RELAXATION SOLUTIONS FOR INVISCID AXISYMMETRIC TRANSONIC FLOW OVER BLUNT OR POINTED BODIES

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Abstract

A finite-difference relaxation method is presented for numerical solution of the full potential equation and exact boundary conditions for general axisymmetric bodies in inviscid, steady transonic flow. Body-normal coordinates are used in the nose region and sheared cylindrical coordinates are used on the afterbody to accommodate corners such as boattails and flares. An improved difference sheme is used which does not require that the flow be nearly alined with a coordinate direction in supersonic regions, and which treats either subsonic or supersonic free streams. Numerical results are illustrated for some simple classical shapes such as spheres and ellipsoids, and for more practical shapes like tangent-ogives with boattails. Special attention is given to bodies which have been studied for area-rule applications. Agreement with available experimental results is good in cases where viscous effects and wind-tunnel wall interference are not important.

Introduction

The problem of axisymmetric transonic flow is of interest not only because of the practical importance of missile and launch vehicle aerodynamics, but also because of its relation to fully three-dimensional flow via the area rule.[1] In fact, there has been considerable activity devoted to experimental investigation of various bodies of revolution at transonic speeds with the aim of determining a longitudinal area distribution which exhibits a high drag-rise Mach number.[2] Presumably, such an optimized area distribution should provide a reasonable starting point in the aerodynamic design of high subsonic aircraft. Clearly, a theoretical analysis method is needed to complement wind-tunnel tests, especially for Mach numbers near 1, where tunnel-wall interference can induce large deviations from free-air conditions even for very small bodies (or blockage ratios).

Bailey[3] and Krupp and Murman[4] obtained numerical solutions for slender pointed bodies of revolution using a relaxation method for the transonic small disturbance potential equation together with the consistent approximation to the surface boundary condition. In this approach, proper use of upwind differencing in supersonic flow regions allows weak discontinuities representing shock waves to occur in the potential solution. In both works, the use of cylindrical coordinates allowed an easy accounting of the effect of a sting attachment. They differed mainly in their treatment of the boundary condition at infinity. Bailey used a simple reciprocal-radius type of transformation to map the unbounded radial coordinate into a bounded computational coordinate, and he retained the option of simulating a porous or solid wind-tunnel wall at a finite radial distance. Krupp and Murman used an asymptotic solution for the far-field potential at the boundary of a finite cylinder surrounding the body. Because the asymptotic solution was derived for subsonic free streams, Krupp and Murman did not consider supersonic free streams. Bailey had no difficulty treating either subsonic or supersonic free streams, since in his approach the boundary conditions are always the same, with only minor modifications at the upstream and downstream boundaries.

Unfortunately, these methods are not well suited to analyze the flow about blunt-nosed axisymmetric bodies which include many shapes of classical and practical interest as well as the area-rule research shapes already mentioned. The assumption of small disturbances is invalid in the region of a blunt nose, and the use of cylindrical coordinates causes further difficulties there. It would seem that for a general treatment of blunt bodies, the small-disturbance assumption must be abandoned and coordinates should be used which more nearly fit the nose. Lipnitskii and Lifshits[5] presented a time-asymptotic, finite-difference method for solving the nonisentropic Euler equations and exact boundary conditions using polar coordinates that were transformed to yield a finite rectangular computational domain. This approach has two drawbacks: first, the polar coordinates are suitable only for smooth closed bodies which are blunt on both ends; and second, the time-asymptotic method for the Euler equations is suspected to require far more computer time and storage than the "steady" relaxation method for the potential equation.[6]

The present paper describes an obvious alternative to the above methods, one which has already been followed in the two-dimensional airfoil problem:[7-9] a relaxation procedure for solving the exact potential equation and boundary conditions, with coordinates that fit the blunt nose. Using this approach with conformal coordinates, Jameson[9] obtained results for closed spheres and ellipsoids in subsonic, supercritical flow. Extension of the fully conformal coordinates to shapes with open tails and corners is not a simple matter; hence, the present method uses coordinates which become a sheared cylindrical system on the afterbody to accommodate such geometric anomalies as flares, boattails, and sting attachments. If previously developed difference schemes are used, however, instability can occur where the local flow is

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