NUMERICAL CALCULATION OF THE THREE DIMENSIONAL TRANSONIC FLOW OVER A YAWED WING

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Abstract

Results are presented of calculations of the three dimensional steady transonic flow over a finite yawed wing. The full potential flow equation is solved in a transformed coordinate system which permits the boundary conditions to be satisfied exactly. The correct directional properties are enforced by rotating the difference scheme to conform with the flow direction, and fast convergence is assured by simulating a time dependent equation designed to settle quickly to a steady state. Computed lift drag ratios are consistent with the results of wind tunnel tests of a yawed wing conducted by R. T. Jones.

1. Introduction

The development of successful numerical methods for calculating two dimensional transonic flows encourages the belief that theoretical methods will eventually be able to play a much more prominent role in the design process than they have in the past, allowing alternative designs to be screened without the need to rely on the wind tunnel. If we are to progress further along the road towards this goal, an essential step must be the development of accurate and reliable methods for calculating three dimensional flows over wings and wing body combinations in the transonic range. This paper reports results of an investigation into the problem of calculating the flow over an isolated yawed wing. This problem was selected for study because its solution appears to be feasible within the capacity of current computers such as the CDC 6600, and may even be brought within the bounds of routine calculation with newer machines such as the CDC 7600. At the same time the model has engineering relevance in support of studies of R. T. Jones' concept of a yawed wing aircraft.(1) The experience gained with this problem should provide a solid platform for an attack on the problem of calculating the more complex flow over a swept wing body combination. An isolated arrow wing appears to be a less favorable model for preliminary study because the singularity at the center line would present a complication which is quite unrealistic, since it would normally be relieved by the presence of the fuselage. By tracking the streamlines near the center section, the results of the present calculation might, on the other hand, be used to obtain an idea of the appropriate body contouring to minimize wing body interference.

The requirements of a good mathematical model are that it should lead to equations which are simple enough to be solved, while continuing to give an accurate representation of the important features of the real flow. For the purposes of this study these requirements seem best met by the use of the potential equation for irrotational flow. Pending the introduction of more powerful computers, it hardly seems possible to treat the full equations in three dimensions, even ignoring viscosity, without exceeding acceptable limits of computer time and cost. A satisfactory aerodynamic design should lead to the presence only of fairly weak shock waves, so that the error in ignoring entropy variations and using an irrotational model should be quite small. The proper treatment of strong shock waves would in any case require a model allowing for the presence of separated flow behind the shock waves, and this is beyond the scope of the present study. Quite extensive trials were made using small disturbance theory. The particular advantage of this theory is that the consistent treatment of the boundary conditions allows the translation of the boundaries to lines parallel with the undisturbed stream, eliminating the difficulty of satisfying a Neumann boundary condition on a curved surface. The small disturbance equations certainly simulate the characteristic features of transonic flow.(2,3) They do not appear, however, to offer the prospect of achieving the desired accuracy in resolving the effects of subtle changes in the wing section, which may be required to achieve shockfree flow.(4) It was therefore decided to use the full potential equation.

The potential equation is invariant under a reversal of the direction of flow, and in the absence of a directional condition corresponding to the condition that entropy can only increase, its solution is not unique. To ensure uniqueness, and to prevent the appearance of expansion shocks, a directional property must be restored in the numerical treatment of the problem. Murman and Cole first devised an effective scheme for treating the small disturbance equation.(2) They showed that the required directional property can be obtained by using retarded difference formulas in the supersonic region, supplying the effect of an artificial viscosity. Variations of the Murman scheme have been successfully used with the full potential equation.(5,6,7) These schemes rely on the identification of one coordinate with the direction of flow, so that a simple switch to retarded difference formulas in this coordinate is all that is required to treat the supersonic zone.

The calculations in this paper employ a coordinate invariant difference scheme in which the proper directional property is obtained by rotating the retarded difference formulas to conform with the local direction of flow. This allows great flexibility in the choice of a coordinate system, so that curvilinear coordinates may be used to improve the accuracy of the treatment of the boundary condition, and mesh points of the computational lattice can be concentrated in the sensitive regions of the flow.

The rotated difference scheme has proved to be consistently stable and convergent both in the case when the flow is subsonic at infinity and in the case of a supersonic free stream. Thus it is possible to provide a uniform treatment of a yawed wing up to flight Mach numbers around 1.2 and yaw angles as great as 60°. This covers the most likely operating range of a yawed wing transport. The accuracy of the calculations, however, decreases at the upper end of the range because the difference scheme is first order accurate in the supersonic zone, which then comprises the bulk of the flow, and because oblique shock waves may be spread over as many as four or five mesh widths.

2. Formulation of the Equations: Curvilinear Coordinates

In line with the considerations set forward in the introduction the governing equation is taken to be the transonic potential flow equation,

$$(a^2-u^2)\phi_{xx} + (a^2-v^2)\phi_{yy} + (a^2-w^2)\phi_{zz} - 2uv\,\phi_{xy} - 2vw\,\phi_{yz} - 2uw\,\phi_{xz} = 0 \qquad (1)$$

in which ϕ is the velocity potential, u, v, and w are the velocity components

$$u = \phi_x , \quad v = \phi_y , \quad w = \phi_z \qquad (2)$$

and a is the local speed of sound. This is determined from the stagnation speed of sound a_0 by the energy equation

$$a^2 = a_0^2 - \frac{\gamma-1}{2}(u^2+v^2+w^2) \qquad (3)$$

where γ is the ratio of the specific heats. Equation (1) is elliptic at subsonic points and hyperbolic at supersonic points. When a supersonic zone is present it is necessary to allow a weak solution, in which shock waves are simulated by jumps across which the normal component of mass flow is conserved.

The boundary condition at the body is

$$\frac{\partial\phi}{\partial\nu} = 0 \qquad (4)$$

where ν denotes the direction normal to the surface. In addition the Kutta condition requires that the circulation at each span station should be such that there is no flow around the sharp trailing edge, leading to the presence of a vortex sheet behind the wing. Using a linearized model in which convection and roll up of the sheet are ignored, the jump Γ in potential should be constant along lines parallel to the free stream behind the trailing edge, and also, in the case of a yawed wing, behind the side edge of the downstream tip. At all points of the sheet the normal component of velocity should also be continuous through the sheet. At infinity, the flow should approach the velocity of the undisturbed stream except in the Trefftz plane far behind the wing, where there will be a two dimensional flow induced by the vortex sheet.

Flows over complicated shapes are most easily treated with the aid of curvilinear coordinates. Then by making the body coincide with a coordinate surface, one can avoid the need for complicated interpolation formulas and prevent a loss of smoothness in the numerical statement of the boundary condition. In two dimensional calculations an effective way of doing this is to map the airfoil to a regular shape, such as a circle, by a conformal transformation. In the three dimensional case no simple method of this type is available. To prevent an excessive growth of computer time it is also important to limit the increase in the number of terms in the equations due to the use of coordinate transformations. A convenient coordinate system for treating wings with straight leading edges can be obtained in two stages. Let x , y and z be Cartesian coordinates with the x-y planes containing the wing sections and the z axis parallel to the leading edge. Then, following a suggestion of Garabedian, the wing is first unwrapped' by a square root transformation of the x-y planes, independent of z,

$$x + iy = \frac{1}{2}(X_1+iY_1)^2 , \quad z = Z_1 \qquad (5)$$

X_1 and Y_1 represent parabolic coordinates in the x-y planes, which become half planes in X_1 and Y_1 , while the wing surface is split open to form a bump on the boundary $Y_1 = 0$. In terms of the transformed coordinates the surface can be represented as

$$Y_1 = S(X_1,Z_1) . \qquad (6)$$

In the second stage of the transformation the bump is removed by a shearing transformation in which the coordinate surfaces are displaced until they become parallel to the wing surface:

$$X = X_1 , \; Y = Y_1-S(X_1,Z_1) , \; Z = Z_1 \qquad (7)$$

The final coordinates X, Y, and Z are slightly non-orthogonal. While the leading edge is restricted to be straight, the wing section can be varied or twisted and the trailing edge can be curved or tapered in any desired manner. To treat a yawed wing the coordinate system is fixed to the wing, and the yaw angle is introduced by rotating the flow at infinity.

Since the potential approaches infinity in the far field, it is necessary to work with a reduced potential G from which the singularity at infinity has been removed. If θ is the yaw angle, and α the angle of attack in the cross plane normal to the leading edge, a suitable dependent variable is

$$G = \phi+\{\tfrac{1}{2}[X^2-(Y+S)^2]\cos\alpha+ X(Y+S)\sin\alpha\}\cos\theta + Z\sin\theta \,. \tag{8}$$

Orthogonal velocity components in the X_1, Y_1 and Z_1 directions are then

$$\begin{aligned} U &= \frac{1}{h}\{G_X-S_XG_Y + [X\cos\alpha+(Y+S)\sin\alpha]\cos\theta\} \\ V &= \frac{1}{h}\{G_Y+ [X\sin\alpha- (Y+S)\cos\alpha]\cos\theta\} \\ W &= G_Z - S_ZG_Y + \sin\theta \,, \end{aligned} \tag{9}$$

where h is the mapping modulus satisfying

$$h^2 = X^2 + (Y+S)^2 \tag{10}$$

The local speed of sound is now given by

$$a^2 = a_0^2 - \frac{\gamma-1}{2}(U^2+V^2+W^2)\,. \tag{11}$$

The governing equation becomes

$$AG_{XX}+ BG_{YY}+ CG_{ZZ}+ DG_{XY}+ EG_{YZ}+ FG_{XZ} = H \tag{12a}$$

where

$$\begin{aligned} A &= a^2 - U^2 \\ B &= a^2(1+S_X^2+h^2S_X^2) - (V-US_X-h^2WS_Z)^2 \\ C &= h^2(a^2-W^2) \\ D &= -2a^2S_X - 2U(V-US_X-h^2WS_Z) \\ E &= -h^2a^2S_Z - 2h(V-US_X-h^2WS_Z)W \\ F &= -2hUW \end{aligned} \tag{12b}$$

and

$$\begin{aligned} H = {} & \{(a^2-U^2)S_{XX}+ h^2(a^2-W^2)S_{ZZ}-2hUWS_{XZ}\}G_Y \\ & + \{(U^2-V^2)\cos\alpha + 2UV\sin\alpha\}\cos\theta \\ & - (U^2+V^2)\{U(\tfrac{X}{h}) + V(\tfrac{Y+S}{h})\} \end{aligned} \tag{12c}$$

The boundary condition on the body takes the form

$$G_Y = - \frac{(S\cos\alpha-X\sin\alpha)\cos\theta+U_1S_X+h^2W_1S_Z}{1 + S_X^2 + h^2S_Z^2} \tag{13a}$$

where

$$\begin{aligned} U_1 &= G_X + (X\cos\alpha + S\sin\alpha)\cos\theta \\ W_1 &= G_Z + \sin\theta \,. \end{aligned} \tag{13b}$$

Finally the parabolic transformation has the advantage that it collapses the height of the disturbance due to the vortex sheet to zero in the Trefftz plane far downstream, so that the far field condition is simply

$$G = 0 \,. \tag{14}$$

3. Numerical Scheme

The success of the Murman difference scheme for the small disturbance equations is attributable to the fact that the retarded difference formulas used for the streamwise derivatives in the supersonic zone lead to the correct region of dependence and also introduce a truncation error which acts like viscosity. Let x be the coordinate in the stream direction. Then the dominant truncation error of the retarded difference formula for ϕ_{xx} is $-\Delta x\phi_{xxx}$, and since the coefficient of ϕ_{xx} is negative in the supersonic zone, this term represents a positive artificial viscosity, which ensures that only the proper type of jump can occur. The term is added smoothly because the coefficient of ϕ_{xx} is zero at the sonic line, where the switch in the difference scheme takes place.

The difference scheme used in the calculations for this paper is designed to introduce correctly oriented retarded difference formulas in a similar smooth manner when the flow direction is arbitrary. The underlying idea is to rearrange the equation as if it were locally expressed in a coordinate system aligned with the flow. Considering first the case of Cartesian coordinates, let s denote the stream direction. Then equation (1) can be written in the canonical form

$$(a^2-q^2)\phi_{ss} + a^2(\Delta\phi-\phi_{ss}) = 0 \tag{15}$$

where q is the stream speed determined from the formula

$$q^2 = u^2 + v^2 + w^2 \tag{16}$$

and $\Delta\phi$ denotes the Laplacian

$$\Delta\phi = \phi_{xx} + \phi_{yy} + \phi_{zz} \,. \tag{17}$$

Since the direction cosines of the stream direction are u/q, v/q, and w/z, the streamwise second derivative can be expressed as

$$\phi_{ss} = \frac{1}{q^2}(u^2\phi_{xx}+ v^2\phi_{yy}+ w^2\phi_{zz} + 2uv\,\phi_{xy} + 2vw\,\phi_{yz} + 2uw\,\phi_{xz}) \tag{18}$$

On substituting the expressions for ϕ_{ss} and $\Delta\phi$, the equation is seen to reduce to the usual form. To carry out this rearrangement the velocity components are first evaluated using central difference formulas to represent the first derivatives, and the local type of the flow is determined from the sign of $a^2 - q^2$. Then at subsonic points all derivatives are approximated by central difference formulas in the conventional manner for treating an elliptic equation. At supersonic points all second derivatives contributing to ϕ_{ss} in the first term are approximated by retarded difference formulas, while all contributions to the remaining terms are approximated by central difference formulas. The retarded formulas are constructed using one-sided difference operators biased in the upstream sense in all three coordinate directions. It is evident that the scheme reduces to the Murman scheme whenever the velocity coincides with one of the three coordinate directions.

When the equation is written in curvilinear coordinates, only the principal part, consisting of the terms containing the second derivatives on the left side of (12a), need be rearranged and split in this way, since the characteristic cone and region of dependence are determined by the coefficients of the second derivatives. Also in the limit of zero mesh spacing the expressions for the second derivatives dominate the finite difference equations. All terms contributing to H on the right side of (12a) are calculated using central difference formulas at both supersonic and subsonic points.

It remains to devise a suitable scheme for solving the difference equations. The original Murman scheme is not easily carried over to the three dimensional case: in order to obtain the correct region of dependence it would call for the simultaneous solution of all the points of a cross plane normal to the flow. The rotated difference scheme in any case forces the use of an iterative method, because it includes downstream points in the central difference formulas contributing to $\Delta\phi - \phi_{ss}$, for which 'old' values of the potential generated during the previous cycle have to be used. To avoid programming difficulties we need a method which calls for the solution of at most a line at a time. Point relaxation, on the other hand, tends to spread the corrections too slowly. Thus line relaxation is the preferred approach. If the iterations are identified as successive levels in an artificial time coordinate, the solution procedure may be regarded as a time dependent process. A detailed theory based on this concept has been advanced elsewhere.[8] The most important features of this theory are that the region of dependence of the time dependent equation simulated by the iterative scheme ought to be compatible with the region of dependence of the time invariant equation, and that the coefficient of ϕ_t ought to be zero at points in the supersonic zone.

Denoting updated values by the superscript +, the typical form of a central difference formula for a second derivative is

$$G_{XX} = \left\{ G^{+}_{i-1,j,k} - (1+r\Delta x)\, G^{+}_{i,j,k} - (1-r\Delta X)\, G_{i,j,k} + G_{i+1,j,k} \right\} / \Delta X^2 \tag{19}$$

where the old value of potential is used on one side because the new value is not yet available, and a linear combination of new and old values is used at the center point. If Δt is the time step this formula may be interpreted as representing

$$G_{XX} - \frac{\Delta t}{\Delta X}\,(G_{Xt} + r\,G_t) \;.$$

Thus the presence cannot be avoided of mixed space-time derivatives in the equivalent time dependent equation. This equation can therefore be written in the form

$$\left(\frac{q^2}{a^2} - 1\right) G_{ss} - G_{mm} - G_{nn} + 2\alpha_1 G_{st} + 2\alpha_2 G_{mt} + 2\alpha_3 G_{mt} = H \tag{20}$$

where m and n are suitably scaled coordinates in the plane normal to the local stream direction s, and H contains all terms except the principal part. The coefficients α_1, α_2 and α_3 depend on the split between new and old values in the difference equations. Setting

$$T = t - \frac{-\alpha_1 s}{\dfrac{q^2}{a^2} - 1} + \alpha_2 m - \alpha_3 n \tag{21}$$

equation (20) becomes

$$\left(\frac{q^2}{a^2} - 1\right) G_{ss} - G_{mm} - G_{nn} - \left(\frac{\alpha_1^2}{\dfrac{q^2}{a^2} - 1} - \alpha_2^2 - \alpha_3^2 \right) G_{TT} = H \tag{22}$$

To avoid producing an ultra hyperbolic equation for which the initial data cannot in general be arbitrarily prescribed,[9] the difference formulas at supersonic points should be organized so that

$$\alpha_1 > \sqrt{(q^2/a^2 - 1)(\alpha_2^2 + \alpha_3^2)} \tag{23}$$

Then the hyperbolic character is retained with s as the timelike direction. The region of dependence of (20) lies entirely behind the current time level except for the single characteristic direction

$$t = 0 \;, \quad m = \frac{\alpha_2}{\alpha_1}\, s \;, \quad n = \frac{\alpha_3}{\alpha_1}\, s$$

The difference equations will have the correct region of dependence provided that the points are ordered so that the upstream part of this line is contained in an updated region. The rate of convergence in the supersonic zone depends on the time which elapses before the most retarded characteristic

$$\left(\frac{q^2}{a^2} - 1\right)t = 2\alpha_1 s \ , \quad m = -\frac{\alpha_2}{\alpha_1} s \ , \quad n = -\frac{\alpha_3}{\alpha_1} s$$

ceases to intersect the initial data, and is maximized by minimizing α_1, subject to condition (23).

These considerations indicate the need to augment the retarded difference formulas for G_{ss} in the supersonic zone with expressions contributing to the term in G_{st}. For this purpose, and also to ensure the diagonal dominance of the equations for the new values on each line, G_{ss} is calculated at supersonic points using formulas of the form

$$G_{XX} = \frac{2G^+_{i,j,k} - G_{i,j,k} - 2G^+_{i-1,j,k} + G_{i-2,j,k}}{\Delta X^2} \qquad (24)$$

The mixed derivatives contributing to G_{ss} are represented by formulas of the form

$$G_{XY} = \left\{G^+_{i,j,k} - G^+_{i-1,j,k} - G^+_{i,j-1,k} + G^+_{i-1,j-1,k}\right\} / \Delta X \ \Delta Y \qquad (25)$$

To meet condition (23) near the sonic line the coefficient of G_{st} can be further augmented by adding a term

$$\varepsilon \frac{\Delta t}{\Delta X} (U\ G_{Xt} + V\ G_{Yt} + h^2 W\ G_{Zt}) \qquad (26)$$

where the mixed derivatives are represented by formulas of the form

$$\frac{\Delta t}{\Delta X} G_{Xt} = \frac{G^+_{i,j,k} - G_{i,j,k} - G^+_{i-1,j,k} + G_{i-1,j,k}}{\Delta X^2} \qquad (27)$$

and ε is a parameter proportional to $\sqrt{q^2/a^2 - 1}$. Usually, however, this term proves unnecessary.

The remaining terms of the principal part at supersonic points are approximated by formulas of the form (19), as are all second derivatives at subsonic points, except on the line which is being updated where new values are used at all three points in the difference formula. In the supersonic zone r is set equal to zero to give a zero coefficient for ϕ_t. At subsonic points the rate of convergence is controlled by the value of r.[10] If ω is the over relaxation factor one takes

$$r\ \Delta x = \frac{2}{\omega} - 1 \qquad (28)$$

At both subsonic and supersonic points the velocities and all terms containing the first derivatives are evaluated by formulas of the form

$$G_X = \frac{G_{i+1,j,k} - G_{i-1,j,k}}{2\Delta X} \qquad (29)$$

using values frozen from the previous cycle. The boundary condition at the body is satisfied by giving appropriate values to G at dummy points behind the boundary, and the standard difference equations are then used at the surface points.

The complete difference scheme leads to a line relaxation algorithm which has proved reliable in practice. The lines to be updated can be in any coordinate direction. The only constraint is the need to march in a direction which is not opposed to the flow in order to obtain a positive coefficient for ϕ_{st}. It has been found best to divide each X-Y plane into 3 strips. Then one marches towards the surface in the central strip, and outwards with the flow in the lefthand and righthand strips.

4. Results

A Fortran computer program has been used to make numerical tests of these ideas. All the runs have been made on the CDC 6600 computer belonging to the AEC Computing and Applied Mathematics Center at New York University, using the FTN compiler. To save computer time, calculations are performed on a sequence of meshes. The solution is first obtained on a coarse mesh. This is then interpolated to provide the starting point for a calculation in which the mesh size is halved in each coordinate direction, leading to 8 times as many mesh cells. Using this procedure the circulation can be roughly determined on the coarse mesh at very low cost. Typically the lattice for the initial calculation contains 64 divisions in the chordwise direction around the transformed surface, each division is normal to the surface, and 16 divisions in the spanwise direction, giving 8192 cells. The refined mesh then has 128×16×32 = 65,536 cells. Generally, 200 cycles on the coarse mesh followed by 100 cycles on the refined mesh are sufficient to reduce the residual to the order of 10^{-5}. Such a calculation takes about 30 minutes on the CDC 6600, and would require about 7 minutes on a CDC 7600.

In order to check the convergence of the method as the mesh size is reduced, a small number of calculations have been made on a sequence of 3 meshes, with 48×8×16 = 6144 cells, 96×16×32 = 49,152 cells, and 192×32×64 = 393,216 cells. Such a large array cannot be contained in the high speed memory, so it is necessary to use the disk for storage. Three disk files are used in rotation: at each cycle the potential is read from one file, updated in the central processor, and written on the next file. Thus a spare copy is always preserved, providing protection of the intermediate results in case of a parity error in reading or writing the disk. To avoid wasted time while the central processor waits for completion of a disk operation, the disk operations are performed in parallel with computation on the central processor, using storage buffers. Calculations with the fine mesh are expensive, requiring about 90 seconds a cycle. With 100 fine mesh cycles a complete calculation takes about 4 hours. Thus for normal purposes, it is necessary to rely on the results of calculations on the mesh with 65,536 cells. The accuracy of these calculations generally appears to be sufficient, and quite realistic lift-drag ratios have been obtained by integrating the surface pressure.

As an example of a fine mesh calculation, Figure 1 shows the result for a partially tapered wing of aspect ratio 8.9 with an NACA 0012 section at 3° angle of attack and Mach .75. The upper surface pressure at successive span stations are plotted above each other at equal vertical intervals. The shock wave is clearly exhibited, and can be seen to curve forward near the tips. The computed lift-drag ratio of 17.7 includes an allowance for a skin friction coefficient equal to .008. The lift-drag ratio calculated on the coarse mesh was 27.6, and on the medium mesh 19.1. Thus the drag is clearly under-estimated on the coarse mesh, but reasonably well indicated on the medium mesh. Figure 2 shows an example for a wing at an appreciable yaw angle and a higher Mach number. In this case the section was one used by R. T. Jones in tests on a yawed wing model. To allow for additional parasite drag the skin friction coefficient was taken to be .01. The angle of attack is the angle in the cross-plane normal to the leading edge. Some twist was introduced, but not enough to equalize the load completely. The shock waves are still quite well captured, as can be seen.

With a supersonic free stream and a large yaw angle, the flow is generally supersonic behind the oblique shock waves which appear on the wing surface. In the calculations the shock waves are then usually less well defined, being spread over 4 or 5 mesh widths. It still appears to be profitable, however, to obtain a useful estimate of the lift-drag ratio. Figure 3 shows some curves of the lift-drag ratio calculated for a partially tapered wing with Jones' section and an aspect ratio of 11.1. The contribution of the spanwise force component has been ignored to avoid difficulties in the region of the tips: better values might be obtained by integrating the momentum and pressure over a suitable control surface. Also the amount of twist was not correctly chosen to equalize the load. Nevertheless the curves show the same trend as the results of tests by R. T. Jones of a yawed wing with an elliptic plan form of aspect ratio 12.7.

The results are encouraging, and demonstrate the feasibility of performing useful 3-dimensional calculations with current computers. There remains the potential for substantial improvement if a second order accurate difference scheme can be devised for the supersonic zone, obviating the need for a very fine mesh to obtain high accuracy. A more accurate treatment of the shock waves is also desirable, if it can be achieved without excessive cost in computer time. The other main line of development is the treatment of more complicated geometries such as wing-body combinations. This may require the patching of different coordinate systems for different regions.

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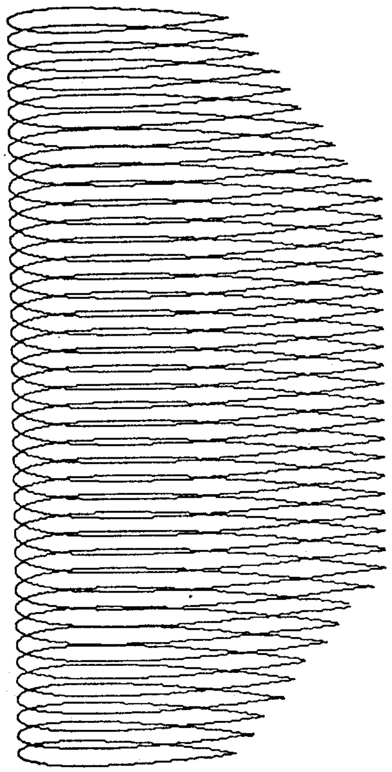

Fig. 1a. View of Wing.
NACA 0012 section AR 8.9 TWIST 0 DEG
M = .750 YAW = 0.00 ALF = 3.00
L/D = 17.75 CL = .4165 CD = .0235

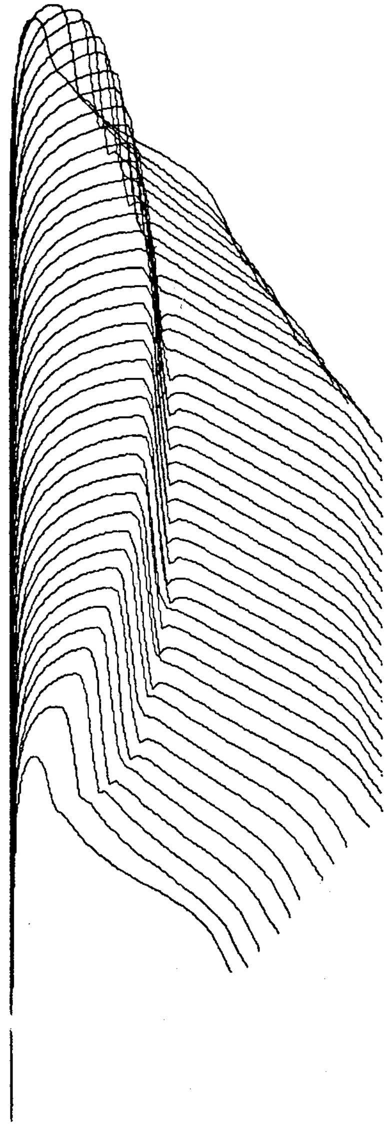

Fig. 1b. Upper Surface Pressure.
NACA 0012 Section AR 8.9 TWIST 0 DEG
M = .750 YAW = 0.00 ALF = 3.00
L/D = 17.75 CL = .4165 CD = .0235

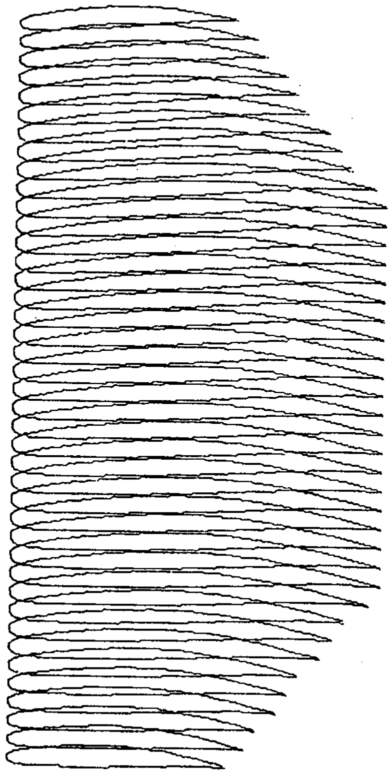

Fig. 2a. View of Wing.
JONES SECTION AR 13.3 TWIST 4 DEG
M = .866 YAW = 30.00 ALF = 3.00
L/D = 16.71 CL = .7474 CD = .0447

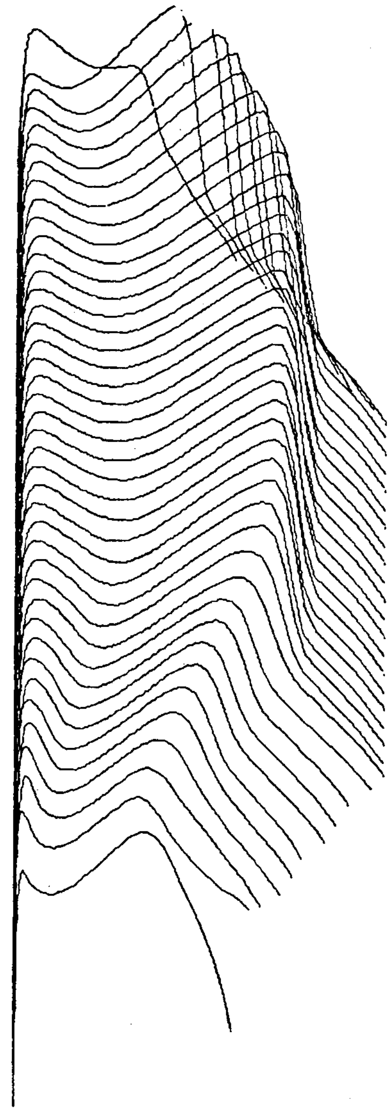

Fig. 2b. Upper Surface Pressure.
JONES SECTION AR 13.3 TWIST 4 DEG
M = .866 YAW = 30.00 ALF = 3.00
L/D = 16.71 CL = .7474 CD = .0447

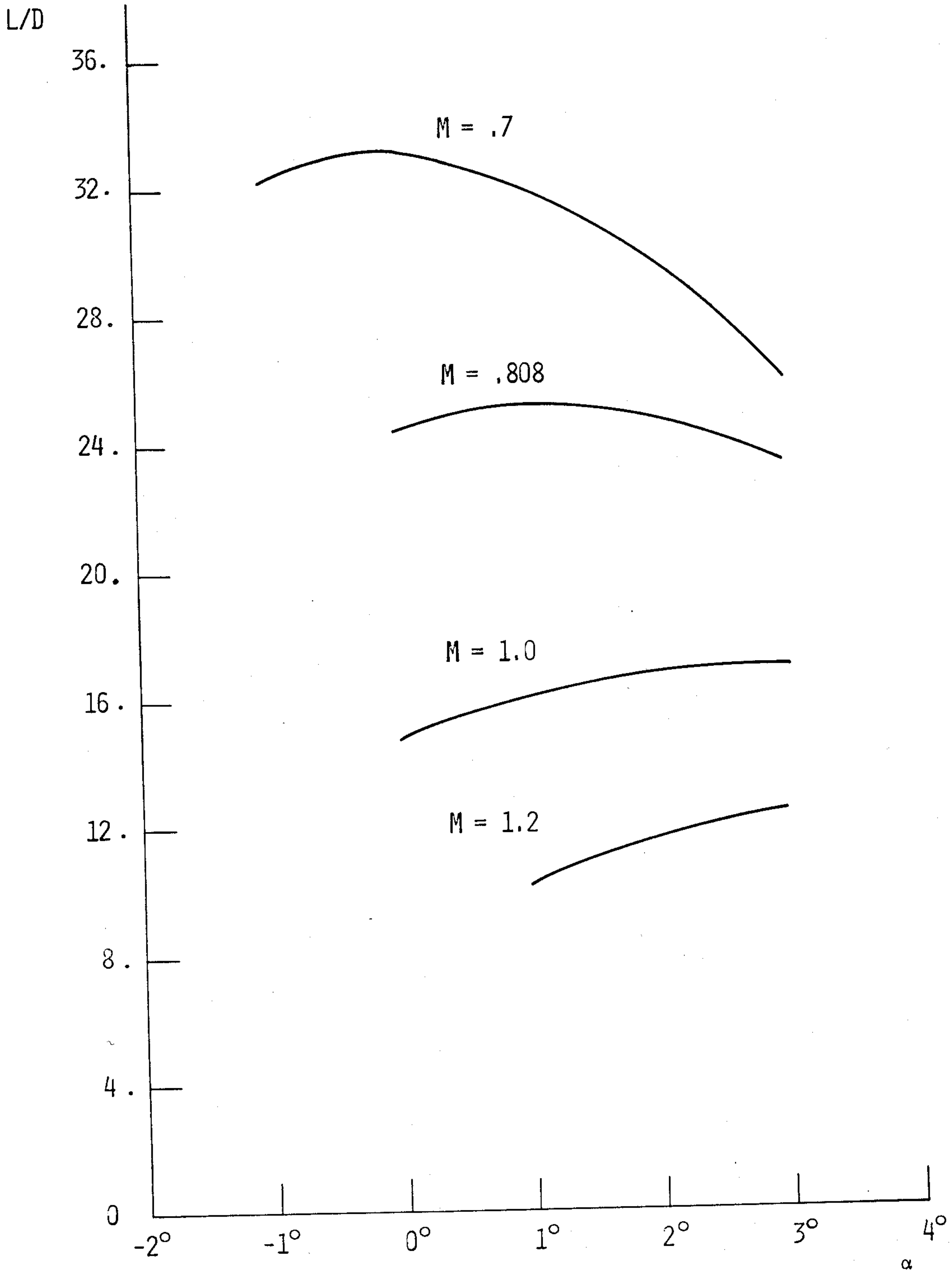

FIG. 3