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Aerodynamic Shape Optimization for the World's Fastest P-51

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In this paper, an Adjoint-based automatic design methodology has been applied to redesign the wing of the P51 "Dago Red", an aircraft competing in the Reno Air Races. The aircraft reaches speeds above 500 MPH and encounters compressibility drag due to the appearance of shock waves. The objective is to delay drag rise without altering the wing structure. Hence the shape modifications are restricted to adding a bump on the wing surface, allowing only outward movement. Moreover, the changes are restricted to a limited chord-wise range. Results indicate that the perturbations created by this bump propagate along the characteristics and are reflected back from the sonic line to weaken the shock. With this bump on the wing, it is expected that Dago Red can reach the speed of 550 MPH, which will create a new World's speed record for a propeller driven airplane.

I. Introduction

The North American P-51 Mustang was one of the greatest WWII fighters. It was the product of two highly advanced technologies: the American advanced structural-and-aerodynamic plane body and the British prestigious Rolls-Royce Merlin engine. The performance and maneuverability of the P-51 outfitted other WWII fighters, resulting in extremely high survivability. A total of 15,686 Mustangs were built since 1944 and about 280 P-51s still exist today,[1] with more than half still airworthy. Among those is the Dago Red, a modified version of the P51-D to compete in the Reno Air Races. With the engine supercharged to gain horsepower and the wing tip cropped to match low attitude flight of the air race, the Dago Red reaches speeds above 500 MPH and suffers from high compressibility drag due to shock formation.

In this paper we propose a wing modification to delay the drag rise without changing the wing structure. Thus we only allow adding material to the wing at specific locations. We implement an adjoint-based optimization to identify the shape of the added material. We will discuss the design methodology in section II, along with results in section III. Results indicate that by adding a small bump near the leading edge of the wing, shock waves can be weaken and the drag rise can be delayed. With this bump, the aircraft may be able to reach the speed of 550 MPH and create a new world speed record.

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II. Design methodology

A. Aerodynamic design without altering the wing structure

During the last decade, adjoint-based optimization for wing design has been extensively studied by Jameson et al[5,4,9,7] The method determines shape modifications that effectively eliminate shock waves. These shape changes can generally be implemented only by modifying the structure. However, it is impossible to enforce additional constraints such that the underlying structure can be preserved. For this purpose we limit the shape modifications to a specific chosrdwise range to avoid affecting the control surfaces, and we restrict them to outward movements only. This should result in a bump which can be added to the wing surface, as illustrated in figure 1 for one wing section.

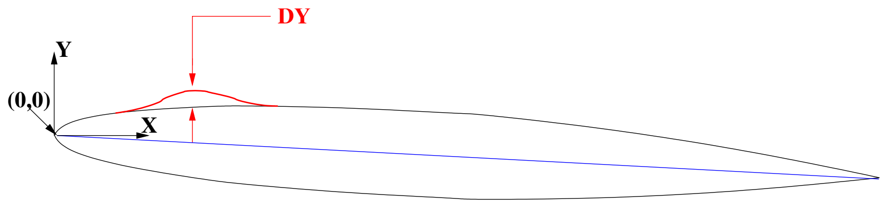


Figure 1. Added bump to achieve shock-free wing.

To design the shape of this bump, we use surface mesh points as the design parameters. There are 4096 surface mesh points on the wing surface and we only choose those that lie between the leading edge and 65 percent chord as the design parameters. We implement the adjoint method to calculate the gradient of these design parameters. Once the gradients are calculated, we implement an additional condition to allow only outward movement;

$$\text{if} \quad g < 0 \quad \text{then} \quad g = 0, \tag{1}$$

assuming a positive g indicates outward movement. Finally, we smooth the gradients in order to preserve the smoothness of a modified wing.

B. The control theory approach to wing design problems

The control theory approach was proposed for shape design by Pironneau in 1974[8] but it did not have much impact on aerodynamic design until its application by Jameson to transonic flow.[3] The major impact arose from its capability to effectively handle a design problem that involves a large number of design variables and is governed by a complex mathematical model, such as fluid flow. The control theory approach is often called the adjoint method since the necessary gradients are obtained through the solution of the adjoint equations of the governing equations.

In the context of control theory, a wing design problem can be considered as:

$$\begin{aligned} Minimizing \quad & I(w,S) \\ w.r.t \quad & S \\ subjected\ \ to \quad & R(w,S) = 0 \end{aligned}$$

where w is the flow variable, S is the vector of wing design parameters, and $R(w,S) = 0$ is the flow equation.

For instance, for a drag minimization problem we can take $I = C_D$ which is an integral of flow w (pressure and shear force) over the wing S (represented by parameters such as airfoils). We modify S (the airfoils) to reduce the drag. The pressure and shear force are obtained from the flow equation $R = 0$ using CFD.

A change in S results in a change

$$\delta I = \left[\frac{\partial I}{\partial w}\right]^T \delta w + \left[\frac{\partial I}{\partial S}\right]^T \delta S, \tag{2}$$

and δw is determined from the equation

$$\delta R = \left[\frac{\partial R}{\partial w}\right] \delta w + \left[\frac{\partial R}{\partial S}\right] \delta S = 0. \tag{3}$$

The finite difference approach attempts to solve δw from equation (3) and substitute it into equation (2) to calculate δI. For a design problem of n design parameters e.g. $\mathcal{O}(S) = n$, this procedure requires a well-converged solution of $n+1$ flow analysis problems to obtain the design sensitivities. Thus it becomes impractical when n becomes large.

In the adjoint approach, we try to avoid solving for δw. This is done by introducing a Lagrange multiplier ψ, and subtracting the variation δR from the variation δI without changing the result. Thus, equation (2) can be replaced by

$$\begin{aligned} \delta I &= \left[\frac{\partial I}{\partial w}\right]^T \delta w + \left[\frac{\partial I}{\partial S}\right]^T \delta S - \psi^T \left(\left[\frac{\partial R}{\partial w}\right] \delta w + \left[\frac{\partial R}{\partial S}\right] \delta S \right) \\ &= \left\{ \left[\frac{\partial I}{\partial w}\right]^T - \psi^T \left[\frac{\partial R}{\partial w}\right] \right\} \delta w + \left\{ \left[\frac{\partial I}{\partial S}\right]^T - \psi^T \left[\frac{\partial R}{\partial S}\right] \right\} \delta S \end{aligned} \tag{4}$$

Choosing ψ to satisfy the adjoint equation,

$$\left[\frac{\partial R}{\partial w}\right]^T \psi = \left[\frac{\partial I}{\partial w}\right], \tag{5}$$

the first term is eliminated, and we find that

$$\delta I = \mathcal{G}^T \delta S, \tag{6}$$

where

$$\mathcal{G}^T = \left[\frac{\partial I}{\partial S}\right]^T - \psi^T \left[\frac{\partial R}{\partial S}\right].$$

The advantage is that equation (6) is independent of δw, with the result that the gradient of I with respect to an arbitrary number of design variables can be determined without the need for additional flow-field evaluations.

Once the gradient vector $\mathcal{G}$ has been established, it may now be used to determine a direction of improvement. The simplest procedure is to make a step in the negative gradient direction (steepest descent method) by setting

$$\delta S = -\lambda \mathcal{G}$$

where λ is positive and small enough that the first variation is an accurate estimate of δI. The variation of the cost function then becomes

$$\begin{aligned} \delta I &= -\lambda \mathcal{G}^T \mathcal{G} \\ &\leq 0 \end{aligned}$$

More sophisticated search procedures might be used such as quasi-Newton methods, which attempt to estimate the second derivative $\frac{\partial^2 I}{\partial S_i \partial S_j}$ of the cost function from changes in the gradient $\frac{\partial I}{\partial S}$ in successive optimization steps. These methods also generally introduce line searches to find the minimum in the search direction which is defined at each step. Reference[2] provides a good description for those techniques. However, not all the techniques are practical for our wing design problem. Line searches, for example, would require extra flow calculations, which we try to avoid. An effective alternative is to redefine the gradient so that it corresponds to a Sobolev inner product. This defines an automatic smoothing procedure for the gradient such that the shape modifications preserve the smoothness of the initial geometry, and also greatly reduce the number of iterations required to determine the optimum shape In the present application we want to maintain smoothness while restricting the shape modifications to a limited chordwise range. For this purpose we define the gradient with respect to an inner product containing second derivatives of the form

$$< u, v >= \int_a^b \left(uv + \epsilon_1 u'v' + \epsilon_2 u''v'' \right) dx$$

The resulting smoothing equation for the gradient is

$$\bar{g} - \frac{\partial}{\partial x}\epsilon_1 \frac{\partial \bar{g}}{\partial x} + \frac{\partial^2}{\partial^2 x}\epsilon_2 \frac{\partial^2 \bar{g}}{\partial^2 x} = g,$$

where g is the original gradient and $\bar{g}$ is the smoothed gradient. Since this equation is of fourth order, it may be solved with the boundary conditions

$$\bar{g} = 0, \quad \bar{g}' = 0.$$

at the two endpoints of the range over which modifications are permitted, thus ensuring that the modifications merge smoothly with the initial profile. The complete details and derivation using the Navier-Stokes equations can be found in reference.[6]

III. Results

We present the geometry of the bump and the benefit of this bump on the drag rise. The bump optimization was performed on the cropped P51-D wing, using the Reynolds-Averaged Navier Stokes Optimizer *Syn-107* on about 1 million mesh cells. The optimization was done at fixed $C_L = .1$ and Mach 0.78, corresponding to a flight condition to make new world speed record at 550 MPH. The original airfoil sections are shown in figure 2. The only available data were a handwritten list of coordinates together with a drawing.

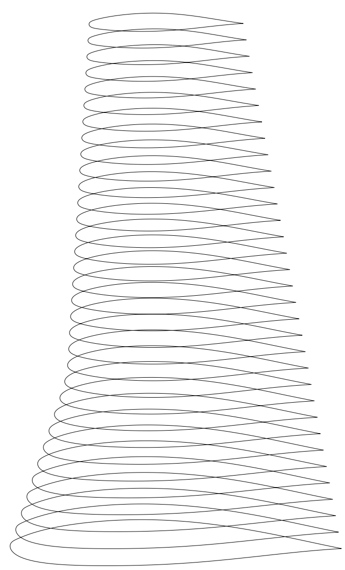


Figure 2. Original wing sections of the P51-D, cropped at 189 inches span station.

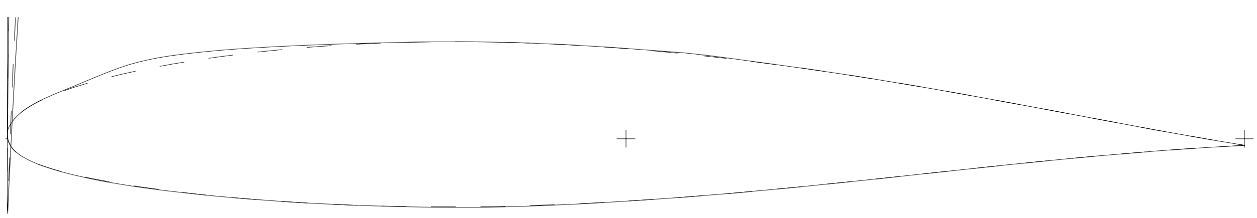
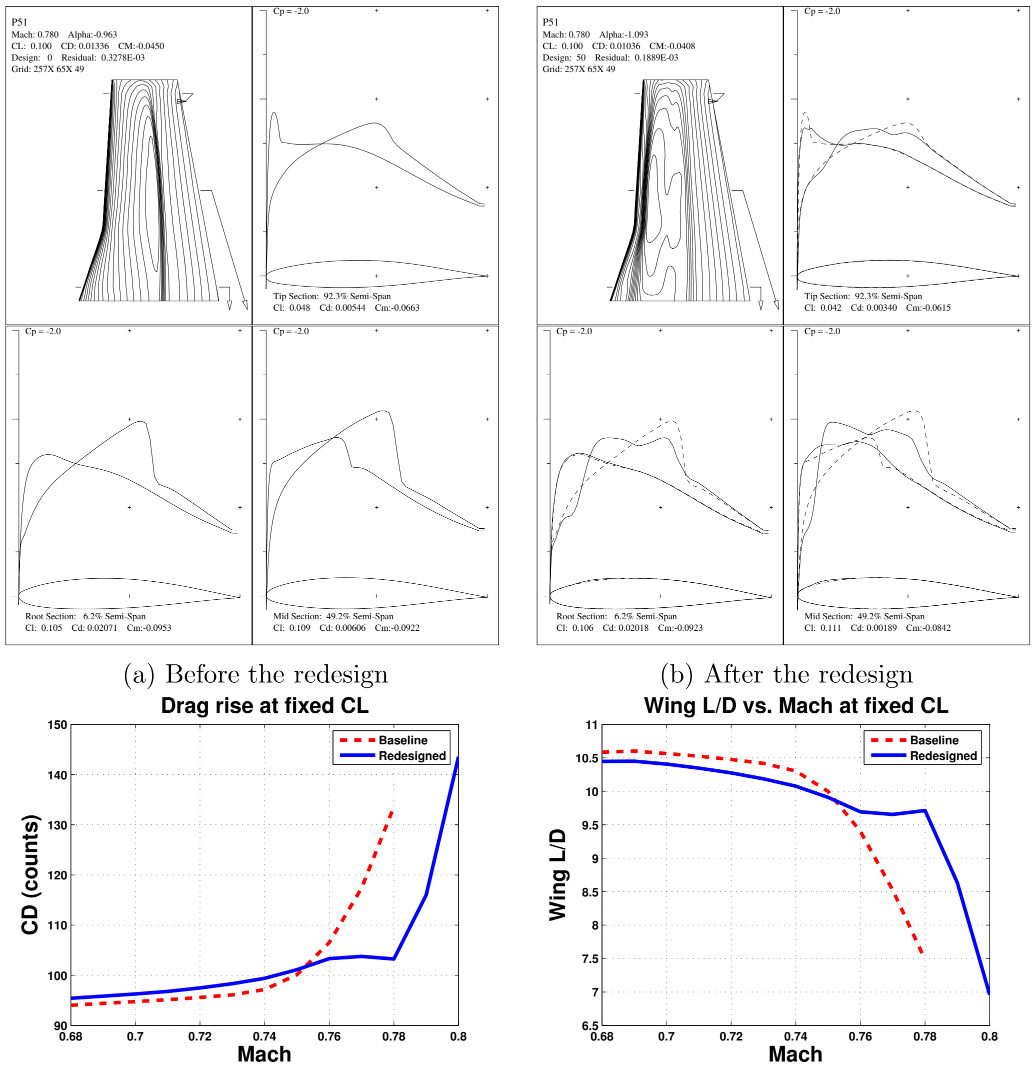


Figure 3. Shape of an optimum bump at the mid-span location. Dashed line represent an original wing section.

These coordinates were too rough and consequently we had to smooth the initial wing sections. Figure 3 shows the optimum shape of the bump. The size of this bump is about three quarters of an inch, which is big enough to be manufactured. The perturbations created by this bump propagate along the characteristics and are reflected back from the sonic line to weaken the shock. The improvement is shown in figure 4. Although we pay a penalty of higher drag below Mach .73, the drag rise can be delayed. This provides an opportunity to increase the maximum speed for the purpose of an attempt to break the world speed record.



(a) Before the redesign

(b) After the redesign

(c) C_D

(d) L/D

Figure 4. Improvements earned by adding a bump on the original P51 wing. (a) C_p distributions of the original wing at Mach .78, (b) C_p distributions of the redesigned wing. The bump eliminates the shock, (c) C_D Vs. Mach number at fixed C_L 0.1, (d) L/D Vs. Mach at fixed C_L 0.1.

IV. Conclusion

By adding a bump near the wing leading edge, it is possible to delay the drag rise of the P51-D without altering the wing structure. The size of this bump is comparatively small, but large enough to be manufactured. The delay of drag rise should benefit the maximum speed that can be attained by the aircraft while using the same engine. The new maximum speed is expected to approach 550 MPH and should be sufficient for a new world speed record. Because this bump can be installed easily, it provides a quick way to improve the aerodynamic performance.

V. Acknowledgment

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