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LIFT OF A BODY OF CIRCULAR SECTION
WITH 3 OR MORE WINGS

by

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SUMMARY

The lift of a body of circular section with 3 or more wings is determined by slender body theory.

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1. In Reference 1 Sacks derives some general results for the lift of slender bodies of arbitrary cross section, using the method of residues to evaluate contour integrals for the force at each section.

2. Sacks retains second order terms in the pressure and finds that the lift depends only on the cross section at the base, the longitudinal variation of cross section being immaterial. This is contrary to the results of Spreiter (Reference 2) for delta wings on parallel and conical bodies, but Spreiter does not retain the second order terms in the pressure.

3. From Table 1 and page 33 of Reference 1 the lift slope for a circular section body with 3 or more wings symmetrically disposed is

\[ C_{L_0} = \frac{5}{3}\left[\frac{2\pi r_0^2}{\text{base}} - S\right] \]

independent of the roll angle, where \( S \) is the section area, \( r_0 \) is the reference area, and \( r_0 \) is the radius of the circle to which the section would be mapped by a conformal transformation. Now the transformation

\[ \frac{w^n}{p^n} + 2 + \frac{r^n}{w^n} = \frac{a^n}{p^n} \left[ \frac{z^n}{a^n} + 2 + \frac{a^n}{z^n} \right] \]

where

\[ \frac{r^n}{a^n} = \frac{1}{2} \left[ \begin{array}{cc} n \frac{2}{2} & n \frac{2}{2} \\ s \frac{2}{2} & s \frac{2}{2} \end{array} \right] \]
maps a circle of radius \( a \) with \( n \) wings of gross semispan \( s \) onto a circle of radius \( p \). It follows that for \( n \geq 3 \) the lift slope of a body of circular section with \( n \) wings, referred to the base area \( \pi a^2 \), is

\[
C_{L_a} = 4 \frac{a^2}{s^2} \left[ \frac{1}{2} \left( 1 + \frac{n}{s^2} \right) \right] \frac{4}{n} - 2 \quad (1)
\]

independent of roll angle, and independent of the longitudinal variation of the cross section. This includes the contribution of the nose cone when the body is parallel at the wings. Spreiter's formula for the lift of \( n \) wings attached to a parallel body with a nose cone (Reference 2)

\[
C_{L_a} = 2 \left( 1 - \frac{a^2}{s^2} + \frac{4}{s^4} \right) \quad (2)
\]

is a special case of formula (1).

4. The lift slope for 3, 4, 5 and 6 wings referred to \( \pi a^2 \) as reference area is shown in Figure 1, together with some results obtained by Hughes (Reference 3). Hughes' results, which do not include second order terms in the pressure, were obtained by numerical integration, and are subject to a small error because of the singularity in the pressure at the leading edges of the wings. Hughes' method, however, enables one to obtain the forces and moments on each individual wing and the body separately (these were in fact obtained but not published) whereas Sacks' method does not, because it depends on the use of the method of residues to evaluate integrals around closed contours. To determine the optimum number of wings in a particular case one would need to know the forces on individual wings, so that one could calculate how thick they would have to be.

5. All these results depend on the assumptions of slender body theory, and their validity needs to be confirmed by experiment.
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<th>REFERENCE No.</th>
<th>AUTHOR</th>
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<tr>
<td>1</td>
<td>Alvin H. Sacks</td>
<td>N.A.C.A. Technical Note 3263. 1954.</td>
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