

HAWKER SIDDELEY DYNAMICS LIMITED

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RADIATION PATTERNS OF

CYLINDRICAL AERIALS

by

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S U M M A R Y

A V.H.F. fuze would be cheaper than a microwave fuze and would need a shorter time to warm up. The length of an aerial to produce the directional radiation needed for a fuze would be of the order of a wavelength, and it would be difficult to incorporate a V.H.F. aerial of this size in a small missile. One solution is to divide the missile case fore and aft and use it as its own fuze aerial. This paper describes calculations to determine the pattern of radiation that could be obtained in this way. It appears that it should be possible to obtain a satisfactory pattern. But since the calculations assume a parallel body without wings or fins, and are for the far field, whereas the fuze would have to operate at close range, this conclusion needs to be confirmed by tests with a rig.

SEPTEMBER, 1966.

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## 2. POLAR DIAGRAMS CALCULATED FOR CYLINDRICAL AERIALS

The calculation of the radiation from a general missile shape is not feasible. The approach adopted here is to calculate the radiation of parallel sided hollow cylinders of the same diameter as the missile case. The electrical resistance of the case walls should be extremely small, and the cylinders are assumed to be perfect conductors. The current distribution is calculated for a step in voltage at an infinitesimal gap which may be anywhere along the length of the cylinder. With a finite gap it is assumed that the current at any distance from the gap is not appreciably affected by the width of the gap, and polar diagrams for different gap widths are obtained by integrating the same current distribution as for an infinitesimal gap over the length of the conducting sections of the cylinder only. The method is due to Hallen and is described in Appendix I. It should be very accurate for cylinders less than  $\frac{1}{50}$  wavelength in diameter. For a cylinder  $\frac{2}{15}$  wavelength in diameter taken here to represent the missile it is likely to give the current to an accuracy of about  $\pm 5\%$ .

An alternative method due to Synge (Refs. 11 and 12) gives an equation for any closed body of revolution with a gap, whereas, Hallen's method gives an equation for an open cylinder. Hallen's method was preferred because of the difficulty of solving Synge's equation to any reasonable degree of accuracy for a thick cylinder, and because of the body of already computed results available with Hallen's method.

The results of the calculations are presented in the figures as polar diagrams for the field at a distance. The radiation is symmetrical about the axis of the cylinder and the polar diagrams show the radiation in any plane containing the axis. For centre-fed aerials the radiation is the same fore and aft, and only the first quadrant need be shown. The diagrams are on a logarithmic scale and each is normalized to the direction of maximum radiation. They are 'one way' diagrams, representing the transmitted power. 'Two way' diagrams representing transmission and reception would show twice the attenuation.

For comparison, polar diagrams for infinitely thin wires are shown in Figs. 1 - 4. The relevant formulae are given in Appendix 2. The number of lobes increases with the length of the wire. A centre-fed wire less than a wavelength long gives only two lobes altogether (Fig. 1). Two forward and two rearward lobes are obtained when the length is between  $1\frac{1}{2}$  and 2 wavelengths (Figs. 2 and 3). Additional forward and rearward lobes appear if the length is increased further (Fig. 4). A length of rather less than 2 wavelengths gives the desired sort of polar diagram, if the lobes can be made to point in the right direction.

Figs. 5 - 8, 10 and 15 show the effect on the polar diagrams of increasing the thickness. In all of these the width of the feeding gap is  $\frac{1}{50}$  wavelength. When the total length is  $\frac{1}{2}$  wavelength, thickness does not have much effect, (Figs. 5, 7, 10) but when it is 2 wavelengths it can be seen that the lobes become wider as the thickness is increased, so that there is an increasing amount of radiation at angles of  $10 - 20^\circ$ .

Figs. 9 - 20 are all for a cylinder radius of  $\frac{1}{15}$  wavelength. A typical missile has a ratio of length to diameter of 15, a ratio which is obtained with a cylinder of this radius when the length is 2 wavelengths, giving a four lobed pattern of the type desired. Figs. 9 and 10 show polar diagrams for a length of  $\frac{1}{2}$  wavelength with gap widths of  $\frac{1}{20}$  and  $\frac{1}{50}$  wavelength. In this case the width of the gap has no appreciable effect. Figs. 11 and 12 show polar diagrams for a length of 1.9 wavelengths and gap widths of  $\frac{1}{20}$  and  $\frac{1}{50}$  wavelength, Figs. 13 - 16 show diagrams for a length of 2 wavelengths and a range of gap widths, while Figs. 17 and 18 show diagrams for a length of 2.1 wavelengths and gap widths of  $\frac{1}{20}$  and  $\frac{1}{50}$  wavelength. It can be seen that as the length is increased through 2 wavelengths bumps appear around  $20^\circ$  in the forward lobes, and that these become more pronounced when the width of the gap is reduced (Figs. 13 - 16). This is because for thick aeriels resonance occurs when the length is slightly less than 2 wavelengths, instead of 2 wavelengths as in the case of a thin wire (Fig. 3), and the bumps are in fact the new lobes which appear beyond resonance (Fig. 4), blurred over because of the thickness. It would be easier to design a structure with a narrow gap, and to prevent too much radiation in a forward direction it would be necessary to use a length of rather less than 2 wavelengths.

Finally, Figs. 19 and 20 show the effect of moving the feeding gap back  $\frac{1}{20}$  wavelength from the centre when the total length is 2 wavelengths, for gap widths of  $\frac{1}{20}$  and  $\frac{1}{50}$  wavelength. The bumps in the forward lobes are now suppressed and the forward lobes are slightly narrower than for a centre-fed aerial with a length of 1.9 wavelengths. This could be the most favourable configuration for the design of the fuze, though it might be difficult to arrange the other components to accommodate such a position for the gap.

### 3. KNOWN EXPERIMENTAL RESULTS

The Radio Research Laboratory at Harvard measured the radiation of a number of cylindrical aeriels during the Second World War. The results are given in Ref. 14. Unfortunately the polar diagrams are printed on a very small (linear) scale, and it is only possible to read them to a low accuracy. They do, however, agree qualitatively with the results of the calculations.

#### 4. CONCLUSION

It appears from the calculations that it may well be possible to use the missile case as the fuze aerial and obtain a satisfactory pattern of radiation, provided that the frequency is chosen so that the total length of the missile is two wavelengths or a little less. Some adjustments can be made by moving the gap away from the centre. The width of the gap will not have much affect on the pattern of radiation, but the calculations show a singularity in the out-of-phase component of the current at an infinitely narrow gap, and one may therefore expect the width of the gap to have a substantial effect on the capacitance of the aerial. A centre-fed aerial about two wavelengths long will have a node in the in-phase component of current at the gap, and will therefore present a high impedance to the source.

These provisional conclusions are subject to two provisos:

- (1) They do not allow for the effect of wings, fins, and tapering of the nose. Some tests at E.M.I. suggest that the effect of these may be appreciable.
- (2) The calculated pattern of radiation is for the far field, it being assumed that the distance is a large number of wavelengths. In fact with a V.H.F. fuze targets might pass within a range of two or three wavelengths. The near field will be modified by two effects. First, the dipole field assumed in the calculations will be modified by near field components. The most important of these is the ordinary component of magnetic field due to a steady current

$$|B_{\phi \text{near}}| = \frac{\mu_0}{4\pi} \frac{ds |\sin \theta|}{r^2}$$

The main component of magnetic field at a distance from a dipole is

$$|B_{\phi \text{far}}| = \frac{\mu_0 \omega}{4\pi c} \frac{ds |\sin \theta|}{r^2}$$

whence for a dipole

$$\left| \frac{B_{\phi \text{far}}}{B_{\phi \text{near}}} \right|^2 = \left( \frac{2\pi r}{\lambda} \right)^2$$

Evidently this effect will not be important at distances of more than a wavelength. The second effect is that at close range it is no longer valid to replace the distance vectors from different parts of the aerial by vectors in a parallel direction. If one regards a cylinder as composed of a set of parallel thin wires distributed around the circumference, one can see that by vector addition of their far field components of  $\underline{E}$ , which are in the  $\theta$  direction, there will be some radiation straight ahead. From the polar diagram of a thin wire 2 wavelengths long, (Fig. 3) one can estimate that, a wavelength ahead of a cylinder of  $\frac{1}{16}$  wavelength radius, the amount of forward radiation due to this effect should be small.

Because of these uncertainties, it would be unwise to proceed with the design of a missile using its case as a fuze aerial, without first constructing a rig and measuring the radiation from a body with wings and fins at ranges typical of engagements.

5. ACKNOWLEDGEMENTS

The method employed in the calculations was developed by Prof. Erik Hallén of the Royal Institute of Technology, Stockholm, and I am indebted to him for his advice and for supplying some numerical results. I am also indebted to Mr. Duncan McDonald for writing the computer programmes for the calculations. The idea of using the missile case as a fuze aerial is due to Mr. H.R. Joiner.



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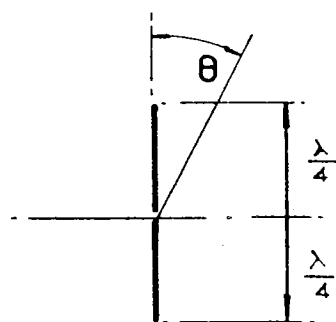
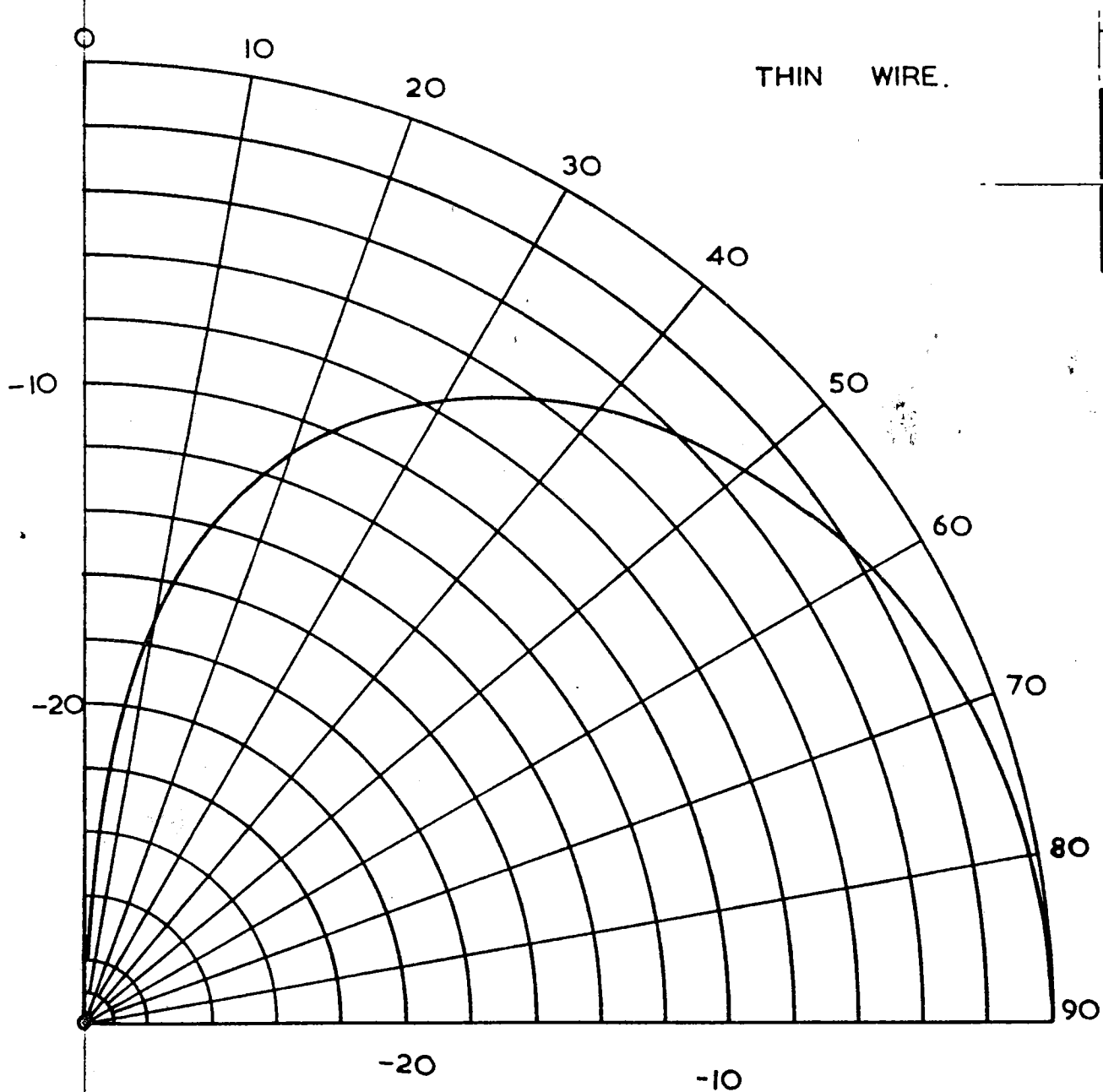
# LIST OF FIGURES

All Figures are Polar Diagrams.

## FIGURE

### No.

1	Centre-fed thin wire;			length $\frac{\lambda}{2}$
2	"			$1.9\lambda$
3	"			$2\lambda$
4	"			$2.1\lambda$
5	Centre-fed cylinder;	radius $\frac{\lambda}{1000}$ ,	gap width $\frac{\lambda}{50}$ ,	length $\frac{\lambda}{2}$
6	"	"	"	$2\lambda$
7	"	$\frac{\lambda}{120}$	"	$\frac{\lambda}{2}$
8	"	"	"	$2\lambda$
9	"	$\frac{\lambda}{15}$	$\frac{\lambda}{20}$	$\frac{\lambda}{2}$
10	"	"	$\frac{\lambda}{50}$	"
11	"	"	$\frac{\lambda}{20}$	$1.9\lambda$
12	"	"	$\frac{\lambda}{50}$	"
13	"	"	$\frac{\lambda}{10}$	$2\lambda$
14	"	"	$\frac{\lambda}{20}$	"
15	"	"	$\frac{\lambda}{50}$	"
16	"	"	$\frac{\lambda}{100}$	"
17	"	"	$\frac{\lambda}{20}$	$2.1\lambda$
18	"	"	$\frac{\lambda}{50}$	"
19	Cylinder with offset feed;	radius $\frac{\lambda}{15}$ ,	gap width $\frac{\lambda}{20}$ ,	$l_1 = 1.05\lambda$ $l_2 = .95\lambda$
20	"	"	$\frac{\lambda}{50}$	"



POLAR DIAGRAM  
DECIBELS.

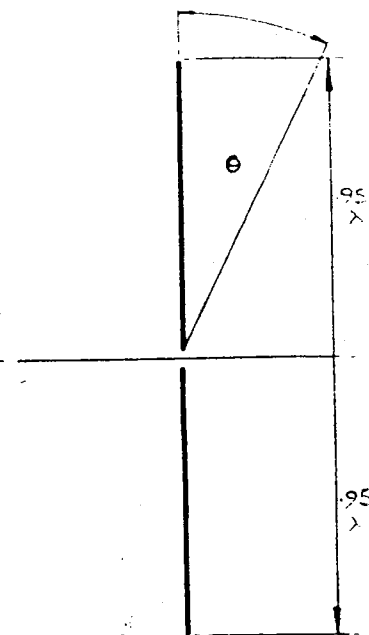
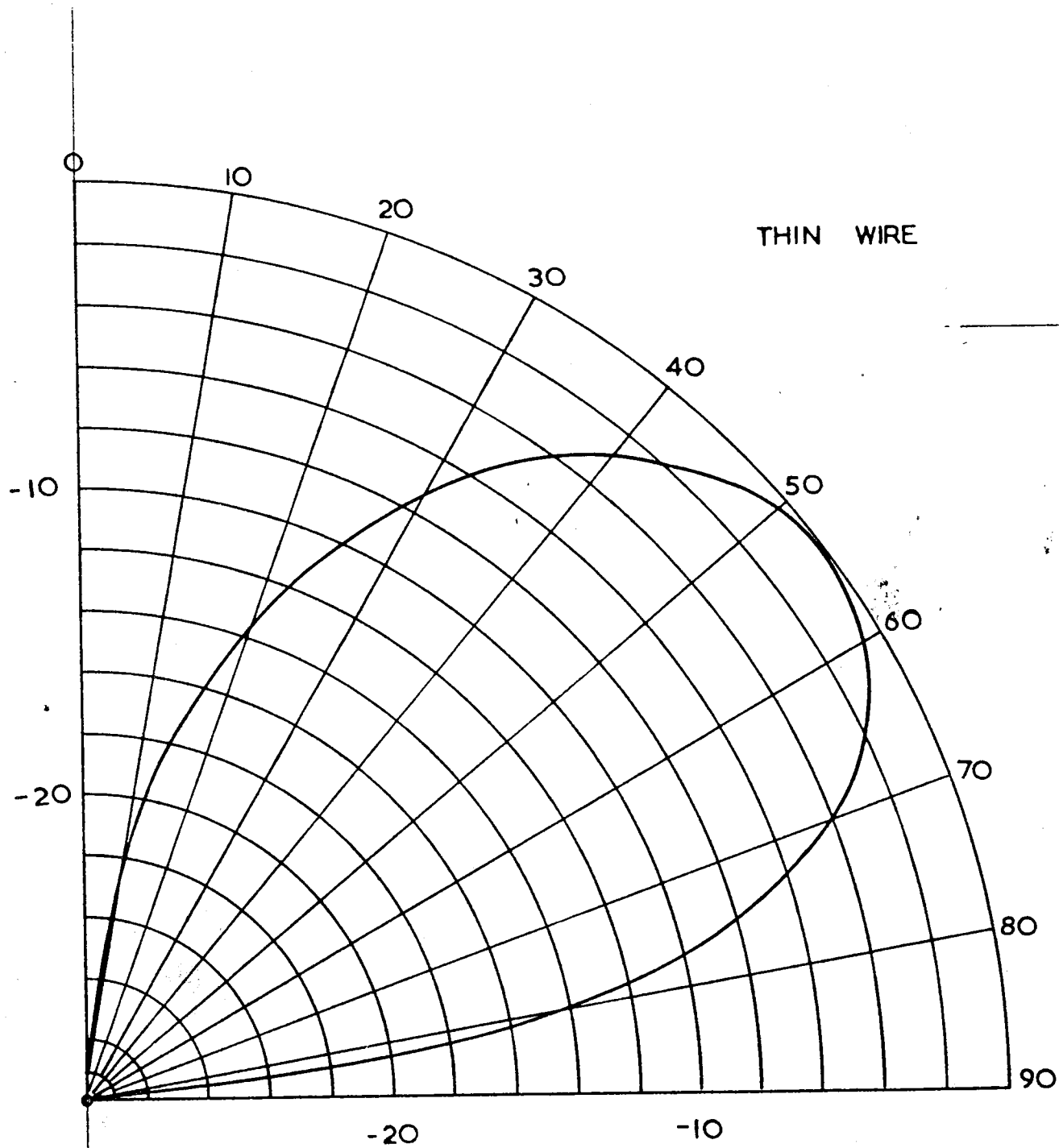
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FIG. No. 1

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POLAR DIAGRAM  
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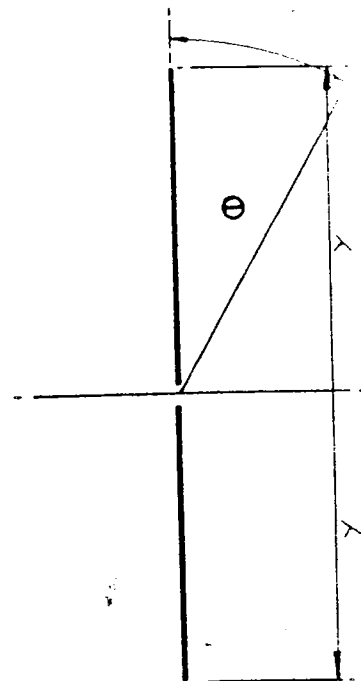
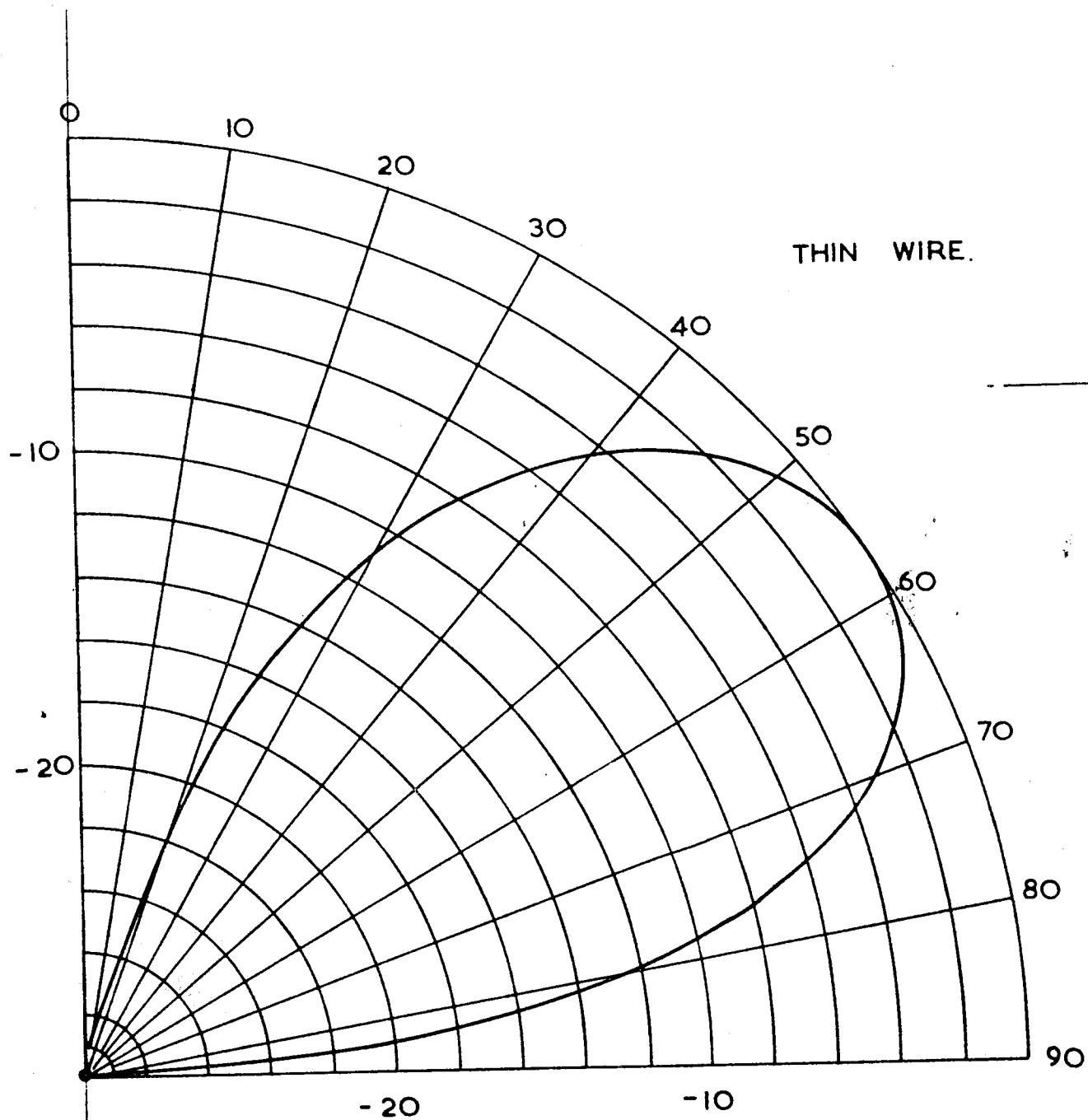
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FIG. No. 2

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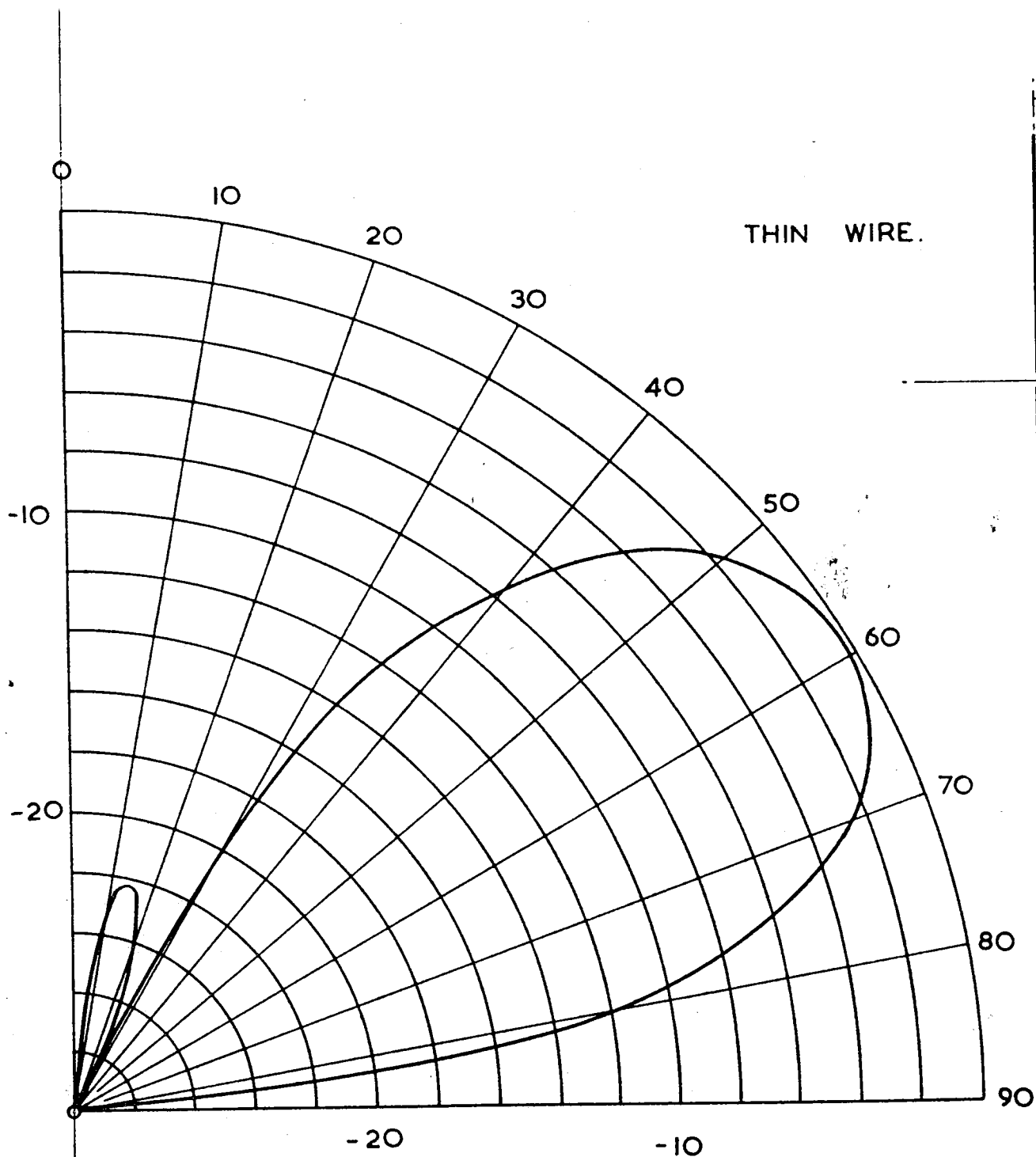
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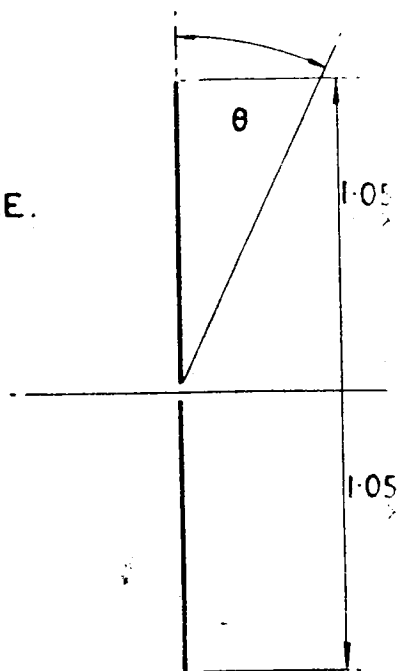
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FIG. No. 3

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THIN WIRE.



POLAR DIAGRAM  
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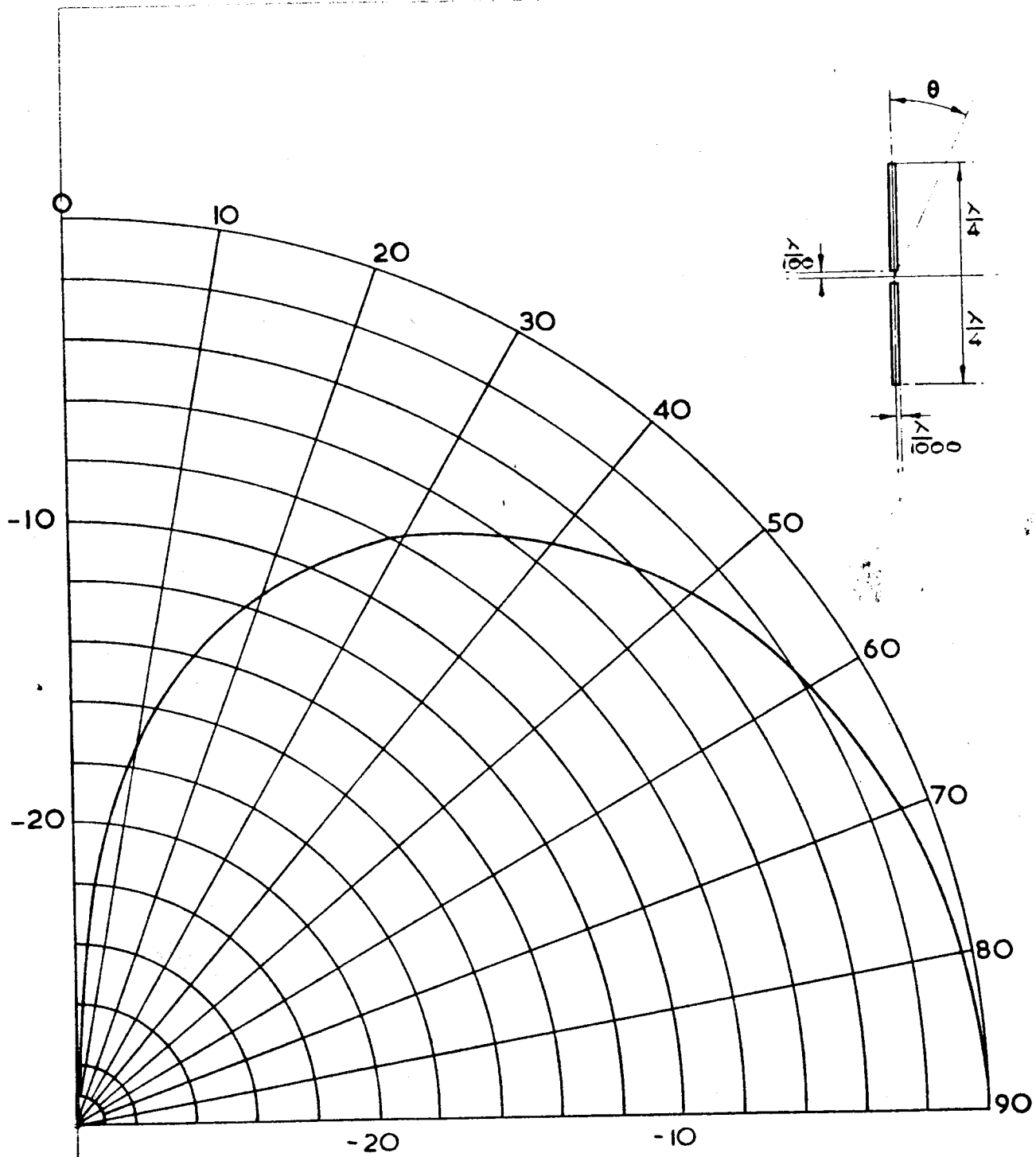
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FIG. No. 4

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POLAR DIAGRAM  
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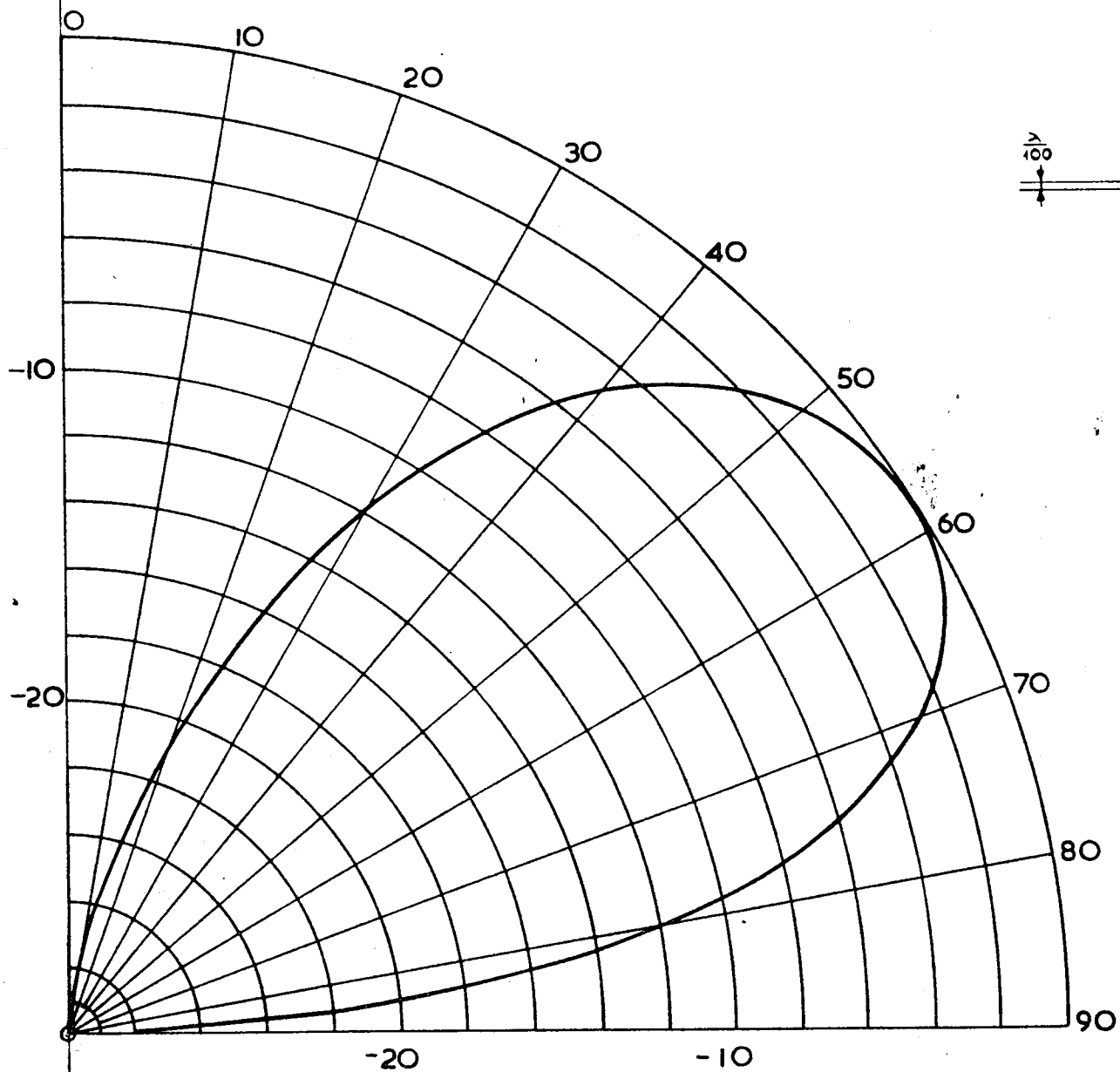
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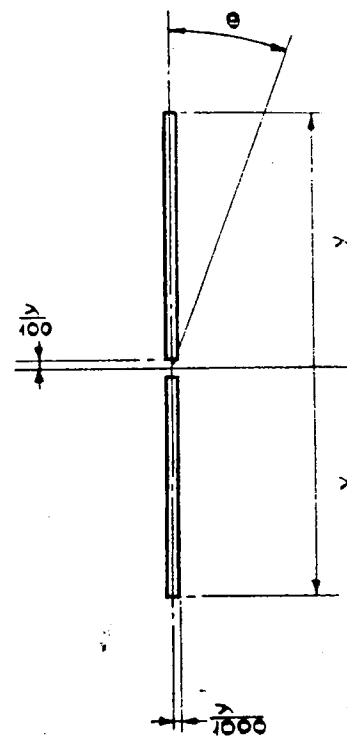
FIG. No. 5

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POLAR DIAGRAM  
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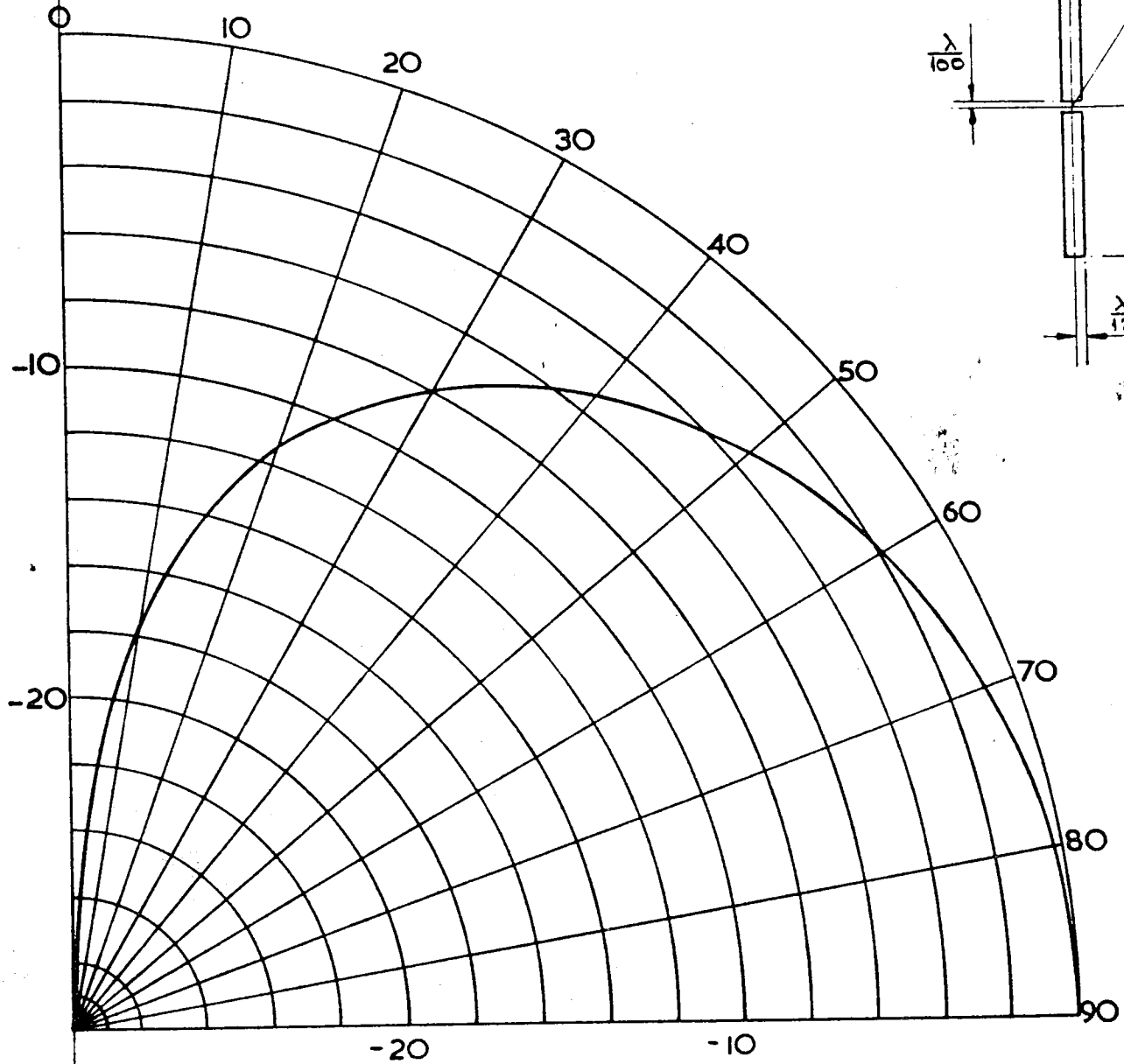
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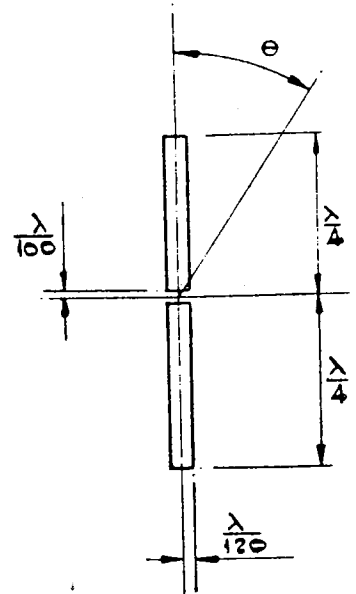
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FIG. No. 6

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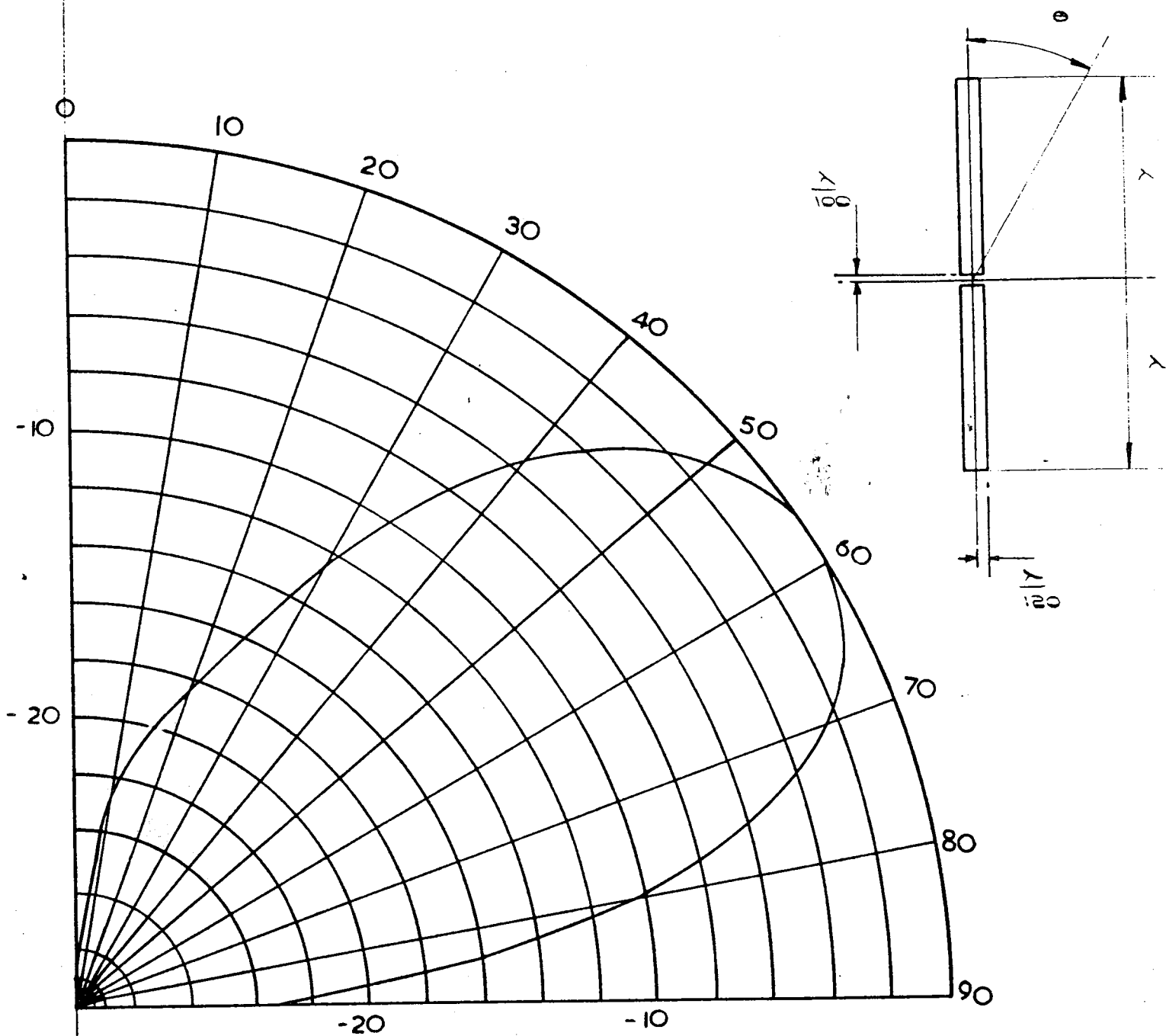
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FIG. No. 7

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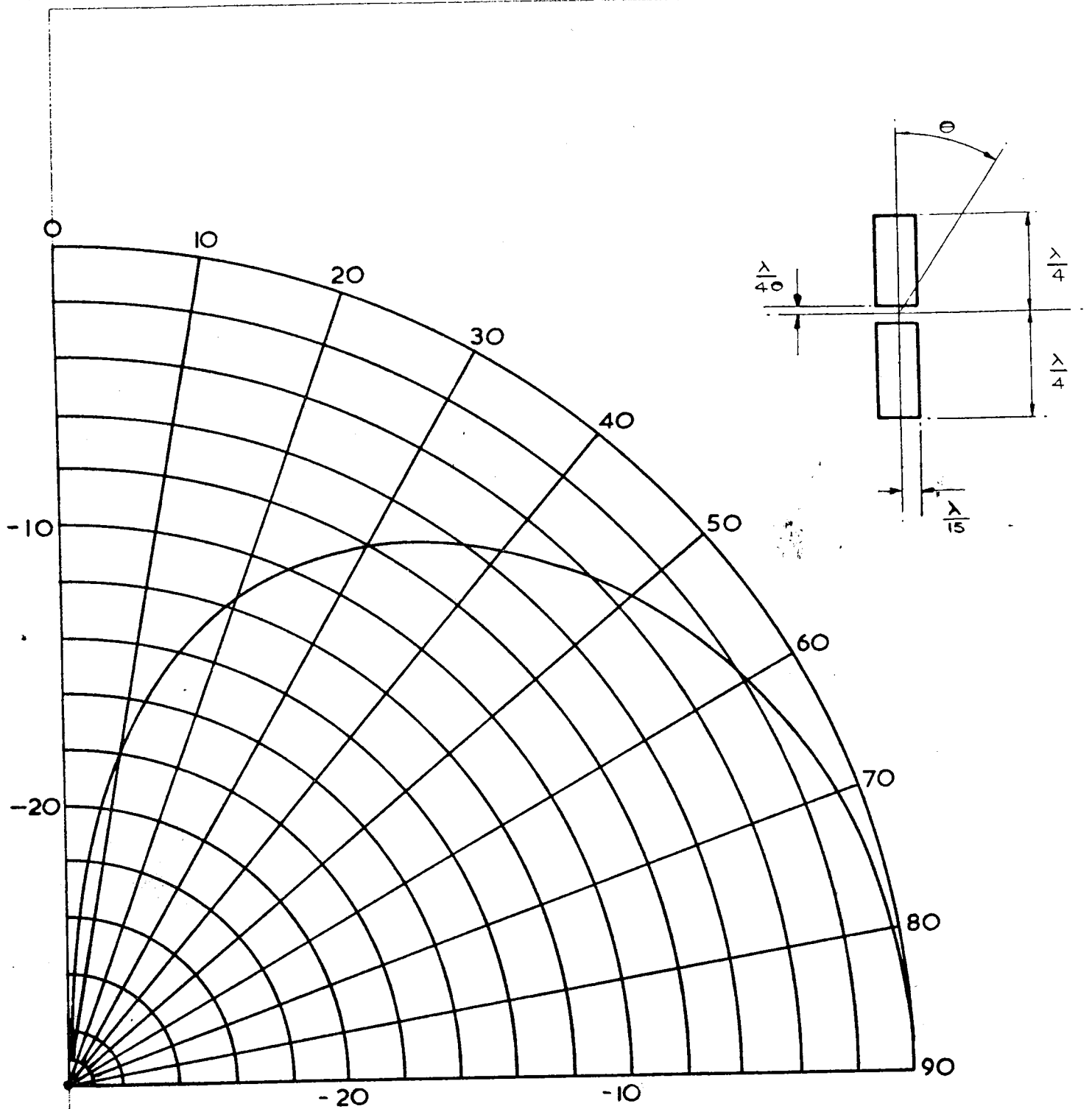
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FIG. No. 8

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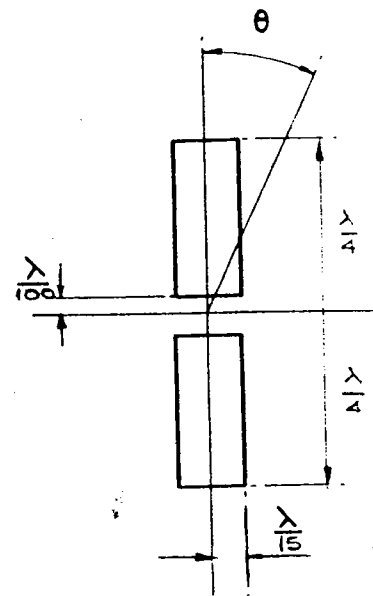
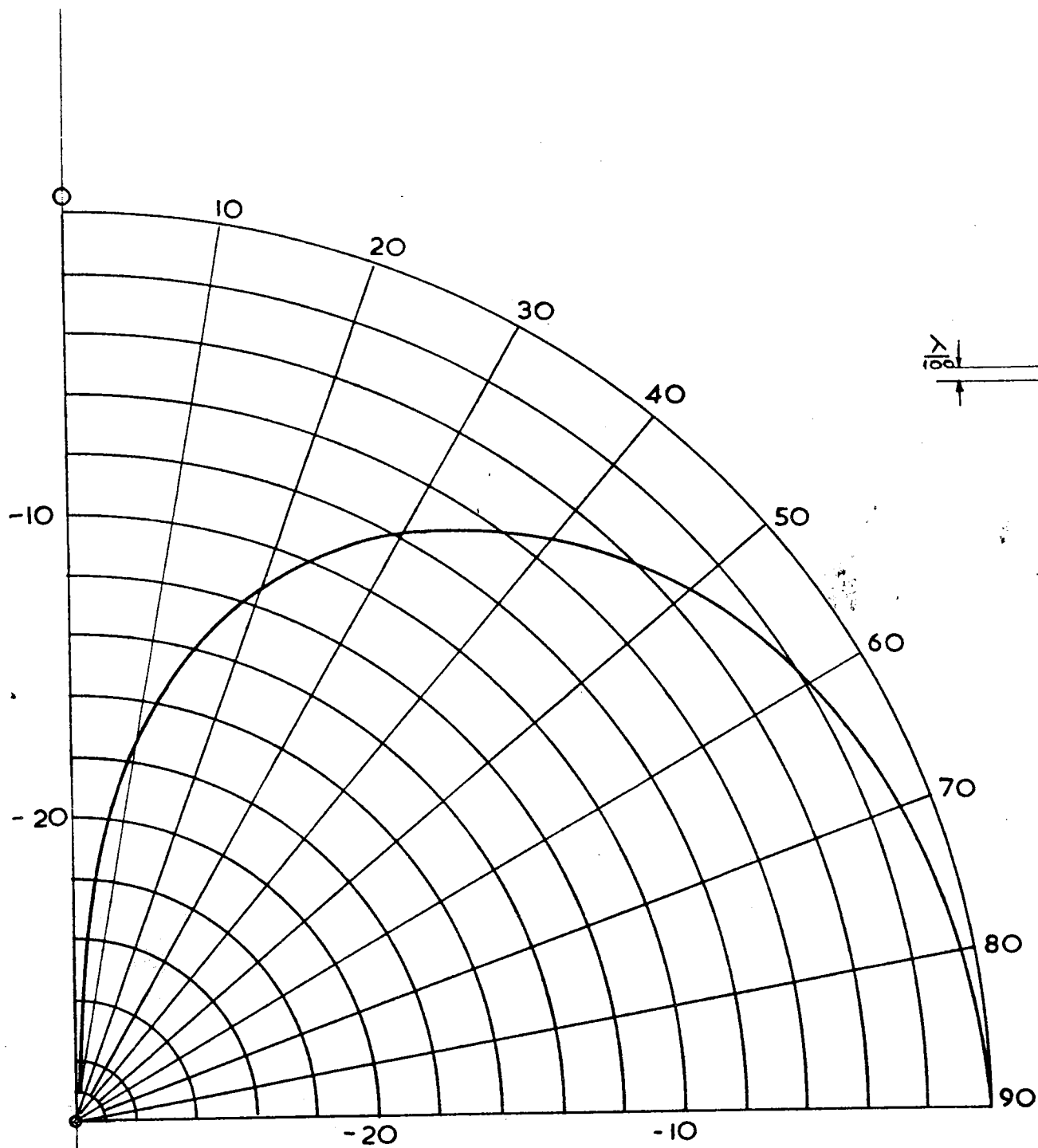
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FIG. No. 9

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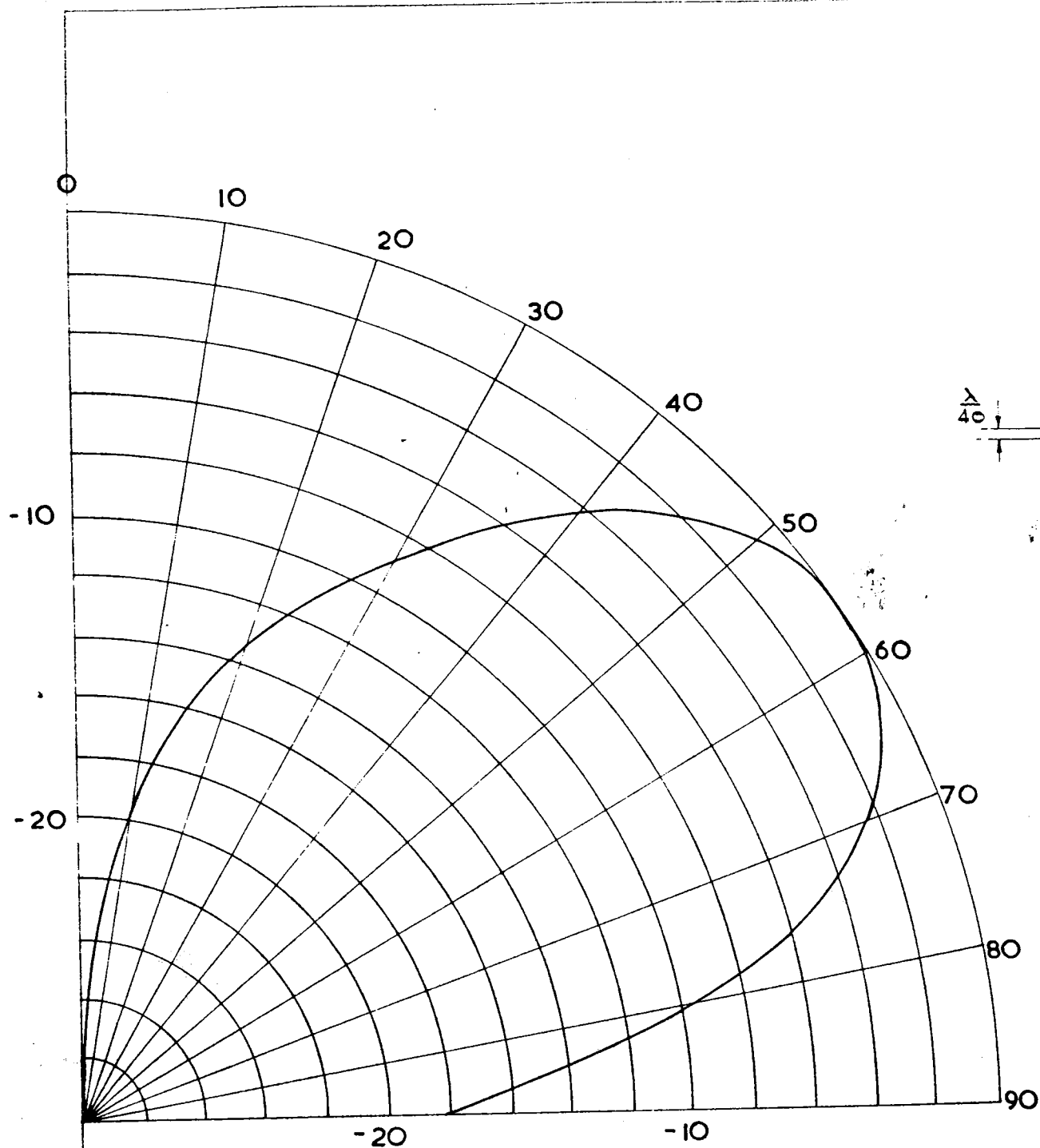
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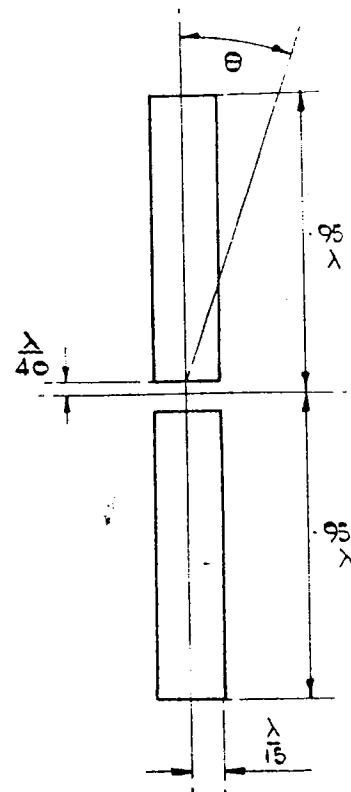
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FIG. No. 10

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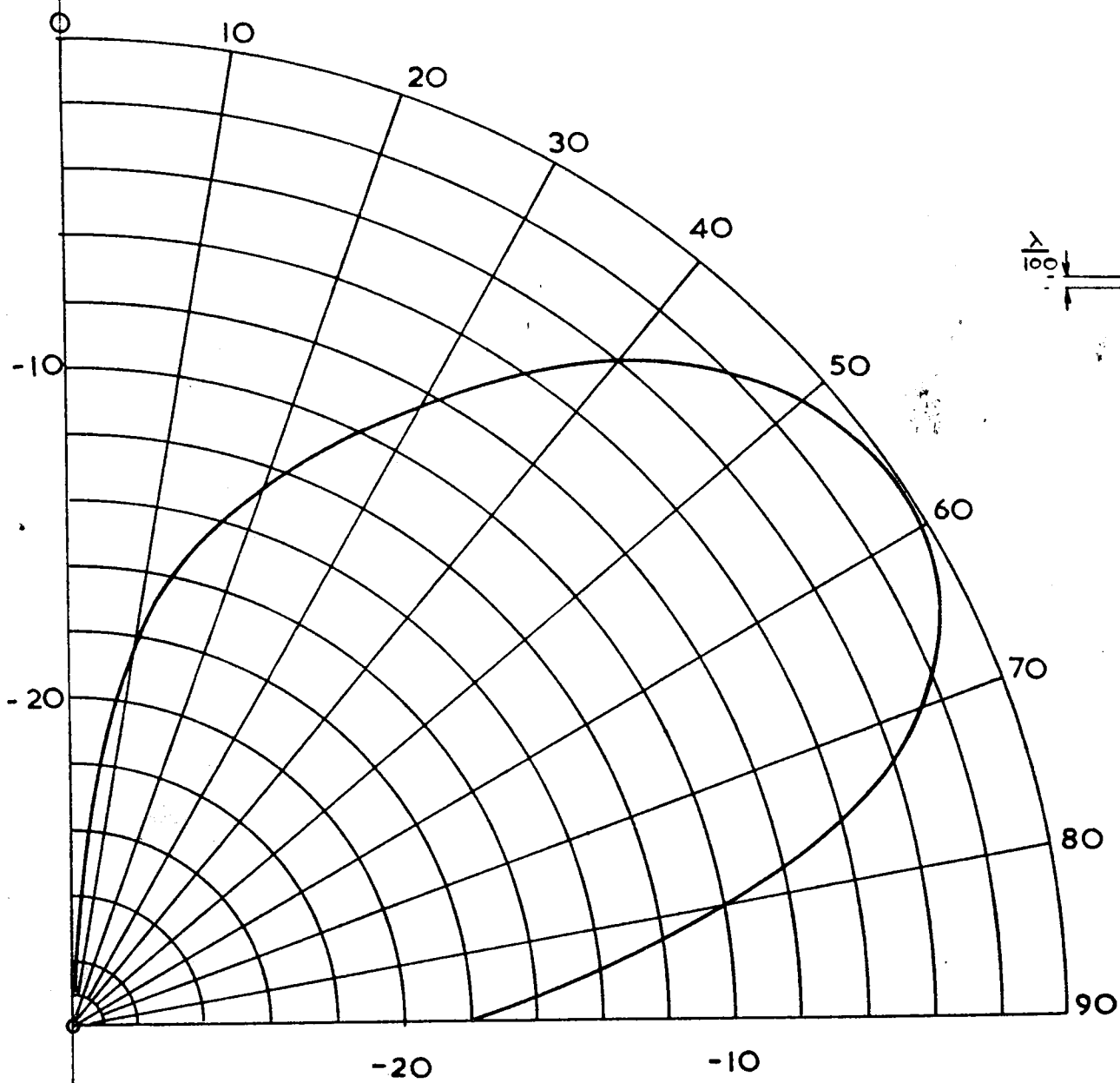
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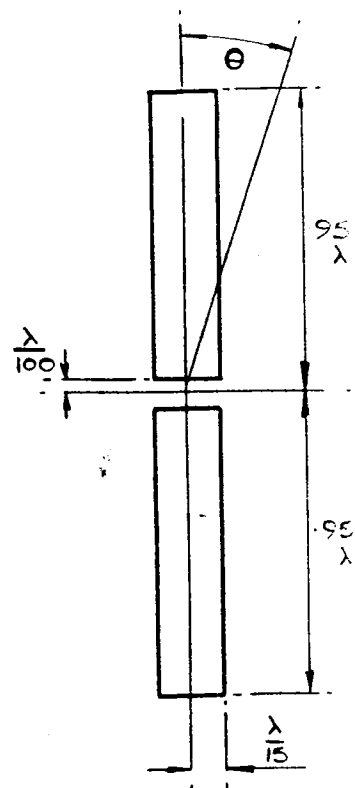
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FIG. No. II

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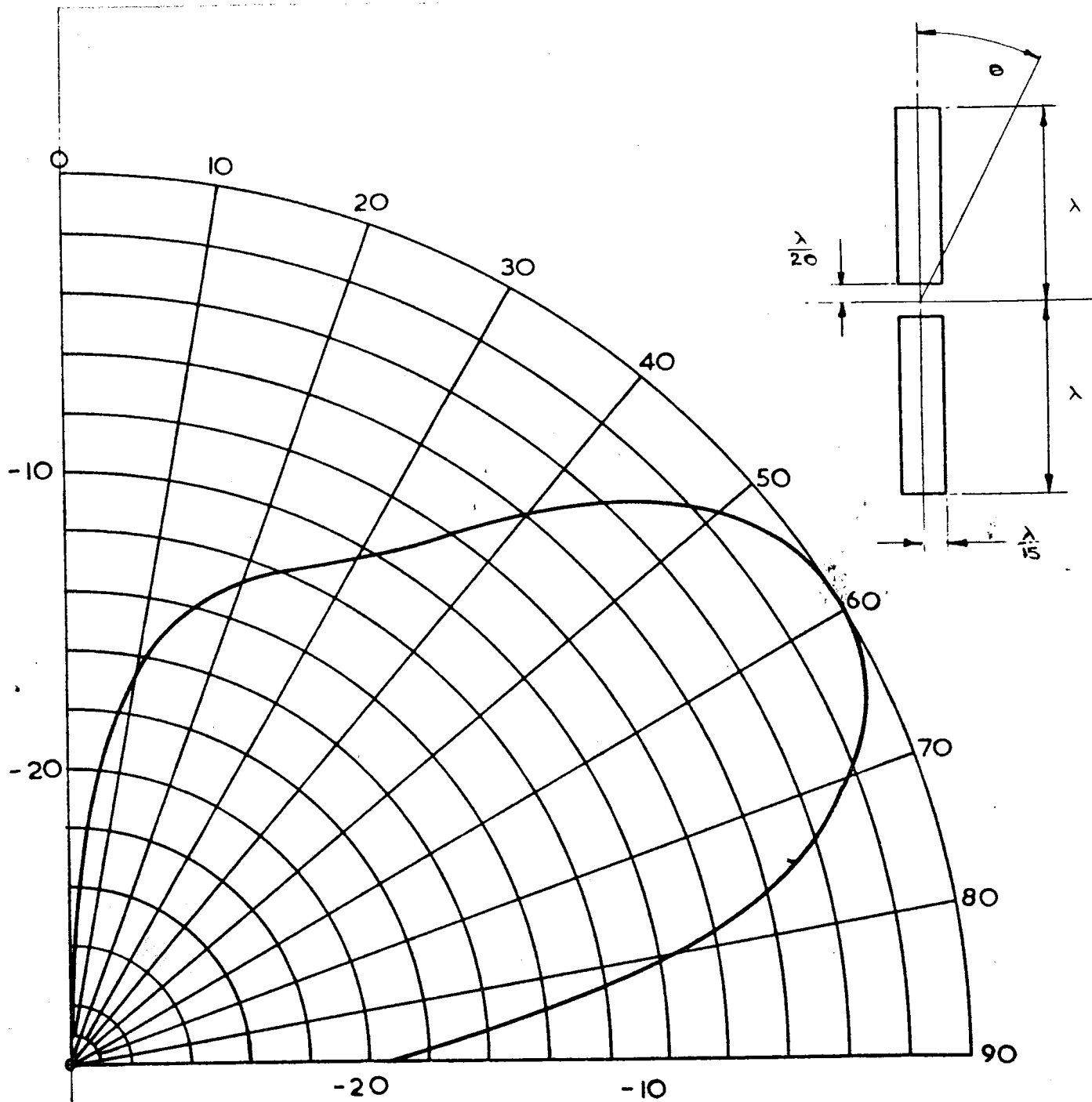
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FIG. No. 12

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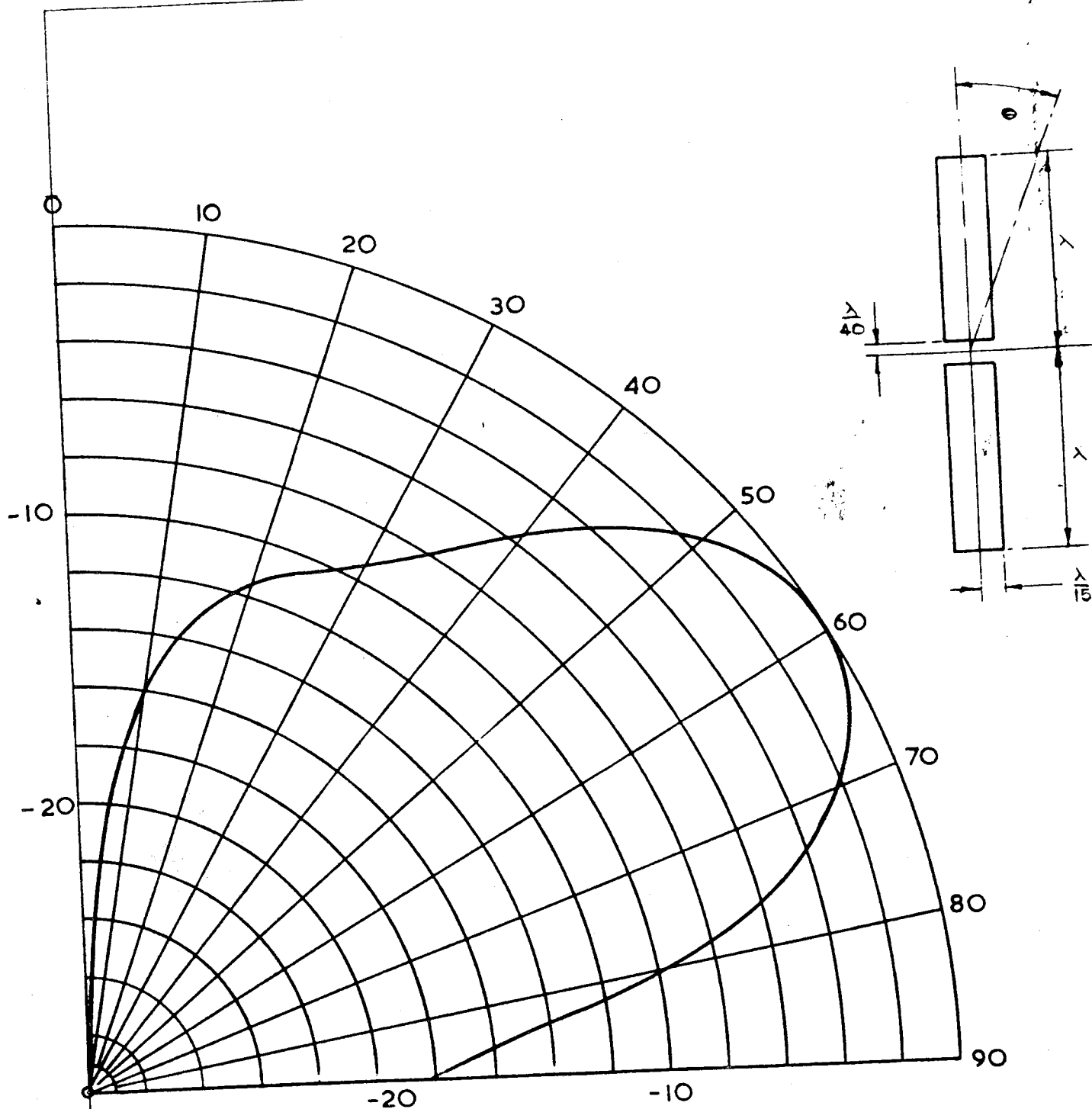
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FIG. No. 13

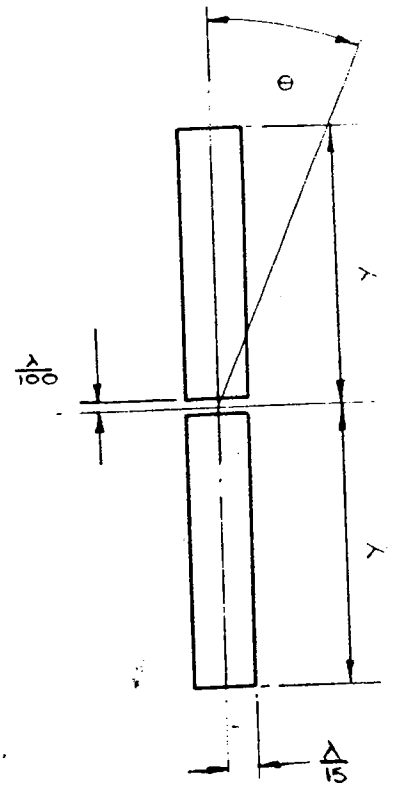
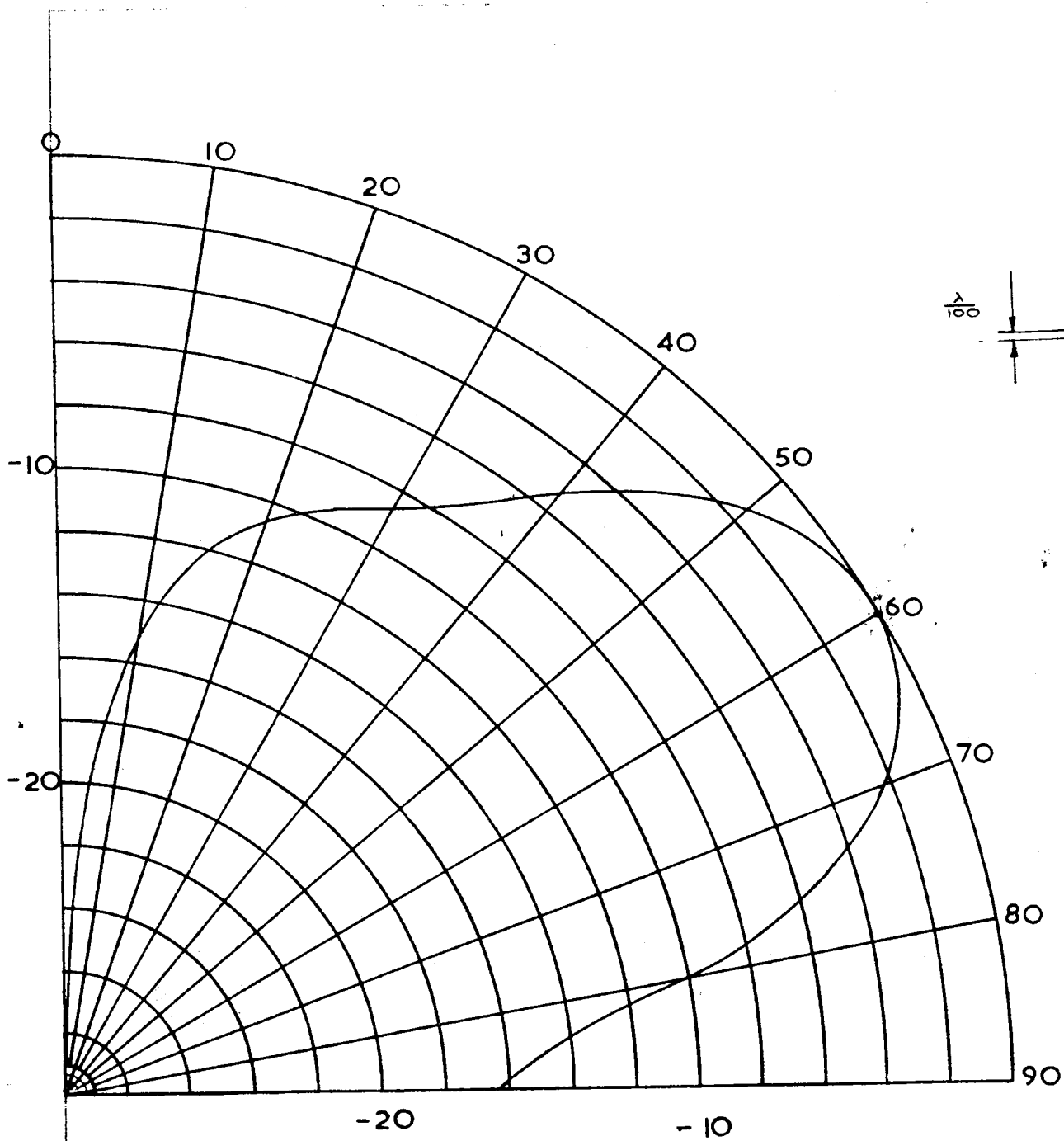
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FIG. No. 14



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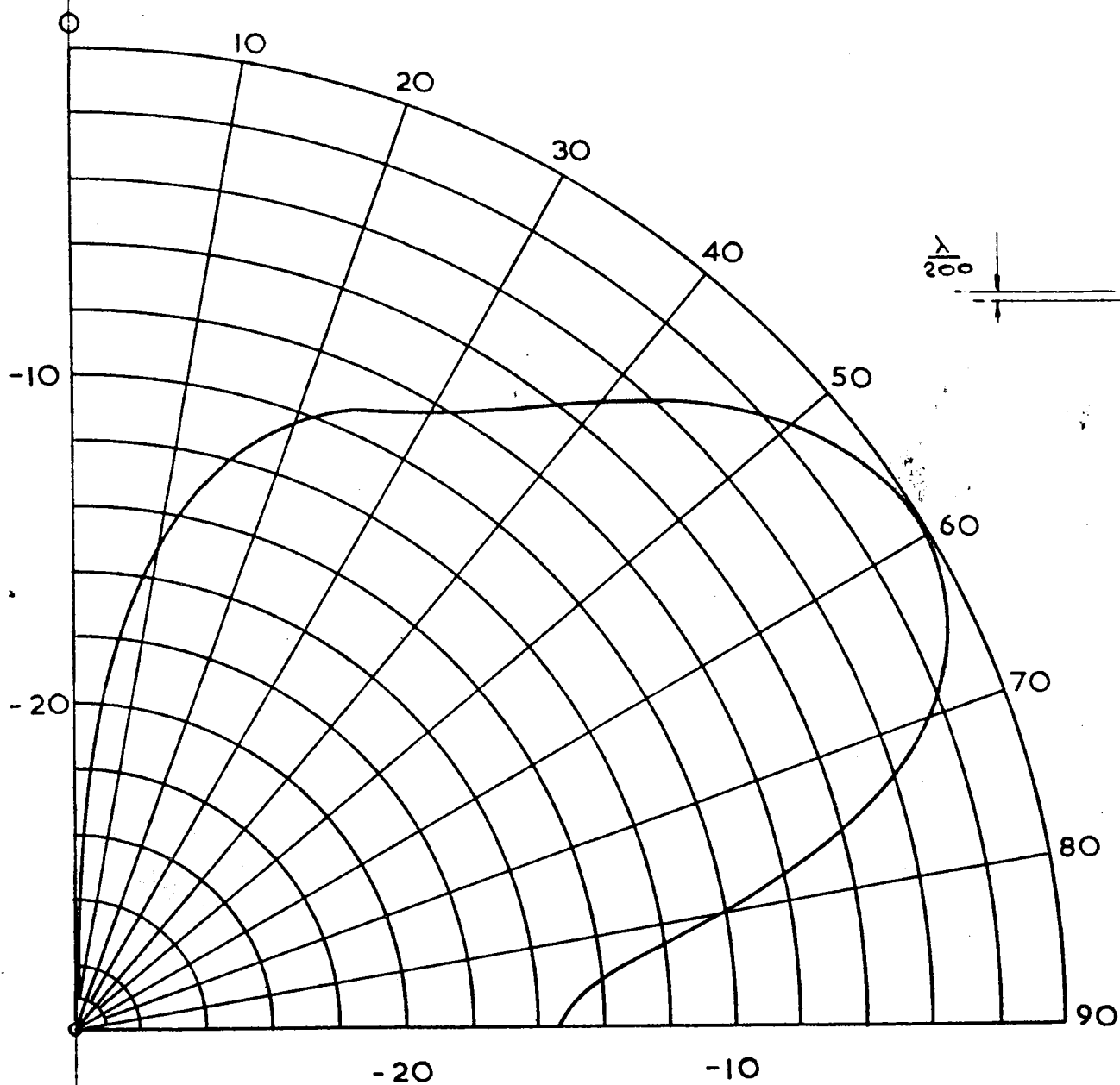
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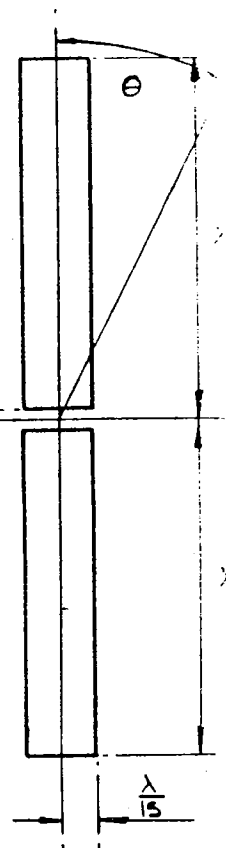
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FIG. No. 15

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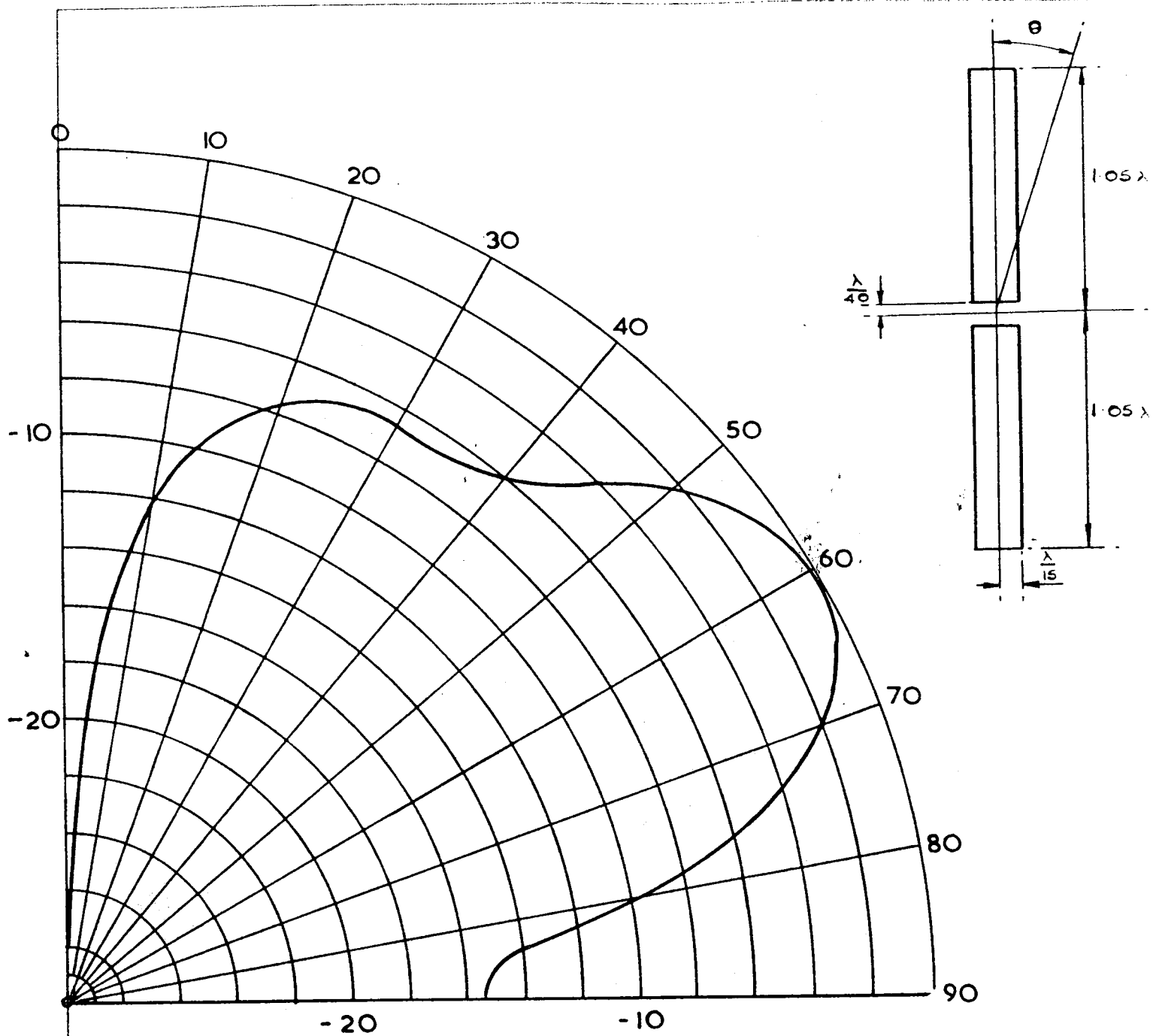
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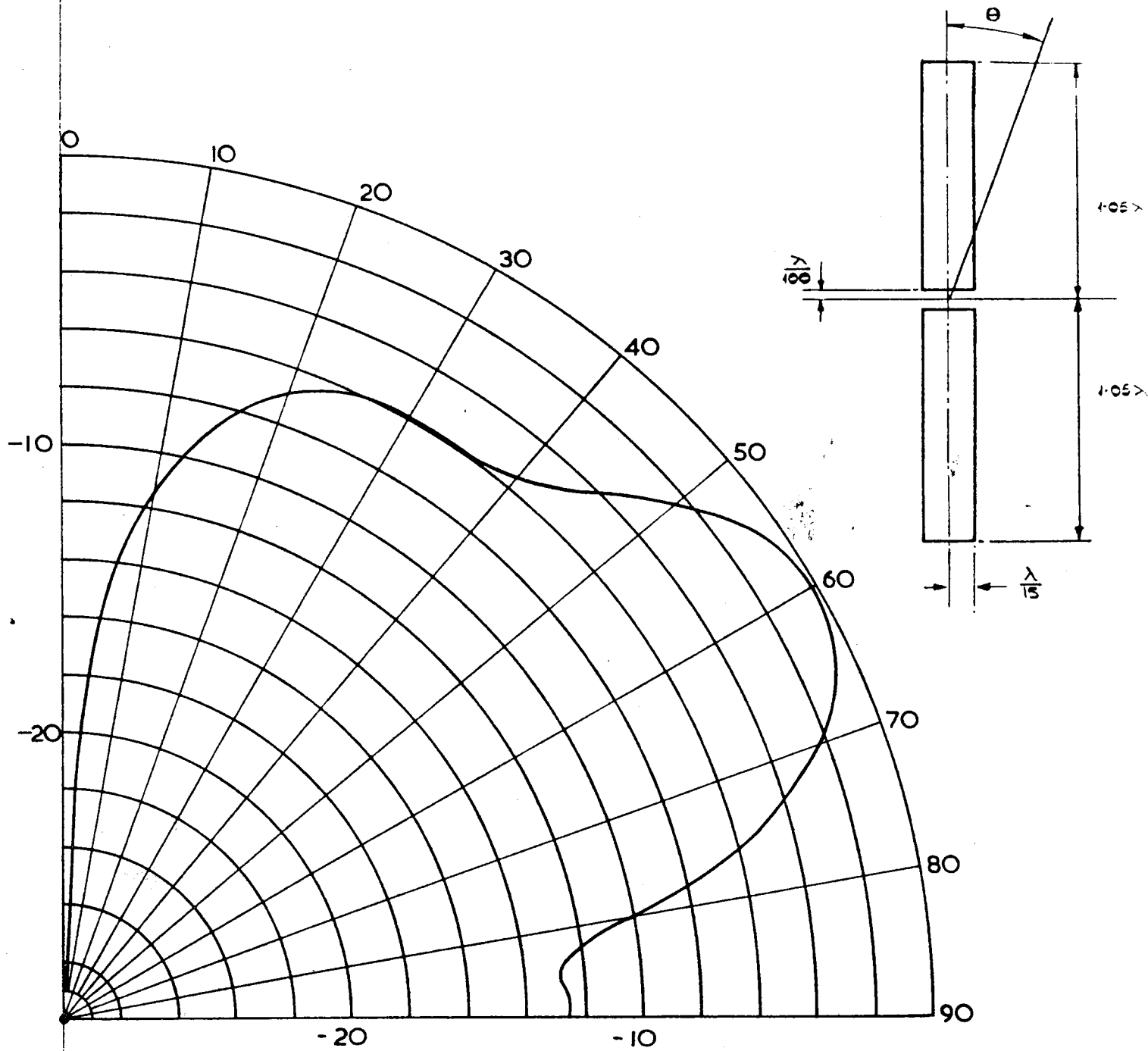
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FIG. No. 17

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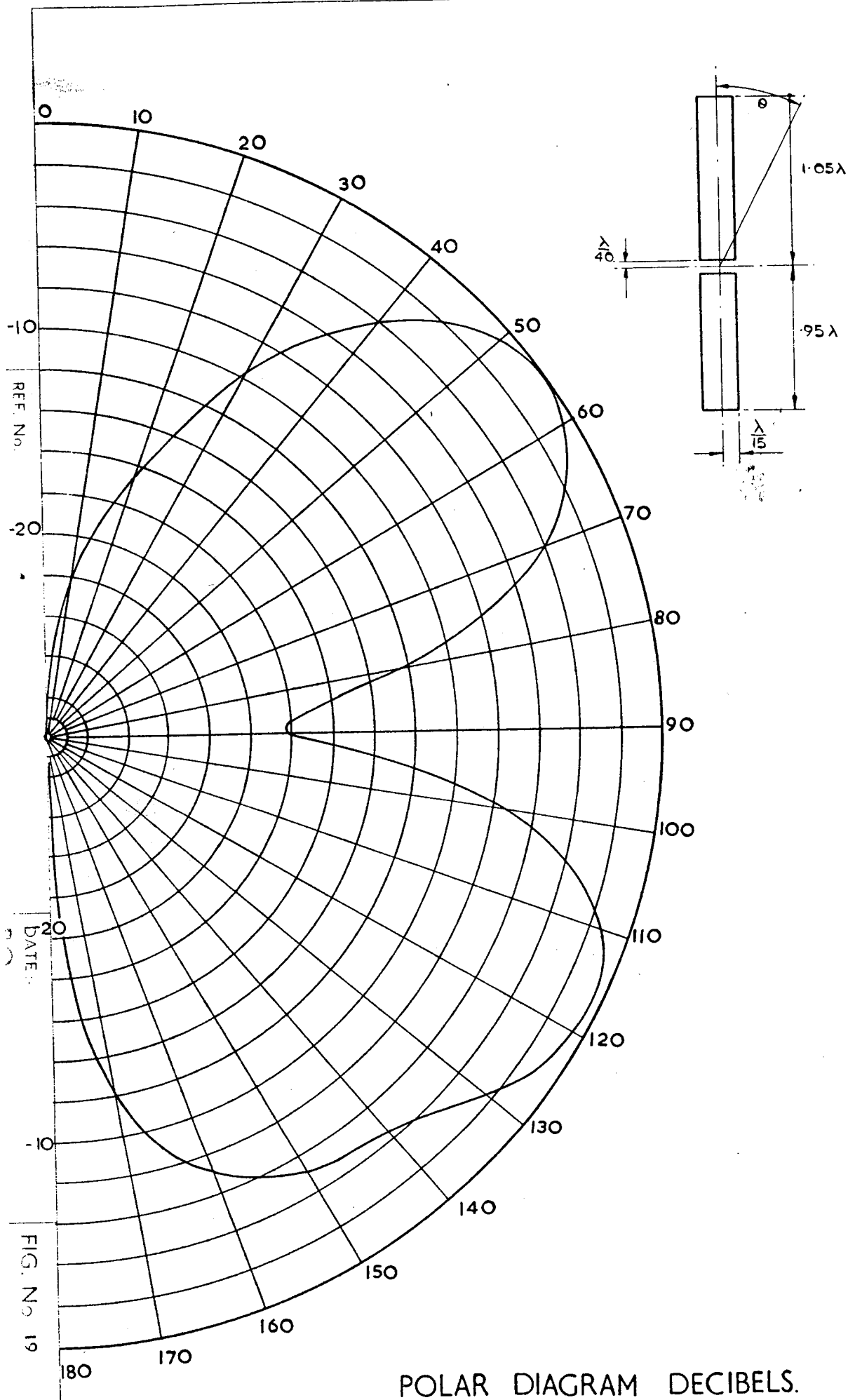
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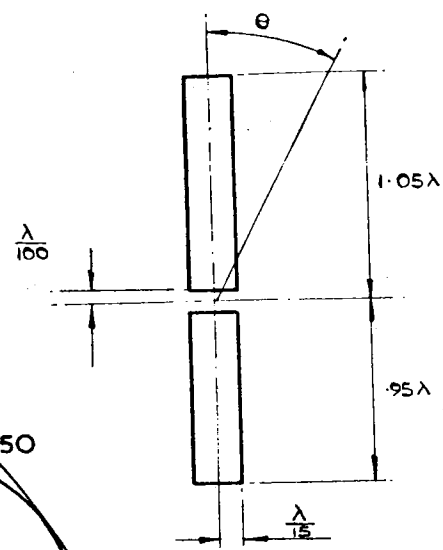
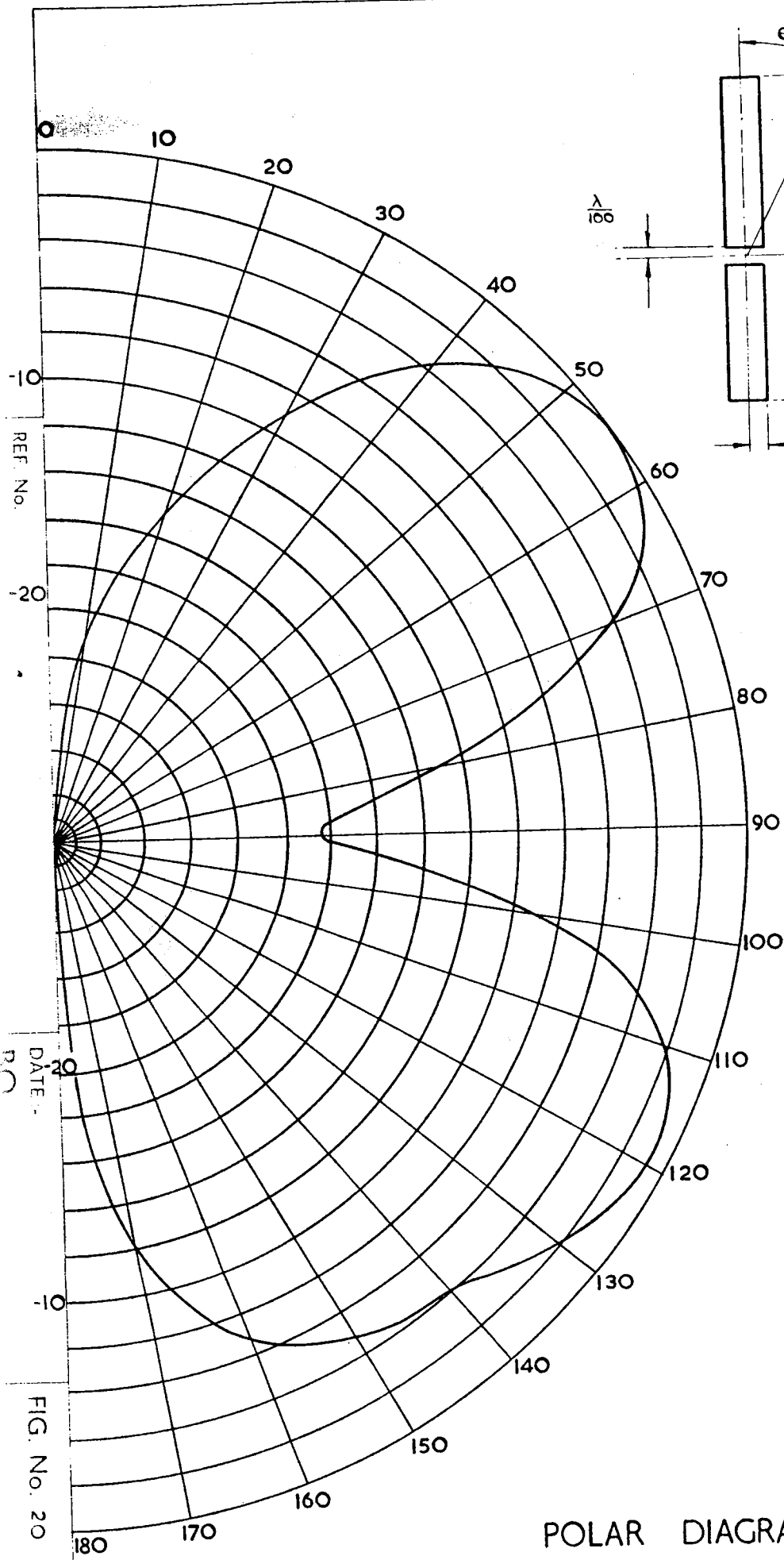
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FIG. No. 18

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FIG. No. 20

POLAR DIAGRAM DECIBELS.

## APPENDIX I

### HALLÉN'S METHOD

The radiation of a cylindrical antenna remains a partially solved problem, and has been the subject of some controversy. The leading worker in the field is Erik Hallén of the Royal Institute of Technology at Stockholm. His original treatment of the problem (Ref. 1) was published in 1938. In 1941 Schellkunoff published another and more physical method (Ref. 9), which he claimed to be more accurate (Ref. 10). Hallén showed that Schellkunoff's method is equivalent to a variation of his own (Ref. 2). Since that time he has developed a more advanced method described in a series of papers (Refs. 3 - 6) and in his book (Ref. 7). This is the method used here. It leads to a solution as an infinite series which would converge to the exact answer were it feasible to evaluate it. In practice some approximations have to be made.

Since by symmetry the current in a parallel cylinder is parallel to the axis, the only component of the vector potential  $\underline{A}$  will be a component  $A_x$  parallel to the axis. Also with a sinusoidal variation in time at an angular frequency  $\omega$

$$\underline{E} = \frac{c}{j\beta} (\text{grad div } \underline{A} + \beta^2 \underline{A})$$

where  $\beta = \frac{\omega}{c}$ .

If the cylinder is a perfect conductor the tangential component of  $\underline{E}$  vanishes at its surface. On the surface of the cylinder  $A_x$  therefore satisfies

$$\frac{\partial^2 A_x}{\partial x^2} + \beta^2 A_x = 0$$

and must consist of a sum of sine and cosine terms. The coefficients of these will generally change at the gap. For a centre-fed antenna there will be symmetry about the gap, and on the surface  $A_x$  will therefore take the form

$$A_x = (A_1 \cos \beta x + A_2 \sin \beta |x|) e^{j\omega t}$$



But

$$j\omega V = -c^2 \frac{\partial A_x}{\partial x}$$

so the condition that there will be a step in voltage,

$$V = \pm V_0 \text{ at } x = 0,$$

gives

$$A_x = \left( \frac{V_0}{c} e^{-j\beta|x|} + A \cos\beta x \right) e^{j\omega t}$$

When  $A_x$  is expressed in terms of the current, Hallen's integral equation is obtained. For a centre-fed cylinder of length  $2l$  and radius  $a$

$$\int_{-l}^l d\xi I(\xi) \frac{1}{2\pi} \int_0^{2\pi} \frac{e^{-j\beta r}}{r} d\phi = \frac{4\pi}{Z_0} (V_0 e^{-j\beta|x|} + A \cos\beta x) \quad (1)$$

where  $Z_0$  is the impedance of free space,  $\phi$  is the angle round the cylinder, and

$$r = \sqrt{(x - \xi)^2 + 4a^2 \sin^2 \frac{1}{2}\phi}$$

The term  $A \cos\beta x$  represents the effect of reflections off the ends of the cylinder and the constant  $A$  has to be determined so that  $I = 0$  when  $x = \pm l$ . Hallen's original method treated equation (1) in terms of standing waves; his new method treats it in terms of travelling waves.

The outgoing wave in the absence of any reflections is the same as for an infinitely long antenna and is treated by setting  $l = \infty$  and  $A = 0$ . The kernel of equation (1) is a function of  $(x - \xi)$

$$K(x - \xi) = \frac{1}{2\pi} \int_0^{2\pi} \frac{e^{-j\beta r}}{r} d\phi$$

For an infinite interval it is therefore possible to obtain a solution by expressing the kernel as a Fourier transform in the variable  $(x - \xi)$

$$K(x - \xi) = \frac{j}{2\pi} \int_{-\infty}^{\infty} d\alpha e^{j\alpha(x - \xi)} \int_{-\infty}^{\infty} e^{j\alpha z} K(z) dz$$

and transferring the term  $e^{j\alpha\xi}$  to give an equation for the Fourier transform of the current. The current may then be obtained by the inversion formula. This procedure yields

$$I(x) = \frac{4\pi}{Z_0} V_0 \frac{j}{2\pi} \int_{\Gamma} \frac{\beta e^{-j\alpha|x|} d\alpha}{(\alpha^2 - \beta^2) I_0 K_0(a\sqrt{\alpha^2 - \beta^2})}$$

where  $I_0$  and  $K_0$  are Bessel functions, and  $\Gamma$  is a path in the complex plane which avoids  $\alpha = \beta$  and must be chosen to eliminate wave-guide terms in the current. If  $\psi$  is defined by

$$I(x) = \frac{4\pi}{Z_0} V_0 \psi \beta(x) e^{-j\beta|x|}$$

This solution may be reduced to

$$\psi = P + Q(x)$$

where

$$P = \frac{1}{\pi} \tan^{-1} \frac{\pi}{2 \left( \log \frac{4}{a\beta\pi} - \gamma \right)} + \frac{2}{\pi^2} \int_0^{\frac{\pi}{2}} \left\{ \frac{1}{\sin\phi [J_0^2(a\beta\sin\phi) + Y_0^2(a\beta\sin\phi)]} - \frac{1}{\phi \left[ 1 + \frac{4}{\pi^2} (\gamma + \log \frac{a\beta\phi}{2})^2 \right]} \right\} d\phi$$

$$Q(z) = \frac{2}{\pi^2} e^{j\beta|x|} \left\{ \int_0^{\frac{\pi}{2}} \frac{e^{-j\beta|x|\cos\phi} - e^{-j\beta|x|}}{\sin\phi [J_0^2(a\beta\sin\phi) + Y_0^2(a\beta\sin\phi)]} d\phi \right. \\ \left. + j \int_0^{\frac{\pi}{2}} \frac{e^{-\beta|x|\cot\phi}}{J_0^2\left(\frac{a\beta}{\sin\phi}\right) + Y_0^2\left(\frac{a\beta}{\sin\phi}\right)} d\phi \right\}$$

and  $\gamma$  denotes Euler's constant .5772.

The use of the Fourier transformation in this way is only effective in the case of an infinite interval. To overcome this difficulty for a finite antenna Hallen separates equation (1) into equations for outgoing and reflected waves, each valid over an infinite interval, by writing the coefficient  $A$  and the current as series

$$A = \sum_{n=1}^{\infty} (-1)^n V_n e^{-j2n\beta l}$$

and

$$I(x) = i_0(x) + \sum_{n=1}^{\infty} (-1)^n e^{-j\beta l(2n-1)} (i_n(1+x) + i_n(1-x))$$

Each successive reflected wave satisfies an equation of the form

$$\int_{-\infty}^{\infty} i_n(\xi) d\xi \frac{1}{2\pi} \int_0^{2\pi} \frac{e^{-j\beta r}}{r} d\phi = \frac{4\pi}{Z_0} V_n e^{-j\beta z}, \quad z > 0 \quad (2)$$

$$\frac{i_n(0) i_{n-1}(2l-z)}{i_{n-1}(2l)}, \quad z < 0.$$

Once equation (2) has been solved the reflection coefficient

$$\frac{V_n}{V_{n-1}}$$

can be determined for each reflection from the condition that

$$i_n(0) = e^{j2\beta l} i_{n-1}(2l)$$

Equation (2) can be solved in turn for each  $i_n$  as a complex integral involving  $i_{n-1}$ , (Ref. 6) but it is not practicable to evaluate this solution. If  $\psi_n$  is defined by

$$i_n(z) = \frac{4\pi}{Z_0} V_n \psi_n(\beta z) e^{-j\beta z}$$

it may be expected that  $\psi_n$  differs very little when  $n > 1$  from the limiting form  $\psi_\infty$  for reflection of a wave approaching from an infinite distance. The replacement of  $\psi_n$  by  $\psi_\infty$  leads to a substantial simplification, because the reflection coefficient then becomes a constant and the series for the current is reduced to a geometric series.  $\psi_\infty$  can be expressed as an integral which again has never been evaluated for general values of  $z$ . Hallén has, however, evaluated the exact value of  $\psi_\infty(0)$ , which gives the exact value of the reflection coefficient. For other values of  $z$  it is necessary to fall back on a solution by iteration of the linearised integral equation for  $\psi_\infty$ . This leads to a series in negative powers of the parameter

$$\Omega_z = \log(2\beta z) - l(2\beta z) - 2 \log \beta a - \gamma - j \frac{\pi}{2}$$

where

$$l(u) = \int_u^\infty \frac{e^{-j(\xi - u)}}{\xi} d\xi$$

Hallén and his associates have tabulated the coefficients of this series, which involve iterated sine and cosine integrals, as far as the term in  $\Omega_z^{-4}$  for cylinders of varying diameter (Refs. 4 and 8). Unfortunately the series begins to oscillate violently when  $z$  approaches zero, and it has been found best to interpolate in the amplitude and phase of  $\psi_\infty$  between the exact values at zero and the values given by the series at a distance  $z$  depending on the diameter.

When  $\psi_{\infty}$  is substituted for  $\psi_n$ , the expression for the current in a cylinder fed at the centre by a voltage  $2V_0$  becomes

$$I(x) = \frac{4\pi V_0}{Z_0} \psi(\beta x) e^{-j\beta|x|}$$

$$- \frac{4\pi V_0}{Z_0} \frac{\psi(\beta l) e^{-j2\beta l}}{\psi_{\infty}(0) + \psi_{\infty}(2\beta l) e^{-j2\beta l}} \times$$

$$\left\{ \psi_{\infty}[\beta(1+x)] e^{-j\beta x} + \psi_{\infty}[\beta(1-x)] e^{j\beta x} \right\}$$

An advantage of the method of travelling waves is that it yields the solution for an off-centre feed without any additional calculation. If the lengths of the two sections are  $l_1$  and  $l_2$  it is only necessary to remember that the outgoing waves travel distances  $l_1$  and  $l_2$  before their first reflections. If both are followed separately the current is found to be

$$I(x) = \frac{4\pi V_0}{Z_0} \psi(\beta x) e^{-j\beta|x|}$$

$$- \frac{4\pi V_0}{Z_0} e^{-j\beta l_2} \frac{\psi_{\infty}(0) \psi(\beta l_2) - \psi_{\infty}[\beta(l_1 + l_2)] \psi(\beta l_1) e^{-j2\beta l_1}}{\psi_{\infty}^2(0) - \psi_{\infty}^2[\beta(l_1 + l_2)] e^{-j2\beta(l_1 + l_2)}} \times$$

$$\left\{ \psi_{\infty}[\beta(l_2 + x)] e^{-j\beta(l_2 + x)} \right\}$$

$$- \frac{4\pi V_0}{Z_0} e^{-j\beta l_1} \frac{\psi_{\infty}(0) \psi(\beta l_1) - \psi_{\infty}[\beta(l_1 + l_2)] \psi(\beta l_2) e^{-j2\beta l_2}}{\psi_{\infty}^2(0) - \psi_{\infty}^2[\beta(l_1 + l_2)] e^{-j2\beta(l_1 + l_2)}} \times$$

$$\left\{ \psi_{\infty}[\beta(l_1 - x)] e^{-j\beta(l_1 - x)} \right\}$$

Once the current has been found the determination of the distant field is relatively simple. The only component of magnetic field is

$$B_{\phi} = \frac{\mu_0}{4\pi c} \iiint \frac{I(t - \frac{r}{c})}{2\pi r} \sin\theta \, dx d\phi$$

where  $r$  is the distance from a point on the cylinder,  $\theta$  the angle between the direction from this point and the axis, and the integral is evaluated over the surface of the cylinder. At a sufficient distance it is possible to substitute

$$r = r_0 - x \cos\theta - a \cos\phi \sin\theta$$

where  $r_0$  is the distance from a fixed point. Then

$$B_{\phi} = \frac{\mu_0}{4\pi c r_0} e^{-j\omega(t - \frac{r_0}{c})} \int_{-l_2}^{l_1} dx \, I(x) e^{j\beta x \cos\theta} \int_0^{2\pi} \frac{d\phi}{2\pi} e^{j\beta a \cos\phi \sin\theta}$$

where the second integral may be recognized as the Bessel function  $J_0(\beta a \sin\theta)$  (Ref. 13 p 149).

These formulae have been computed on Pegasus.

## APPENDIX 2

### RADIATION FROM AN INFINITELY THIN WIRE

It is shown by Synge in Ref. 12 that the current distribution in a thin wire with lengths  $l_1$  and  $l_2$  either side of the gap is

$$I(x) = \frac{\sin(l_1 - x)}{\sin l_1}, \quad x > 0$$

$$\frac{\sin(l_2 + x)}{\sin l_2}, \quad x < 0$$

Integrating along the length of the wire one obtains for the far field

$$\begin{aligned} E &= \sin\theta \left[ \int_{-l_2}^0 \frac{\sin(l_2 + x)}{\sin l_2} e^{jx \cos\theta} dx + \int_0^{l_1} \frac{\sin(l_1 - x)}{\sin l_1} e^{jx \cos\theta} dx \right] \\ &= \frac{1}{\sin\theta} \left[ \frac{\cos(l_1 \cos\theta) - \cos l_1}{\sin l_1} + \frac{\cos(l_2 \cos\theta) - \cos l_2}{\sin l_2} \right] \\ &\quad + \frac{j}{\sin\theta} \left[ \frac{\sin(l_1 \cos\theta)}{\sin l_1} - \frac{\sin(l_2 \cos\theta)}{\sin l_2} \right] \end{aligned}$$

Since the current is everywhere at the same phase, a reversal of direction makes no alteration to the phase differences of the field from different parts of the wire, and hence no alteration to the field strength. Since there is also rotational symmetry about the axis of the wire, it follows that the field strength is equal at  $\theta$ ,  $-\theta$ ,  $\pi - \theta$ ,  $\pi - \theta$ .

If the total length of the wire is equal to a whole number of half wavelengths, a shift in the feed point does not in general alter the distribution of current, since the condition that the current shall be equal on either side of the feed point continues to be fulfilled. If, however, the length is equal to an even number of half wavelengths there is a different distribution when the feed point is exactly at the centre, the phase of the current in one half of the wire being reversed.



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