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REACTION TIME AND

THE SEQUENTIAL INTERCEPTION OF

MULTIPLE TARGETS

by

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SUMMARY

A formula is found for the minimum time between the arrival of successive targets, which will allow all of them to be intercepted. This time, which is a function of the number of targets, may be negative, indicating that the targets could be engaged in reverse order.

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A system capable of engaging only one target at a time achieves its maximum rate of fire when each target is intercepted at the minimum possible range, so that the time of flight of each missile is at a minimum. For a limited period, however, it is possible to intercept targets with a frequency of arrival greater than the maximum rate of fire by intercepting the first target at the maximum possible range, and later targets successively closer in.

Assume

(1) that the minimum time between the completion of an engagement and the launch of the next missile is $T_0$: this time may be determined by the time required to disengage the tracking radar and train it on the next target or by the time required to warm up the missile

(2) that the interception is near head on so that the sum of the target speed $V_T$ and the missile speed $V_M$ is a sufficient approximation to the closing speed

(3) that $V_T$ is constant

(4) that $V_M$ is constant after an initial period of acceleration and that at the minimum range of interception the full missile speed is already achieved.

For interception at a range $X$ the time of flight is then

$$\frac{X}{V_M} + T_1$$

where $T_1$ represents the time penalty due to acceleration (Fig. 1).

The total time delay or 'reaction time' is

$$T = T_0 + T_1$$

and the total time of interception is thus

$$t = \frac{X}{V_M} + T$$

(1)

Also if the interception begins when the target is at a range $R$

$$X = R - tV_T$$

and

$$t = \frac{R + V_M T}{V_T - V_M}$$

(2)
If $X_0$ is the minimum range of interception, the minimum time of interception is

$$
t_0 = \frac{X_0}{V_M} + T
$$

and this is the minimum time interval between targets which can be sustained indefinitely.

Suppose, however, that the number of targets to be intercepted is limited, and that the time interval between the arrival of each target is $t$. If the interception of the $n^{th}$ target begins when it is at a range $R_n$, and takes a time $t_n$ (Fig. 2), then

$$
R_{n-1} - R_n = V_T (t_{n-1} - T)
$$

whence from (2)

$$
t_{n-1} - t_n = \frac{V_T}{V_T + V_M} (t_{n-1} - t)
$$

Writing

$$
k = \frac{V_M}{V_T + V_M}
$$

it follows that

$$
t_n - t = k(t_{n-1} - t) = k^{n-1}(t_1 - t) \quad (3)
$$

The $n^{th}$ interception will not be completed if the time available is less than the minimum time of interception, so the $n^{th}$ target can be intercepted only if

$$
t_n \geq t_0
$$

or

$$
t \geq \frac{t_0 - t_1 k^{n-1}}{1 - k^{n-1}} \quad (4)
$$

If the first target is intercepted at a range

$$
X_1 = r X_0
$$

then substituting from (1), it can be seen that when there are $n$ targets, the minimum time interval between the arrival of successive targets which will allow all of them to be intercepted is

$$
t = T + \frac{X_0}{V_M} \frac{1 - r k^{n-1}}{1 - k^{n-1}} \quad (5)
$$
Evidently $t$ is least when the first interception takes place at maximum range. If $t$ is negative the targets could be intercepted in reverse order, the last target first.

Since $k < 1$, it can be seen from (4) that as the number of targets increases, so the minimum acceptable time interval between targets approaches the minimum time $t_0$ for a single interception. This is illustrated in Figs. 3 and 4 for different values of the ratio of the time $t_1$ of the first interception to $t_0$. This ratio is a measure of the difference between the range of the first interception and the minimum range of interception.

Figs. 5 - 12 show the application of (5) to some particular cases. Figs. 5 and 6 show the effect of varying the missile speed when the maximum and minimum ranges of interception are 15000 and 3000 ft., for target speeds of 1500 and 3000 f.p.s. Figs. 7 - 10 show the effect of varying the maximum range of interception when the minimum range of interception is 3000 ft., for different combinations of target and missile speeds. It can be seen that against fast moving targets, once the missile speed is equal to the target speed, there is little advantage to be gained from a further increase of speed, and it is then better to increase the maximum range of interception. Finally, Figs. 11 and 12 illustrate the capabilities of missiles with speeds of 1500 and 3000 f.p.s. when their maximum and minimum ranges of interception are 15000 and 3000 ft.
TARGET SPEED 3000 FT/SEC
MAXIMUM INTERCEPTION RANGE 15000 FT
MINIMUM INTERCEPTION RANGE 3000 FT
REACTION TIME T
TARGET SPEED 1500 FT/SEC.
MISSILE SPEED 1500 FT/SEC.
MINIMUM INTERCEPTION RANGE 3000 FT
REACTION TIME T.
TARGET SPEED: 3000 FT/SEC
MISSILE SPEED: 1500 FT/SEC
MINIMUM INTERCEPTION RANGE: 3000 FT
REACTION TIME: T

MINIMUM TIME BETWEEN TARGETS

NUMBER OF TARGETS

MAXIMUM INTERCEPTION RANGE: FT
T-16 3000 6000 9000 12000 15000 18000 21000 2400

FIG. No. 9

REF. No.
ISSUE No.
DATE:
AQ. No.
TARGET SPEED 3000 FT./SEC.
MISSILE SPEED 3000 FT./SEC.
MINIMUM INTERCEPTION RANGE 3000 FT
REACTION TIME T.

MAXIMUM INTERCEPTION RANGE FT.
T-163000 6000 9000 12000 15000 18000 21000 24000

REF. No. ISSUE No. DATE:
FIG. No. AQ. No.
NUMBER OF
TARGETS:

MINIMUM TIME:
BETWEEN TARGETS

MISSILE SPEED 3000 FT/SEC
MAXIMUM INTERCEPTION RANGE 15000 FT
MINIMUM INTERCEPTION RANGE 3000 FT
REACTION TIME T

TARGET SPEED FT/SEC
500 1000 1500 2000 2500 3000 3500

FIG. No. 12