HAWKER SIDDELEY DYNAMICS LIMITED

COVENTRY

H.S.D./COV. TECHNICAL MEMORANDUM R.228

REACTION TIME AND THE SEQUENTIAL INTERCEPTION OF MULTIPLE TARGETS

by

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SUMMARY

A formula is found for the minimum time between the arrival of successive targets, which will allow all of them to be intercepted. This time, which is a function of the number of targets, may be negative, indicating that the targets could be engaged in reverse order.

SEPTEMBER, 1966.

REACTION TIME AND THE SEQUENTIAL INTERCEPTION OF MULTIPLE TARGETS

A system capable of engaging only one target at a time achieves its maximum rate of fire when each target is intercepted at the minimum possible range, so that the time of flight of each missile is at a minimum. For a limited period, however, it is possible to intercept targets with a frequency of arrival greater than the maximum rate of fire by intercepting the first target at the maximum possible range, and later targets successively closer in.

Assume

(1) that the minimum time between the completion of an engagement and the launch of the next missile is T_0: this time may be determined by the time required to disengage the tracking radar and train it on the next target or by the time required to warm up the missile

(2) that the interception is near head on so that the sum of the target speed V_T and the missile speed V_M is a sufficient approximation to the closing speed

(3) that V_T is constant

(4) that V_M is constant after an initial period of acceleration and that at the minimum range of interception the full missile speed is already achieved.

For interception at a range X the time of flight is then

$$\frac{X}{V_M} + T_1$$

where T_1 represents the time penalty due to acceleration (Fig. 1).

The total time delay or 'reaction time' is

$$T = T_0 + T_1$$

and the total time of interception is thus

$$t = \frac{X}{V_M} + T \tag{1}$$

Also if the interception begins when the target is at a range R

$$X = R - tV_T$$

and

$$t = \frac{R + V_M T}{V_T - V_M} \tag{2}$$

If X_o is the minimum range of interception, the minimum time of interception is

$$t_o = \frac{X_o}{V_M} + T$$

and this is the minimum time interval between targets which can be sustained indefinitely.

Suppose, however, that the number of targets to be intercepted is limited, and that the time interval between the arrival of each target is t. If the interception of the n^{th} target begins when it is at a range R_n, and takes a time t_n (Fig. 2), then

$$R_{n-1} - R_n = V_T\,(t_{n-1} - T)$$

whence from (2)

$$t_{n-1} - t_n = \frac{V_T}{V_T + V_M}\,(t_{n-1} - t)$$

Writing

$$k = \frac{V_M}{V_T + V_M}$$

it follows that

$$t_n - t = k(t_{n-1} - t) = k^{n-1}(t_1 - t) \qquad (3)$$

The n^{th} interception will not be completed if the time available is less than the minimum time of interception, so the n^{th} target can be intercepted only if

$$t_n \geqslant t_o$$

or

$$t \geqslant \frac{t_o - t_1 k^{n-1}}{1 - k^{n-1}} \qquad (4)$$

If the first target is intercepted at a range

$$X_1 = r\,X_o$$

then substituting from (1), it can be seen that when there are n targets, the minimum time interval between the arrival of successive targets which will allow all of them to be intercepted is

$$t = T + \frac{X_o}{V_M}\,\frac{1 - rk^{n-1}}{1 - k^{n-1}} \qquad (5)$$

Evidently t is least when the first interception takes place at maximum range. If t is negative the targets could be intercepted in reverse order, the last target first.

Since $k < 1$, it can be seen from (4) that as the number of targets increases, so the minimum acceptable time interval between targets approaches the minimum time t_0 for a single interception. This is illustrated in Figs. 3 and 4 for different values of the ratio of the time t_1 of the first interception to t_0. This ratio is a measure of the difference between the range of the first interception and the minimum range of interception.

Figs. 5 - 12 show the application of (5) to some particular cases. Figs. 5 and 6 show the effect of varying the missile speed when the maximum and minimum ranges of interception are 15000 and 3000 ft., for target speeds of 1500 and 3000 f.p.s. Figs. 7 - 10 show the effect of varying the maximum range of interception when the minimum range of interception is 3000 ft., for different combinations of target and missile speeds. It can be seen that against fast moving targets, once the missile speed is equal to the target speed, there is little advantage to be gained from a further increase of speed, and it is then better to increase the maximum range of interception. Finally, Figs. 11 and 12 illustrate the capabilities of missiles with speeds of 1500 and 3000 f.p.s. when their maximum and minimum ranges of interception are 15000 and 3000 ft.

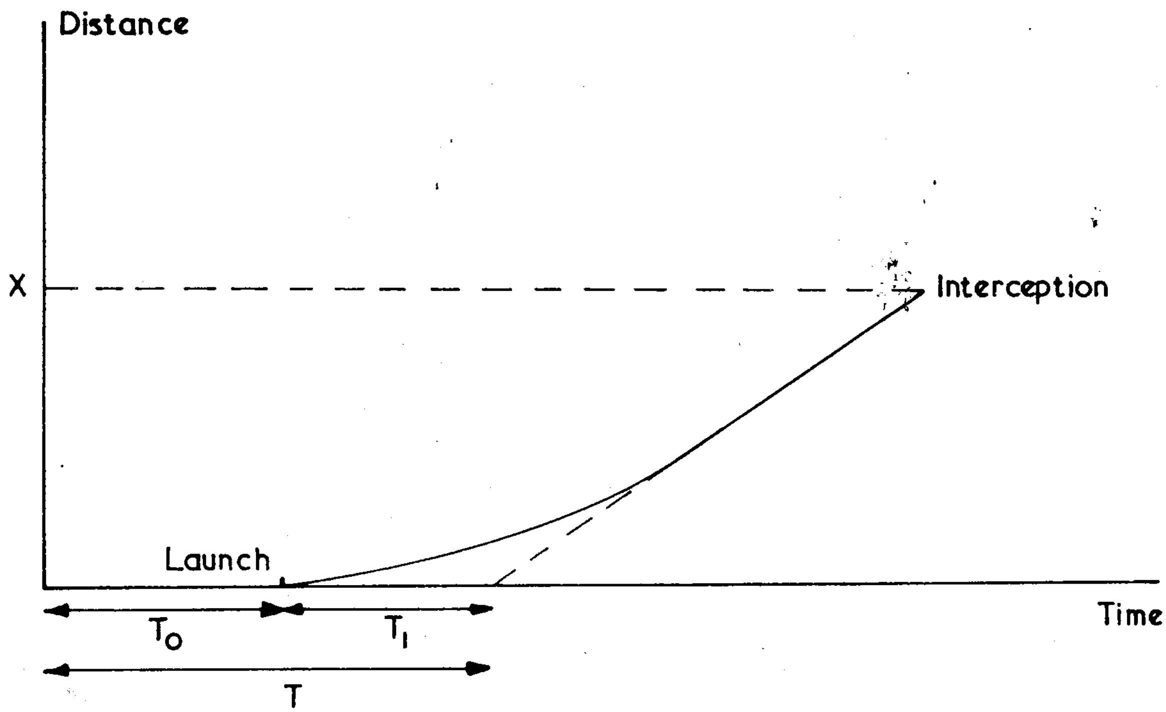

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FIG. No. 1.

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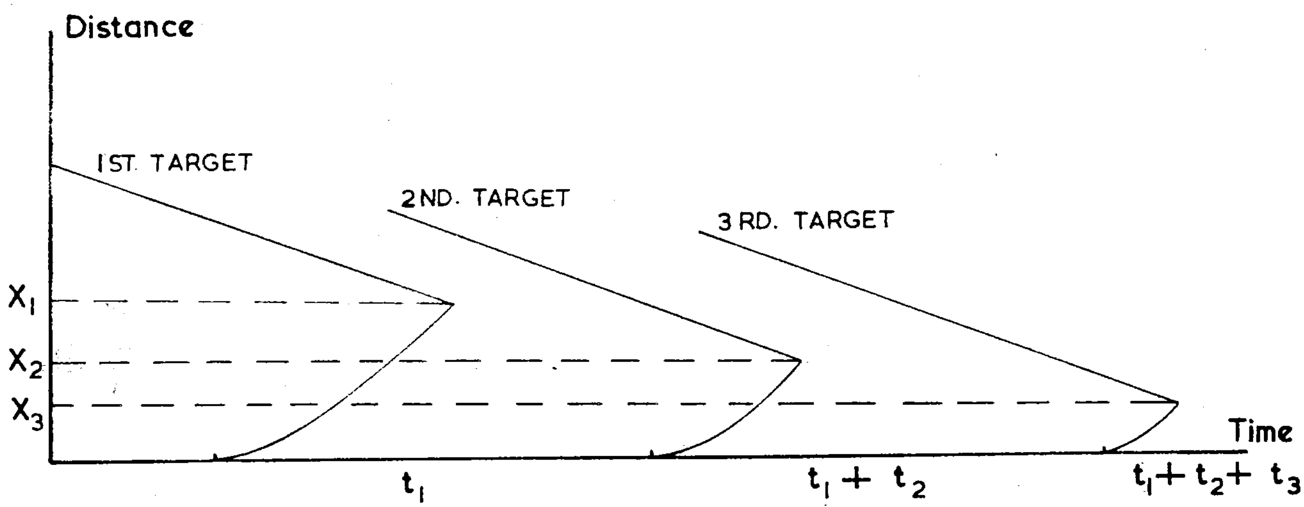
Distance
1ST. TARGET
2ND. TARGET
3RD. TARGET
X_1
X_2
X_3
Time
t_1
$t_1 + t_2$
$t_1 + t_2 + t_3$

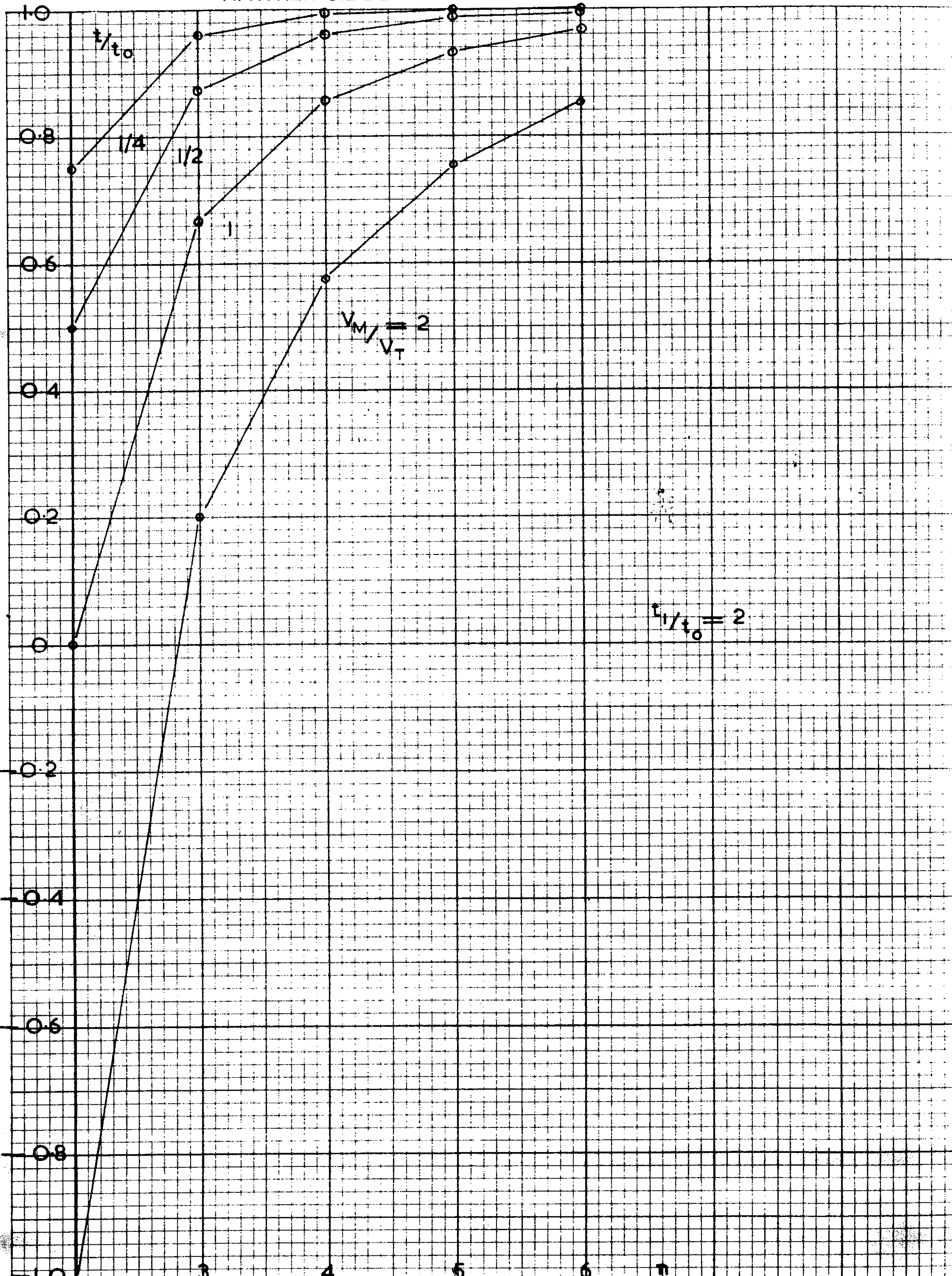

FIG. No. 3

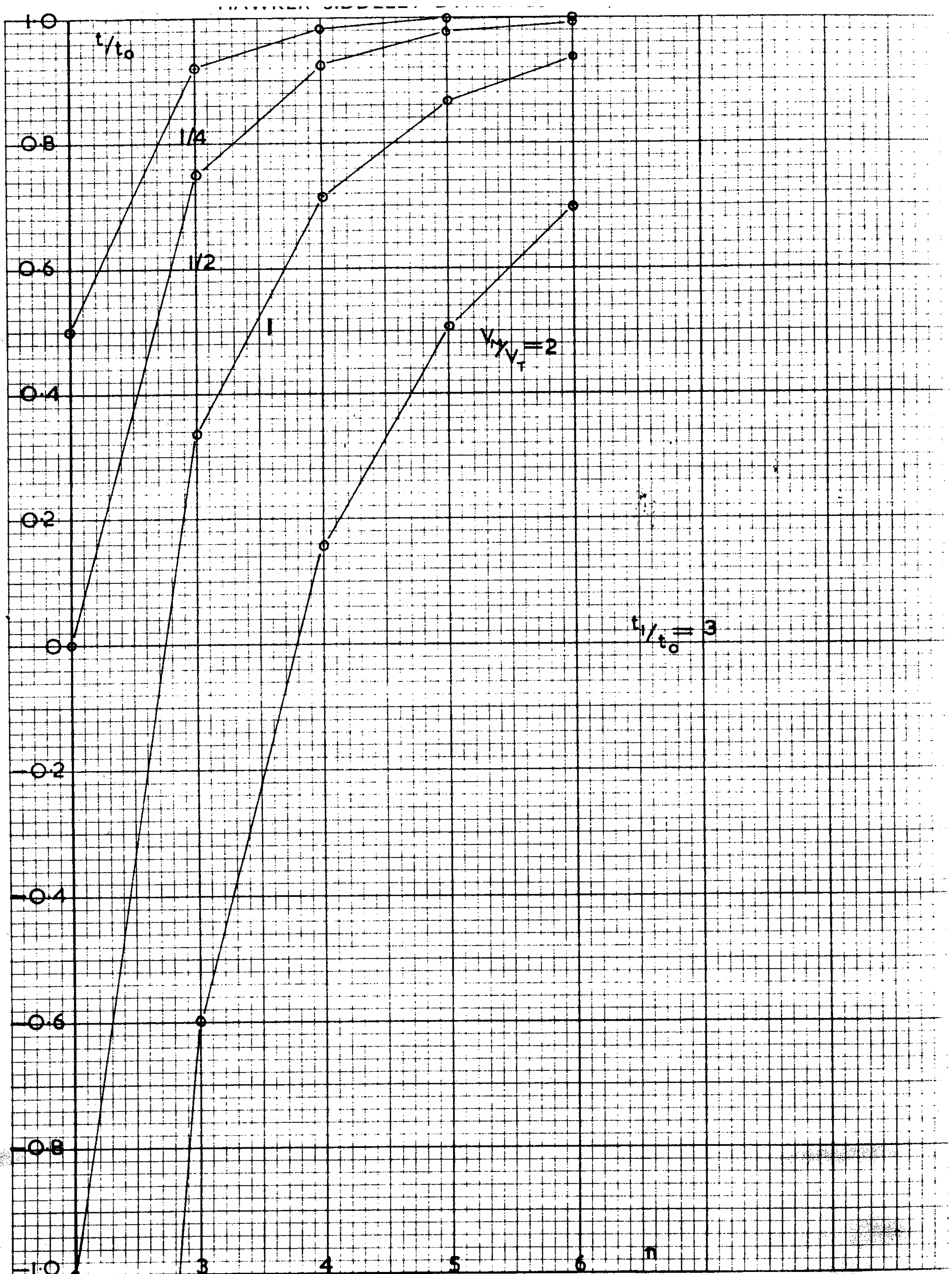

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FIG. No. 4

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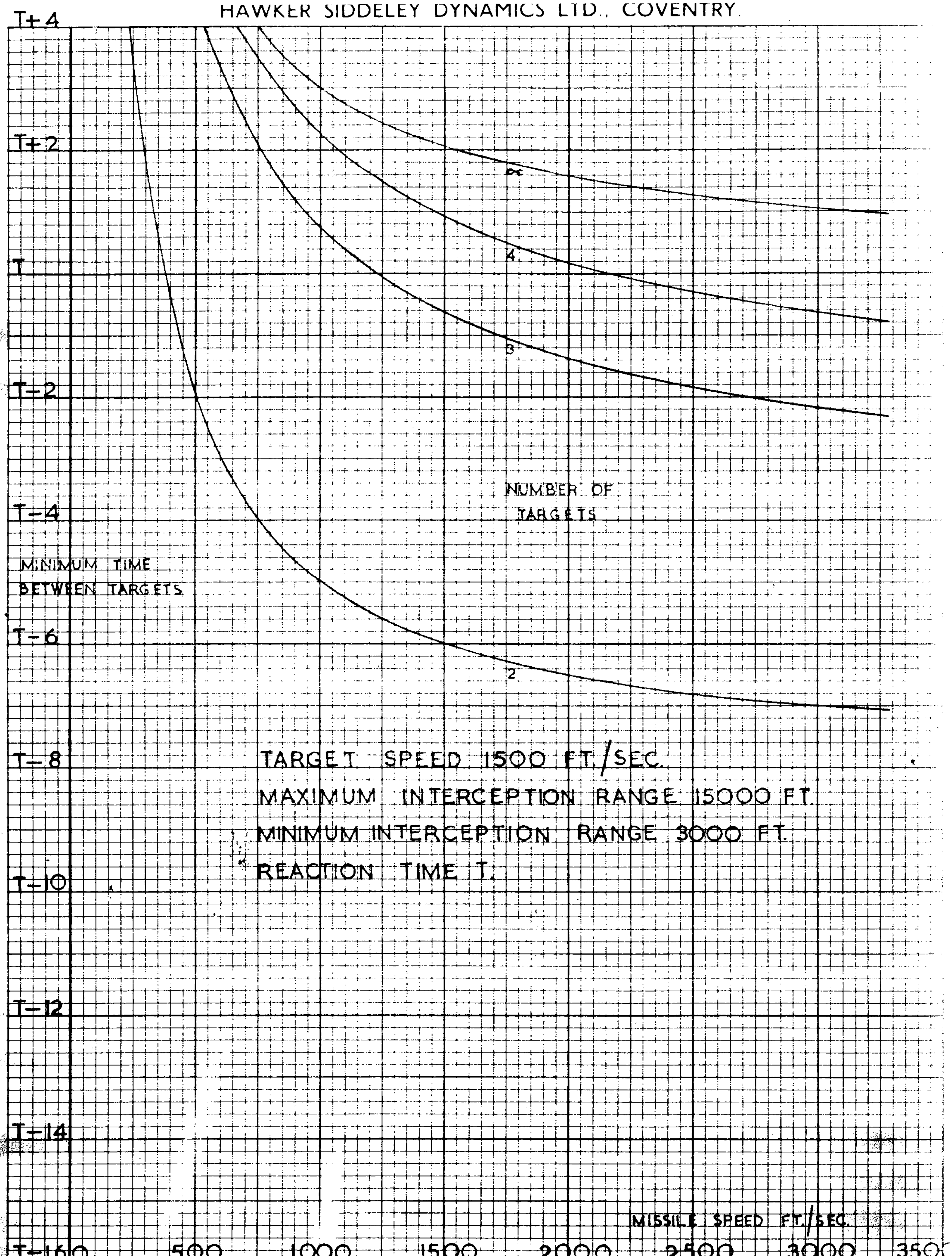

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FIG. No. 5

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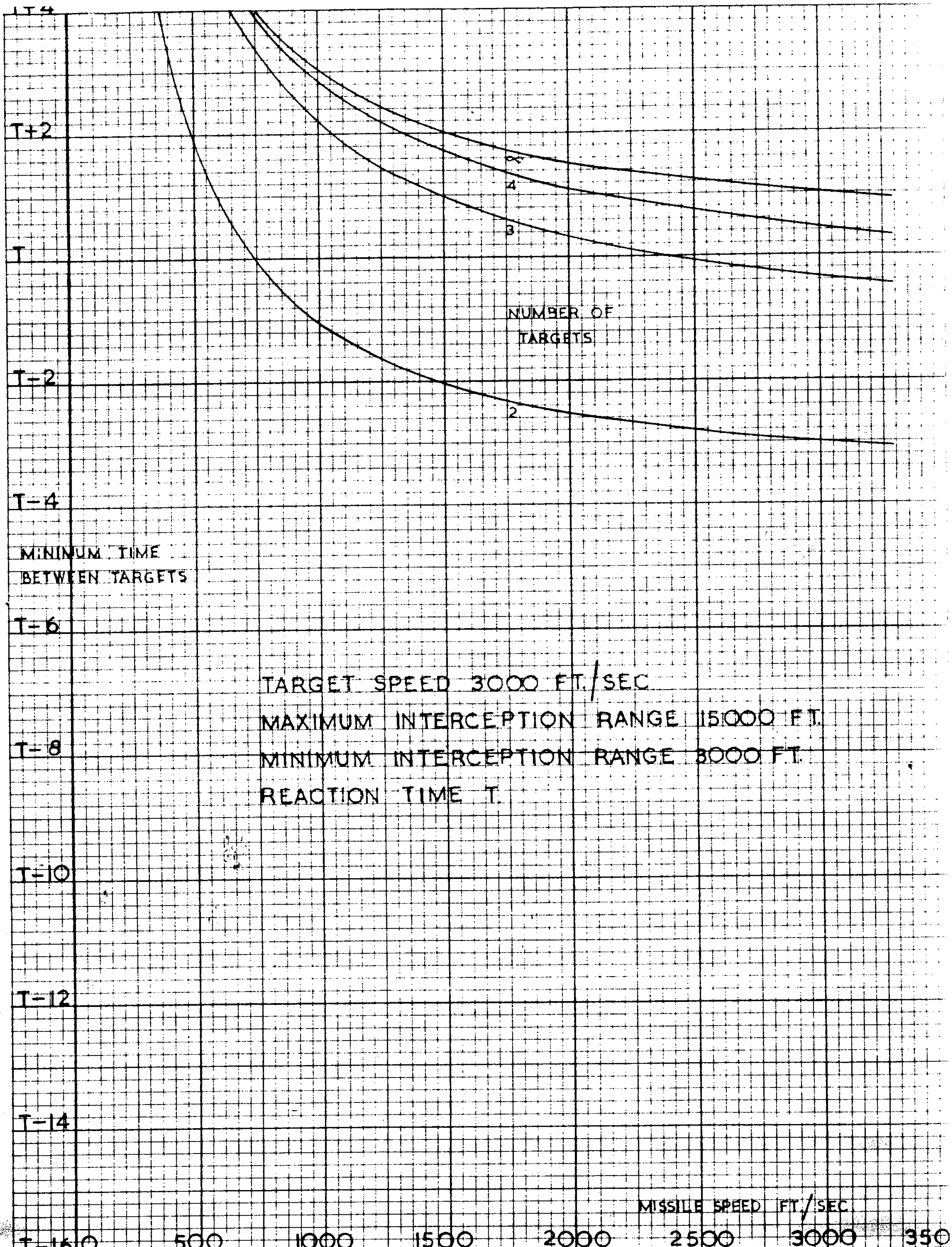

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FIG. No. 6

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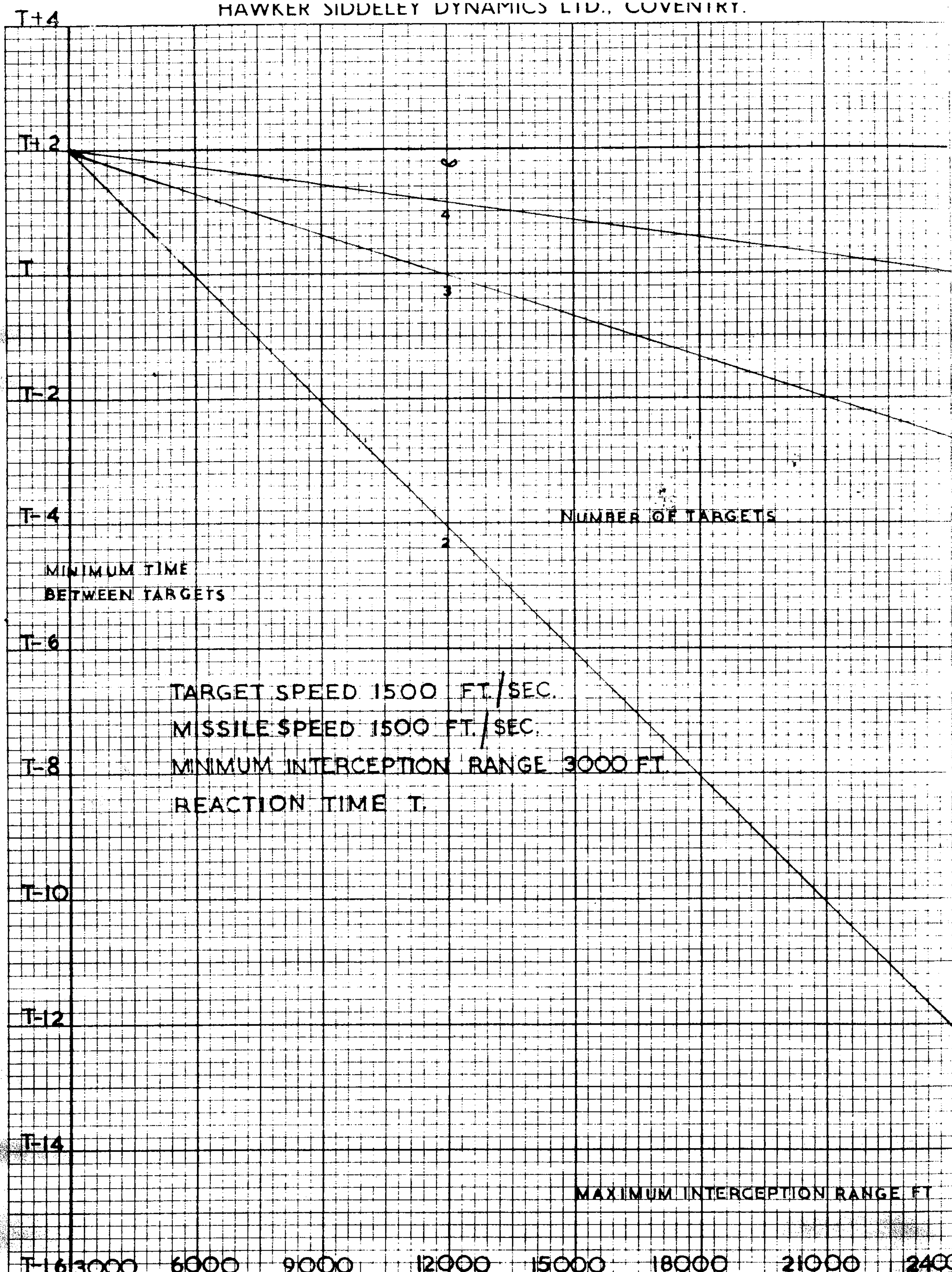

T+4
T+2
T
T-2
T-4
T-6
T-8
T-10
T-12
T-14
T-16
∞
4
3
2
NUMBER OF TARGETS
MINIMUM TIME
BETWEEN TARGETS
TARGET SPEED 1500 FT./SEC.
MISSILE SPEED 1500 FT./SEC.
MINIMUM INTERCEPTION RANGE 3000 FT.
REACTION TIME T.
MAXIMUM INTERCEPTION RANGE FT
3000
6000
9000
12000
15000
18000
21000
24000

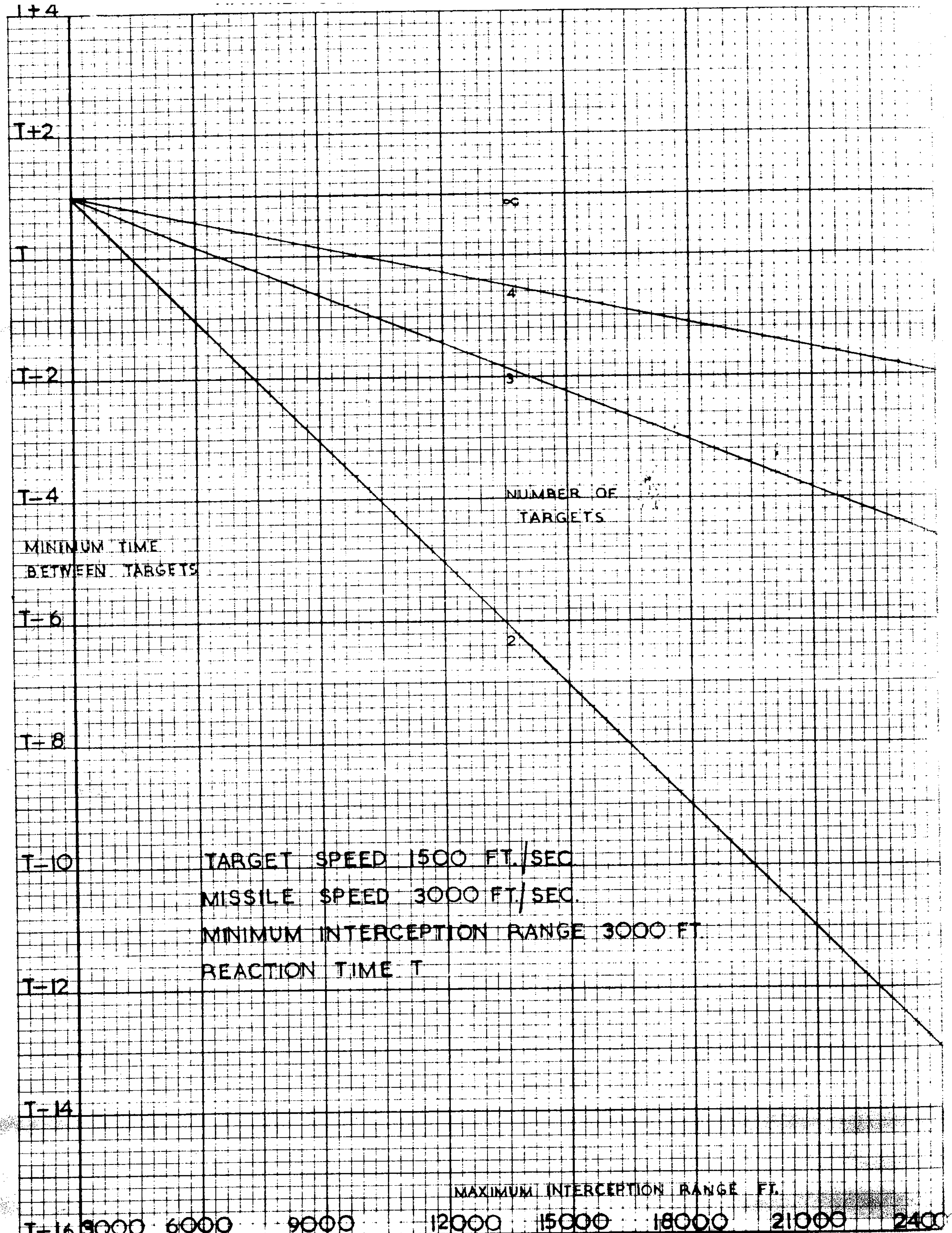

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FIG. No. 8

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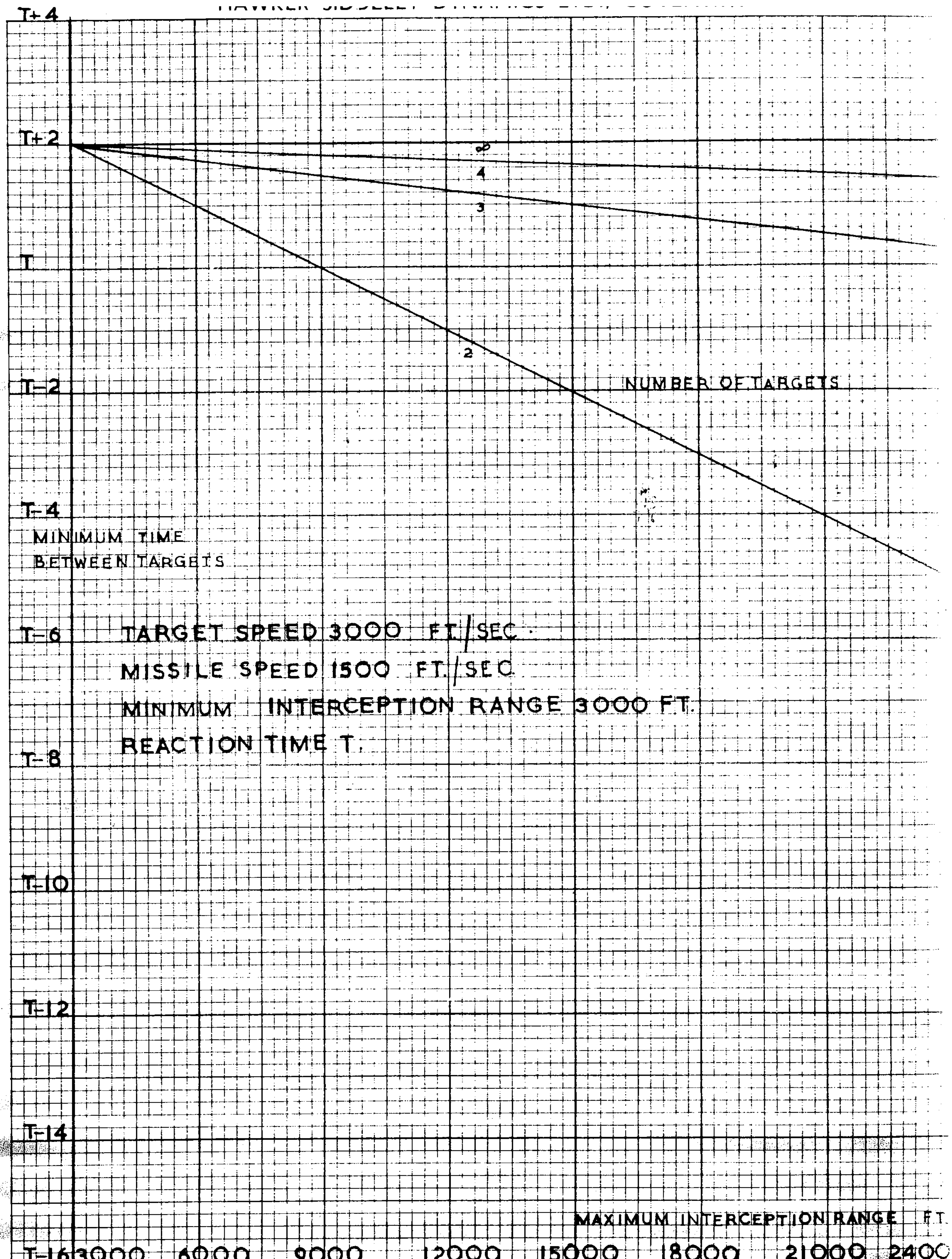

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FIG. No. 9

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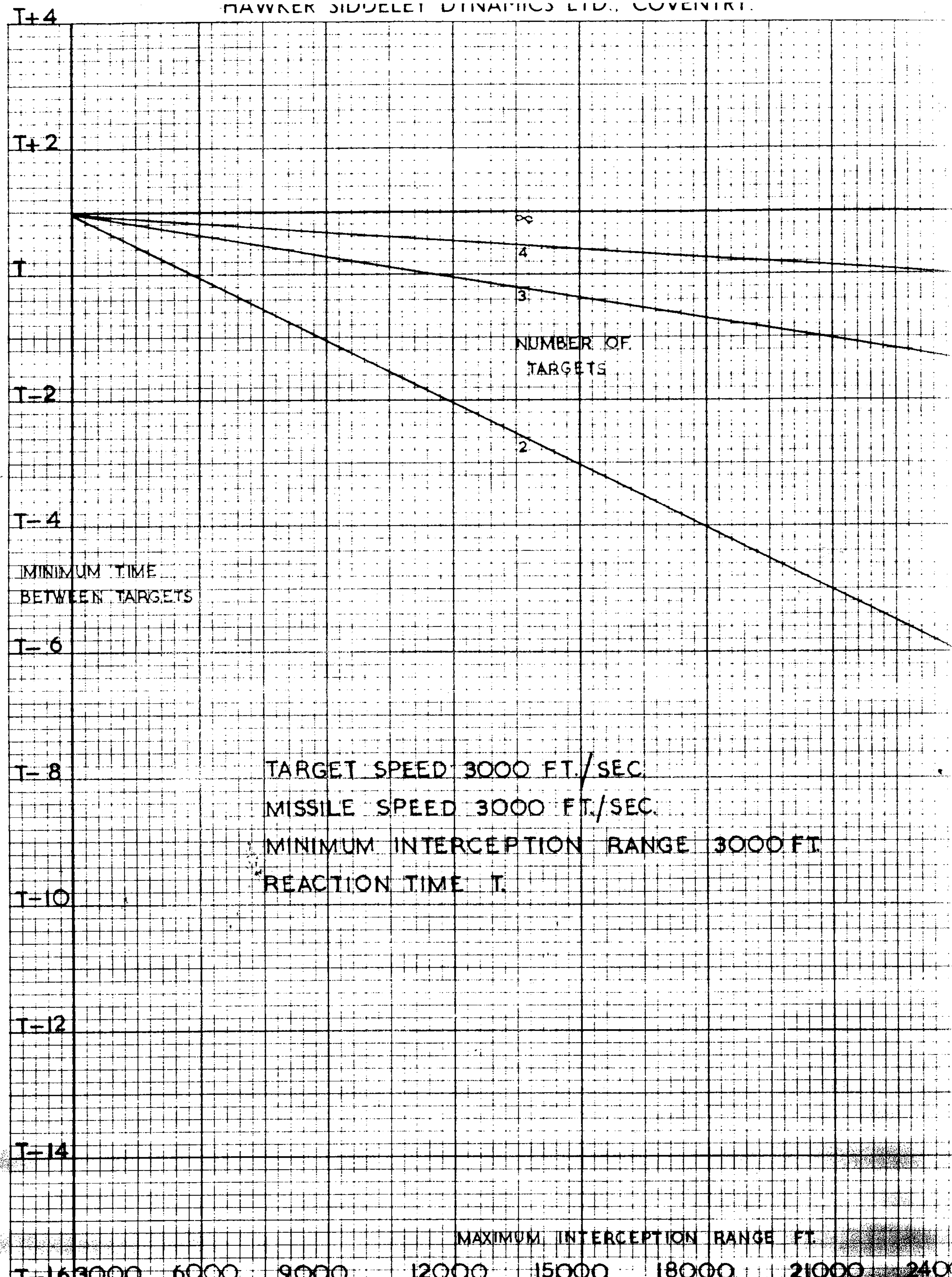

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FIG. No. 10

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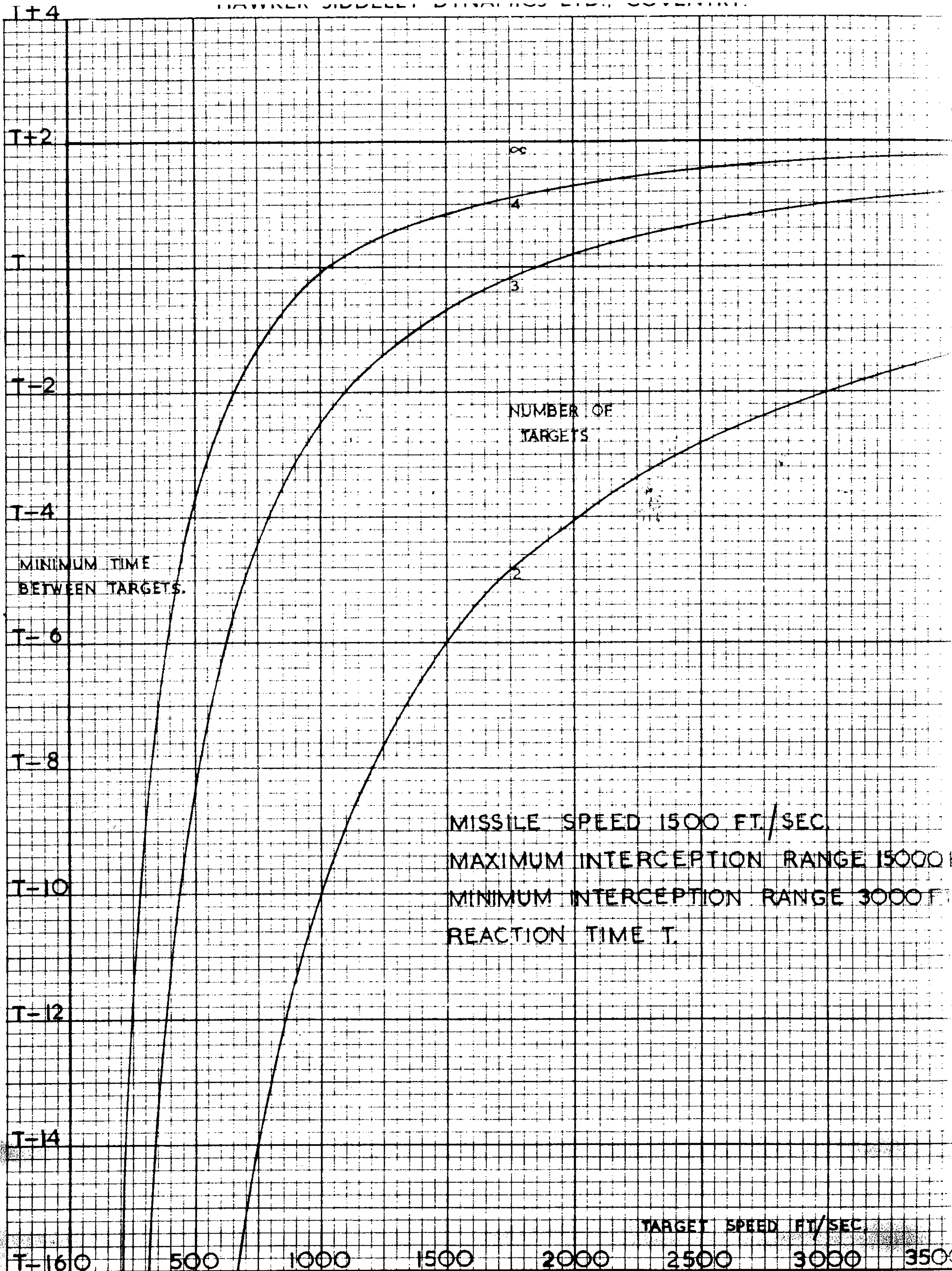

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FIG. No. 11

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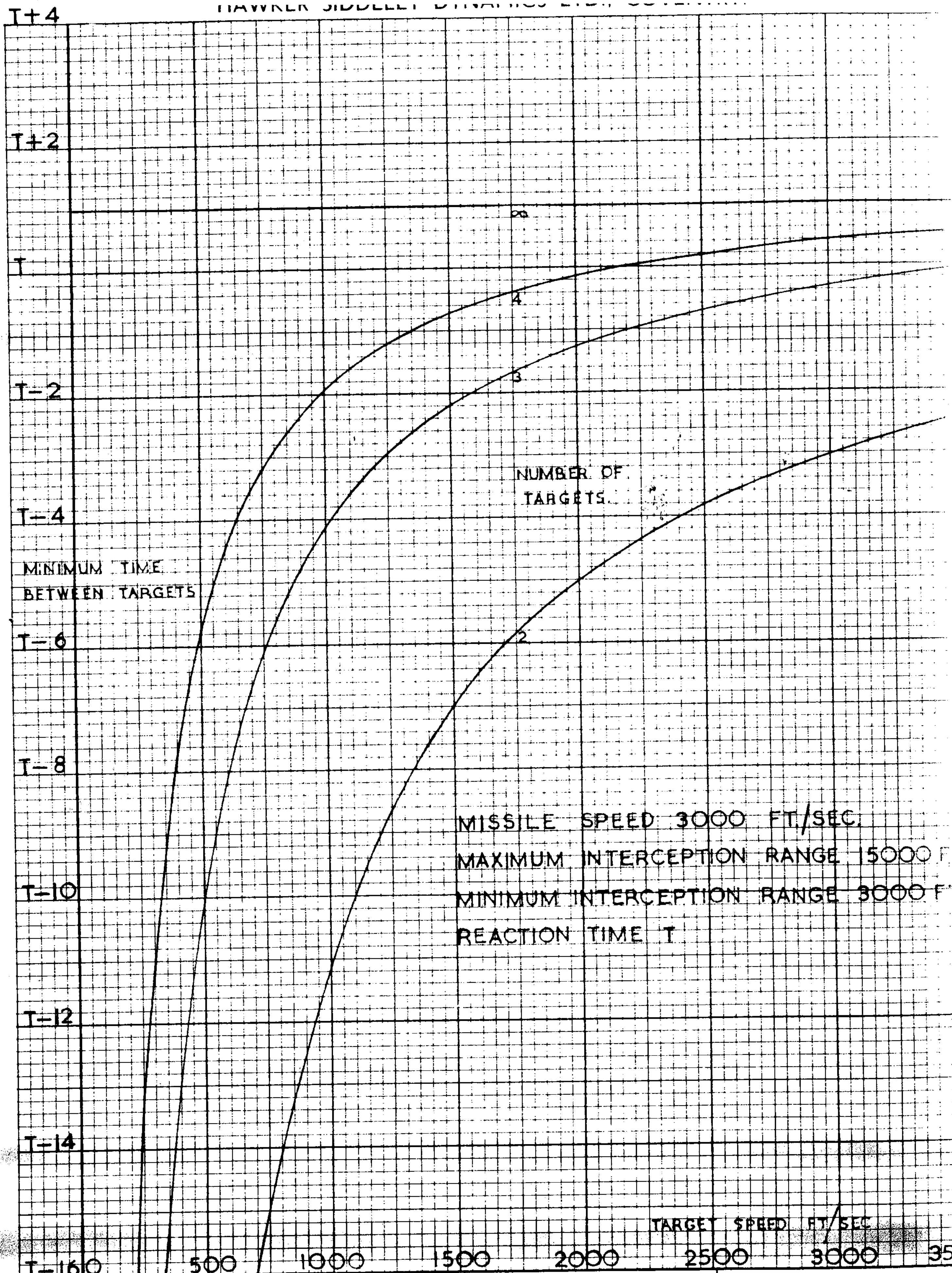

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FIG. No. 12

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