enough optimization intervals should be used. For the stability augmentation of piloted aircraft, this is an important reason for carrying out the optimization in the time domain where both finite and infinite intervals may be considered. Analyses which are restricted to the frequency domain as well as those involving the usual concept of stability exclude this option. As a matter of fact, a roots analysis, which considered the stability of individual poles only, would surely lead one to select the 100 second gains upon examining Table 8. Roughly speaking, optimal gains for the essentially infinite optimization interval produce identical poles as those for the short interval, aside from reversing the sign of the spiral root.

Simplification of the Kalman System

Implementation of the full Kalman solution would result in a complex control structure as evidenced by the block diagram of Figure 1. Furthermore, the values of the feedback gains vary with the flight condition. However, it was found that the Kalman solution could be used as a guide for the design of a fixed gain system with a reduced number of feedbacks which resulted in near optimal performance throughout the flight range. Since an eventual comparison with the existing E2A design which uses feedbacks to the rudder only was desired, only the "rudder only" optimal system was considered. Examine Tables 1, 2, and 3. The feedbacks from δ_a, r_W, p_D, ϕ_D are consistently so small that their effect on a "typical" rudder signal can be ignored. The feedback from δr has the effect of speeding up the rudder actuator. Since the speed of this actuator was a condition of the problem, this feedback must be ignored. Furthermore the remaining feedbacks from β, p, ϕ and r are relatively invariant with flight condition. It was thus possible to choose fixed values for these four feedbacks as a reasonable sub-optimal system. The block diagram for this system is shown in Figure 3.

For comparison with this suboptimal system, the existing E2A system, which is displayed in Figure 4, can be examined. In the E2A system, which uses feedbacks from the side force, roll rate and yaw rate to the rudder, the side force and roll rate gains are adjusted for changes in dynamic pressure by nonlinear potentiometers. The rudder gearing is increased by a factor of 2.5 when the pedals are allowed 20° of authority during the power approach. Moreover, the side force loop has a low pass (turn coordination) filter and the yaw rate loop has a high pass (washout) filter. The feedback gains shown in Table 9A were not optimized according to some precise performance index: they varied during the flight test program until the pilots found the aircraft's handling qualities to be acceptable. Ignoring filter dynamics the existing E2A feedbacks are equivalent to feedbacks from β, p, r, and δr which are shown in Table 9B.

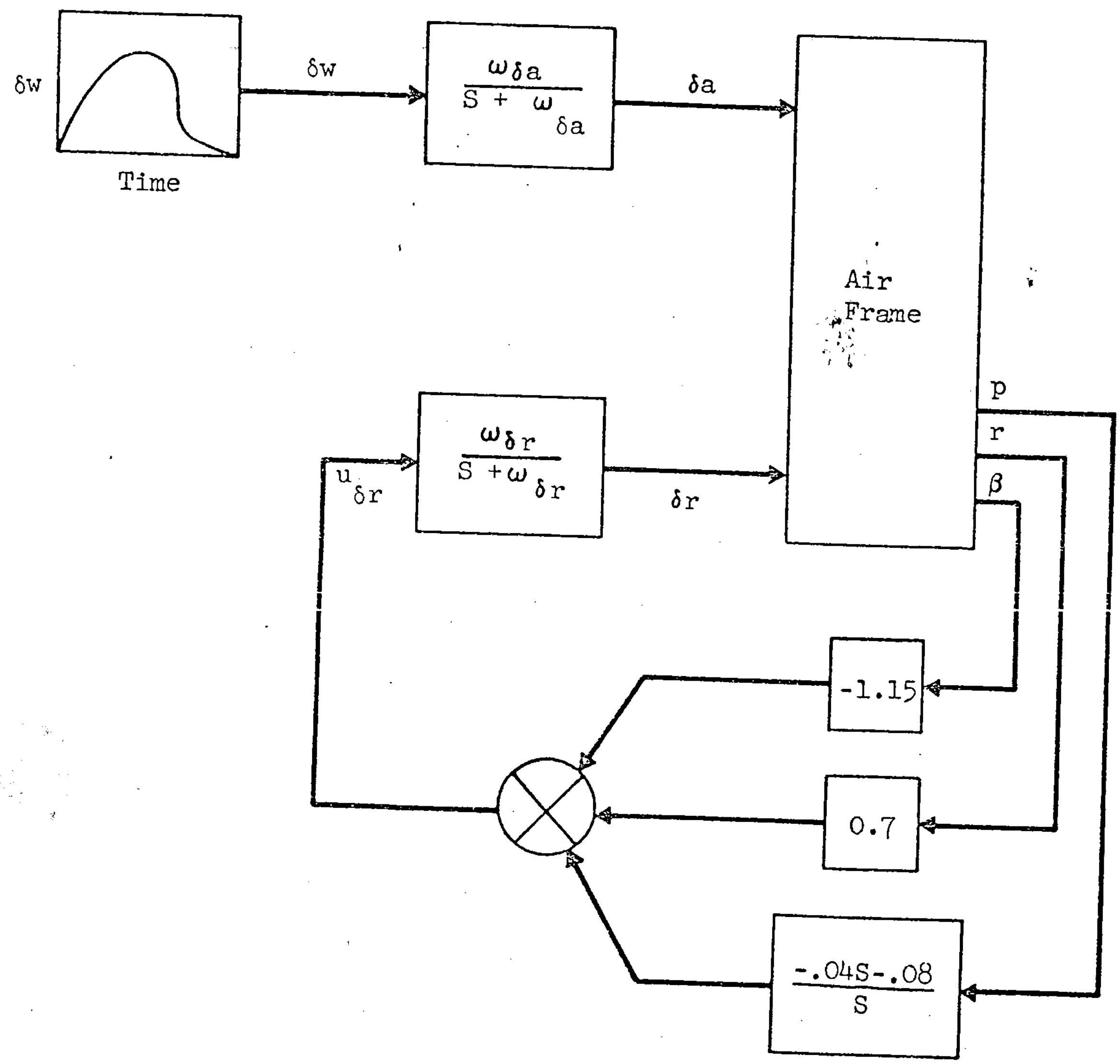

Figure 3 A Suboptimal Fixed Gain System of Constrained Configuration

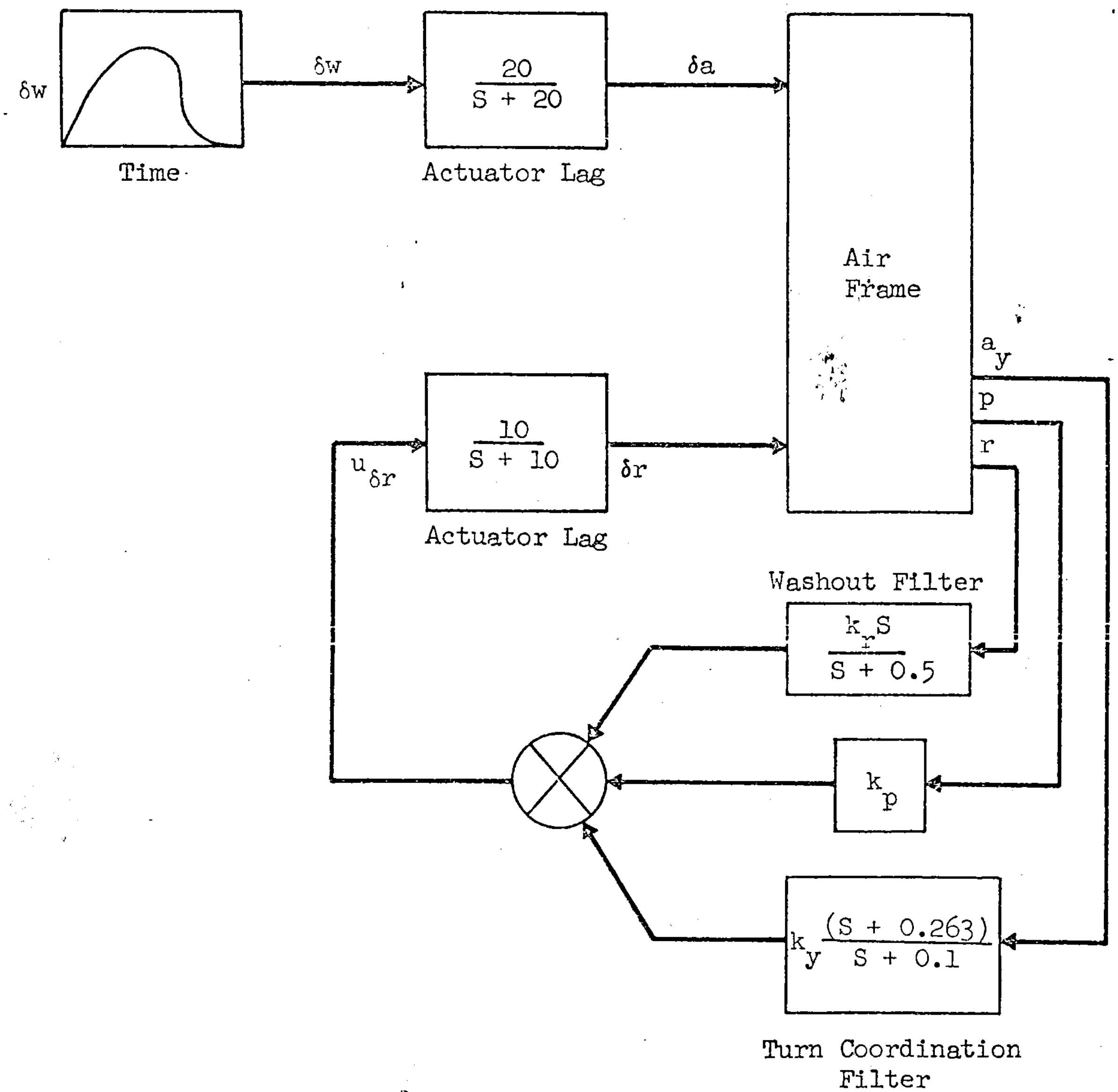

Figure 4 Actual E2A Lateral Directional Stability Augmentation System

Assuming that the time history of the control wheel is specified, comparison trajectories can be computed corresponding to several candidate control systems. In this case three candidates were selected: 1) No feedback control, 2) the Nominal E2A System, 3) a possible Suboptimal fixed gain system suggested by the results of the Kalman method. A summary of this comparison is shown in Table 10. It should be emphasized that the particular suboptimal system chosen for comparison was in no way optimized. Methods for optimizing systems with constrained configuration for a set of flight conditions are currently being pursued, Reference 7.

TABLE 1

E2A Optimal Gains Weights: $Q_1 = 10$, $Q_2 = 1$, $Q_3 = 1$, $R_1 = 30$, $R_2 = 10$

Flight Condition	1 PA	2 CR 10000	3 CR 30000	4 P 10000	5 P 10000
A. Rudder Feedbacks					
β	-.245	-.260	-.279	-.237	-.279
p	-.0080	.0072	.0087	-.0074	.0015
ϕ	.0263	.0240	.0196	.0099	.0126
r	.365	.331	.360	.221	.331
δr	-.0734	-.130	-.148	-.206	-.165
δa	-.0036	-.0003	.0015	.0021	.0020
rw	-.0385	-.0234	-.0178	-.0091	-.0136
p_D	-.130	-.860	-.0660	-.0356	-.0511
ϕ_D	-.0916	-.0626	-.0481	-.0266	-.0375
Aileron Feedbacks					
β	-.153	.0711	.155	.240	.199
p	-.177	-.197	-.236	-.171	-.228
ϕ	-.354	-.312	-.311	-.312	-.312
r	.148	.0237	-.0034	-.0327	-.0223
δr	-.0217	-.0016	.0088	.0124	.0118
δa	-.0361	-.0780	-.101	-.193	-.124
rw	.0927	.0770	.0732	.0516	.0669
p_D	.360	.374	.378	.364	.376
ϕ_D	.267	.297	.305	.313	.309

TABLE 2

E-2A Optimal gains; $Q_1 = 100$, $Q_2 = 1$, $Q_3 = 1$, $R_1 = 30$ $R_2 = 10$

Flight Condition	1 PA	2 CR 10000	3 CR 30000	4 P 10000	5 P 10000
A. Rudder Feedbacks					
β	-1.242	-1.273	-1.321	-1.250	-1.333
p	- .0677	- .0177	- .0114	- .0257	- .0176
ϕ	- .108	- .0458	- .0292	- .0310	- .0302
r	.942	.810	.847	.569	.787
δr	- .179	- .294	- .322	- .477	- .363
δa	- .0182	- .0144	- .0101	- .0031	- .0071
rw	- .0289	- .0163	- .0120	- .0027	- .0077
p_D	- .103	- .0673	- .0513	- .0139	- .0349
ϕ_D	- .0740	- .0510	- .0313	- .0112	- .0272
B. Aileron Feedbacks					
β	-.798	- .352	- .179	.236	- .0039
p	-.233	- .211	- .245	- .174	- .234
ϕ	-.471	- .349	- .331	- .318	- .324
r	.614	.264	.184	- .0010	.0967
δr	-.109	- .0866	- .0609	- .0185	- .0426
δa	-.0495	- .0859	- .106	- .194	- .127
rw	.101	.0812	.0758	.0531	.0689
p_D	.382	.384	.385	.369	.382
ϕ_D	.282	.303	.309	.316	.313

TABLE 3

E-2A Optimal gains; $Q_1 = 1000$, $Q_2 = 1$, $Q_3 = 1$, $R_1 = 30$ $R_2 = 10$

Flight Condition	1 PA	2 CR 10000	3 CR 30000	4 P 10000	5 P 10000
A. Rudder Feedbacks					
β	-4.831	-4.918	-5.021	-4.899	-5.061
p	- .143	- .0474	- .0347	- .0516	- .0407
ϕ	- .347	- .167	- .115	- .0961	- .105
r	2.119	1.769	1.822	1.277	1.700
δr	- .380	- .582	- .622	- .923	- .697
δa	- .0371	- .0331	- .0248	- .0015	- .0162
rw	- .0201	- .0114	- .0081	- .0014	- .0033
p_D	- .0752	- .0516	- .0387	- .0052	- .0207
ϕ_D	- .0551	- .0402	- .0306	- .0039	- .0169
B. Aileron Feedbacks					
β	-2.432	-1.578	-1.111	.640	- .420
p	- .281	- .223	- .253	- .174	- .238
ϕ	- .607	- .395	- .355	- .317	- .336
r	1.262	.626	.461	.0347	.234
δr	- .223	- .199	- .149	- .0093	- .0974
δa	- .0615	- .0936	- .110	- .196	- .129
rw	.107	.0836	.0773	.0535	.0699
p_D	.401	.391	.389	.370	.385
ϕ_D	.294	.308	.312	.316	.315

TABLE 4

Relevant Peak and Final Values of Aircraft State Variables for Different Settings of Q_1 for Rudder and Aileron Control on a 10 Second Rolling Maneuver

Flight Condition	Q_1	1 PA	2 CR 10000	3 CR 30000	4 P 10000	5 P 30000
Peak Slideslip (Degree)	10 100 1000	2.8 .7 .2	1.8 .4 .1	1.3 .3 .1	1.7 .3 .1	1.3 .3 .1
Peak Rudder (Degree)	10 100 1000	6.2 5.0 5.4	3.4 2.6 2.8	2.5 1.9 2.1	1.1 1.0 .9	1.8 1.5 1.6
Peak Aileron (Degree)	10 100 1000	19.6 21.2 22.0	14.2 14.9 15.1	12.8 13.2 13.3	9.1 9.3 9.3	11.5 11.8 11.9
Peak Roll Rate (Deg/Sec)	10 100 1000	23.9 23.6 23.0	35.9 35.7 35.4	39.1 38.9 38.7	41.0 40.8 40.7	40.5 40.4 40.2
Final Roll Angle (Degree)	10 100 1000	27. 26. 26.	40. 40. 40.	42. 42. 42.	43. 43. 43.	43. 43. 43.
Final Yaw Rate (Deg/Sec)	10 100 1000	4.1 4.3 4.3	4.3 4.2 4.2	3.3 3.2 3.2	2.8 2.9 2.9	2.9 2.9 2.9

TABLE 5

Comparison of Free Air Frame Roots and Optimal Roots
Using Optimal Feedbacks to Rudder and Aileron

$Q_1 = 100,\ Q_2 = 1,\ Q_3 = 1,\ R_1 = 30,\ R_2 = 10$

Flight Condition	System	1 PA	2 CR 10000	3 CR 30000	4 P 10000	5 P 30000
Dutch Roll Damping	FAF OPT	.287 .628	.131 .614	.074 .616	.126 .604	.072 .618
Dutch Roll Frequency	FAF OPT	.94 2.11	1.35 2.82	1.34 2.88	1.99 4.33	1.44 3.18
Spiral Root	FAF OPT	+ .034 - .368	+ .012 - .684	+ .009 - .818	+ .002 - .825	+ .005 - .844
Roll Subsidence Root	FAF OPT	- 2.79 - 3.10	- 2.70 - 3.70	- 2.09 - 3.43	- 5.37 -10.15	- 2.45 - 4.36
Rudder Actuator Root	FAF OPT	-10.00 -10.00	-10.00 -10.02	-10.00 -10.02	-10.00 -10.52	-10.00 -10.04
Aileron Actuator Root	FAF OPT	-20.00 -19.96	-20.00 -19.82	-20.00 -19.79	-20.00 -17.80	-20.00 -19.65

TABLE 6

E2A Optimal Gains; Stability Augmentation by Rudder Only

$Q_1 = 100$, $R_1 = 30$

Flight Condition	T (Sec)	1 PA	2 CR 10000	3 CR 30000	4 P 10000	5 P 10000
β	10 100	-1.307 -1.226	-1.277 -1.311	-1.312 -1.344	-1.246 -1.250	-1.322 -1.345
p	10 100	-.109 .084	-.0467 -.005	-.0388 -.001	-.0298 -.028	-.0350 -.019
ϕ	10 100	-.195 .370	-.0988 .013	-.0691 -.013	-.0421 -.031	-.0574 -.019
r	10 100	1.003 1.004	.817 .835	.848 .860	.568 .569	.784 .793
δr	10 100	-.189 .208	-.296 -.305	-.322 -.328	-.476 -.477	-.361 -.365
δa	10 100	-.0270 .0123	-.0267 -.010	-.0225 -.010	-.0086 -.006	-.0173 -.008
rw	10 100	.0004 .263	-.0012 .048	-.0003 .024	-.0014 .006	.0005 .019
p_D	10 100	-.0483 .631	-.0309 .120	.0153 .058	-.0025 .018	-.0081 .048

TABLE 7

Relevant Peak and Final Values of Aircraft Variables
for Rudder Control on a 10 Second Rolling Maneuver

$Q_1 = 100$, $R_1 = 30$

Flight Condition	T (Sec)	1 PA	2 CR 10000	3 CR 30000	4 P 10000	5 P 30000
Peak Sideslip (Degree)	0 10 100	17.2 2.4 9.2	9.8 1.2 2.5	7.1 .8 1.2	4.3 .5 .4	6.7 .6 1.1
Peak Rudder (Degree)	0 10 100	0. 6.5 16.9	0. 3.3 4.7	0. 2.0 2.8	0. 1.2 9.0	0. 1.7 2.4
Peak Aileron (Degree)	0 10 100	20.1 20.1 20.1	13.5 13.5 13.5	9.0 9.0 9.0	9.0 9.0 9.0	9.0 9.0 9.0
Peak Roll Rate (Deg/Sec)	0 10 100	24.9 24.1 17.8	36.4 36.2 33.4	33.3 32.8 31.1	40.9 39.5 39.2	38.3 37.4 35.9
Final Roll Angle (Degree)	0 10 100	20. 66. 10.	22. 52. 24.	20. 43. 25.	31. 41. 37.	32. 50. 34.
Final Yaw Rate (Deg/Sec)	0 10 100	1.2 11.6 1.6	2.4 4.8 2.4	1.6 3.4 1.8	2.8 2.8 2.4	2.2 3.4 2.2

TABLE 8

Comparison of Roots of Free Airframe with Optimal Roots
Using 10 Second and 100 Second Optimization Intervals
for $Q_1 = 100$ $R_1 = 30$ Using Rudder Feedbacks Only

Flight Condition	System	1 PA	2 CR 10000	3 CR 30000	4 P 10000	5 P 30000
Dutch Roll Damping	FAF 10 100	.29 .63 .63	.13 .61 .61	.07 .62 .61	.13 .68 .68	.07 .62 .62
Dutch Roll Frequency	FAF 10 100	.94 2.18 2.18	1.35 2.75 2.75	1.34 2.83 2.83	1.99 4.30 1.44	1.44 3.13 3.13
Spiral Root	FAF 10 100	.034 .089 -.095	.012 .043 -.044	.009 .029 -.030	-.002 +.009 -.002	.005 .002 -.002
Roll Subsidence Root	FAF 10 100	-2.79 -2.79 -2.79	-2.70 -2.67 -2.67	-2.09 -2.02 -2.02	-5.35 -5.35 -5.35	-2.45 -2.38 -2.38
Rudder Actuator Root	FAF 10 100	-10.	-10.	-10.	-10.	-10.

TABLE 9

Nominal Feedback Gains for the Existing E2A - Rudder Feedback Only

A. In Terms of a_y, p, r

Flight Condition	1 PA	2 CR 10000	3 CR 30000	4 P 10000	5 P 30000
a_y	71.6	15.3	14.8	7.9	12.9
p	-.324	-.070	-.067	-.036	-.059
r	1.27	.38	.38	.38	.38

B. In Terms of β, p, r, δr

Flight Condition	1 PA	2 CR 10000	3 CR 30000	4 P 10000	5 P 30000
β	-1.075	-.301	-.308	-.398	-.325
p	.296	.065	.064	.034	.057
r	1.34	.39	.39	.39	.39
δr	.53	.19	.20	.24	.21

TABLE 10

Relevant Peak and Final Values of Aircraft Variables
for Control Systems Using:
No Control (NO)
Nominal E2A Control (NOM)
A Fixed Gain Suboptimal Control (SUB)

Flight Condition	Control System	1 PA	2 CR 10000	3 CR 30000	4 P 10000	5 P 30000
Peak Sideslip (Degree)	NO NOM SUB	11.2 2.7 1.6	9.2 2.7 .8	4.5 1.0 .7	2.5 .9 .3	4.1 1.1 .6
Peak Rudder (Degree)	NO NOM SUB	0. 5.05 4.06	0. 2.70 3.05	0. 1.45 1.45	0. .63 .71	0. 1.26 1.24
Peak Aileron (Degree)	NO NOM SUB	16.1	14.0	7.0	6.8	6.9
Peak Roll Rate (Deg/Sec)	NO NOM SUB	16.1 19.7 18.4	33.7 36.3 36.5	22.9 25.0 23.9	29.6 30.4 29.5	26.9 28.8 27.4
Final Roll Angle (Degree)	NO NOM SUB	-1.6 28.2 26.1	21.6 38.1 49.1	2.2 14.0 19.2	11.9 14.8 21.7	11.2 20.6 27.8
Final Yaw Rate (Deg/Sec)	NO NOM SUB	-1.17 5.06 4.32	1.96 4.13 5.40	-.52 1.05 1.50	.96 1.01 1.53	-1.09 1.40 1.96

REFERENCES

1. R. E. Kalman, Contributions to the Theory of Optimal Control, Bol. Soc. Mat. Mex., March 1960.

2. R. E. Kalman, When is a Linear Control System Optimal?, J. Basic Eng Trans ASME Series D 86, March 1964.

3. E. G. Gilbert, Controllability and Observability in Multivariable Control Systems, J. SIAM on Control.

4. E. Kreindler, Closed Loop Sensitivity Reduction of Linear Optimal Control Systems, IEEE Tran AC, 1968.

5. B. D. O. Anderson, The Inverse Problem of Optimal Control, Stanford Electronics Laboratory Report 66-08, 1966.

6. Grumman Aircraft W2F-1, Final Aerodynamic Stability and Control Status and Estimated Flying Qualities Report, Grumman Report XA123-152-5.

7. A. Jameson and M. J. Rossi, The Optimization of the Average Response of Linear Systems of Constrained Configuration, Grumman Advanced Development Report to be published.

APPENDIX

BASIC AIRCRAFT DATA

Inertial and Kinematic Data

Area - 700 ft.^{2} Span - 80.6 ft.

Flight Condition	1 PA	2 CR 10000	3 CR 30000	4 P 10000	5 P 30000
Configuration	PA	CR	CR	P	P
Speed (nominal)	1.4 Vspa	1.4 Vscr	1.4 Vscr	.8 V_H	.8 V_H
Velocity (ft/sec)	180.9	280.9	417.9	472.0	465.0
Mach number	.162	.268	.420	.438	.467
Altitude (1000 ft)	Sea Level	10	30	10	30
Dynamic Pressure (lbs/ft^2)	38.9	73.3	78.3	196.0	97.0
Weight (lbs)	40660.	43087.	43087.	43087	43087
Angle of Attack (deg.)	3.3	6.7	6.0	1.6	4.4
Roll Moment * of Inertia-Ix (slug-ft^2)	116000.	118600.	118600.	118600.	118600.
Yaw Moment of * Inertia-I (Slug - ft^2)	232700.	235300.	235300.	235300.	235300.
Roll/Yaw * Cross Product of Inertia - Ixz (slug - ft^2)	14300.	14200.	14200.	14200.	14200.

*Body Axis

Dimensionless Aerodynamic Derivatives

Flight Condition	1 PA	2 CR 10000	3 CR 30000	4 P 10000	5 P 30000
$C_{Y\beta}$(1/rad)	-1.29	-.946	-.940	-.905	-.917
C_{Yp}(1/rad/sec)	.1514	.094	.090	.045	.075
C_{Yr}(1/rad/sec)	.3734	.315	.313	.303	.310
$C_{Y\delta r}$(1/rad)	.631	.602	.596	.547	.602
$C_{Y\delta a}$(1/rad)	0	0	0	0	0
$C_{\ell\beta}$(1/rad)	-.0672	-.0955	-.0959	-.0955	-.0959
$C_{\ell p}$(1/rad/sec)	-.6656	-.534	-.547	-.657	-.577
$C_{\ell r}$(1/rad/sec)	.3405	.204	.210	.105	.180
$C_{\ell\delta r}$(1/rad)	-.031	-.0258	-.0206	.0090	-.0092
$C_{\ell\delta a}$(1/rad)	.2116	.2324	.2370	.2606	.2472
$C_{n\beta}$(1/rad)	.0648	.0850	.0831	.0779	.0800
C_{np}(1/rad/sec)	-.0887	-.0629	-.0605	-.0280	-.0496
C_{nr}(1/rad/sec)	-.1577	-.1160	-.1145	-.1050	-.1100
$C_{n\delta r}$(1/rad)	-.216	-.234	-.233	-.216	-.231
$C_{n\delta a}$(1/rad)	-.0286	-.0150	-.0134	-.0054	-.0104

FORMULAS for the Dimensional Derivatives

Letting $m = \frac{\text{weight}}{32.2}$ the formulas relating basic aircraft data to the dimensional derivatives are:

$$Y_\beta = \frac{qs}{m} C_{Y\beta} \qquad \ell_\beta = \frac{qsb}{Ix} C_{\ell\beta} \qquad n_\beta = \frac{qsb}{Iz} C_{n\beta}$$

$$Y_p = \frac{qsb}{2mV} C_{Yp} \qquad \ell_p = \frac{qsb^2}{2IxV} C_{\ell p} \qquad n_p = \frac{qsb^2}{2IzV} C_{np}$$

$$Y_r = \frac{qsb}{2mV} C_{Yr} \qquad \ell_r = \frac{qsb^2}{2IxV} C_{\ell r} \qquad n_r = \frac{qsb^2}{2IzV} C_{nr}$$

$$Y_{\delta r} = \frac{qs}{m} C_{Y\delta r} \qquad \ell_{\delta r} = \frac{qsb}{Ix} C_{\ell\delta a} \qquad n_{\delta r} = \frac{qsb}{Iz} C_{n\delta r}$$

$$Y_{\delta a} = \frac{qs}{m} C_{Y\delta a} \qquad \ell_{\delta a} = \frac{qsb}{Ix} C_{\ell\delta a} \qquad n_{\delta a} = \frac{qsb}{Iz} C_{n\delta a}$$