Future Research Avenues in Computational Engineering and Design

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Abstract

This paper provides an overview of the development of robust algorithms for use in the area of computational aerodynamic analysis and design and relates our industrial experience with the use of these algorithms. In addition, we present our view of the direction of the field of multidisciplinary design and make suggestions as to possible avenues of research that will facilitate the development of new, integrated, computational design environments in the future.

1 Introduction

The objective of this paper is to present a coherent view of future avenues of research in computational engineering and science based on recent accomplishments in accurate and robust algorithm development, and on techniques for the automatic design of systems governed by partial differential equations. In light of these developments, we propose our view of a computational engineering environment for the design of multidisciplinary aerospace systems based on high-fidelity methods.

Our discussion is mostly focused on the fields of aerospace design and applied mathematics from which we derive most of our experience. Our predictions are based on both the current status of computational science and on reasonable extrapolations for existing computational methods in analysis and design, including the development and promise of new algorithmic ideas, the maturation of massively parallel computing architectures, and the appearance of software environments that facilitate the integration of multiple computational procedures.

Our vision is best articulated in light of the statements contained in the 1975 paper of Chapman, Mark, and Pirtle [1] in which the authors envisioned the concept of a *numerical wind tunnel* that would make the aerodynamic analysis of flight vehicles at arbitrary flight conditions possible. Their vision has been largely accomplished in principle, although, in practice, several important issues remain to be resolved. Among them it is worth mentioning inaccuracies in existing turbulence models, slow mesh generation for complex configurations, lack of high-resolution higher-order accurate algorithms, and unacceptably low rates of convergence for high Reynolds number computations. All of these items contribute to the high cost of numerical simulation. If Navier-Stokes level modeling is to be used within the framework of a *numerical wind tunnel*, the cost of single computations has to be reduced to a point where hundreds or thousands of analyses can be completed within the time frame of a few days or hours.

With the added perspective of 25 years of progress it is evident that the role of the *numerical wind tunnel* is complementary to that of the real one. With the current status of computational algorithms, there are regions of the design envelope for which numerical analysis can yield fast and inexpensive turnaround, but there are certainly quite a few others (typically those in which separated flows and extremely complex geometric configurations are present) for which the real wind tunnel can provide more accurate and less expensive information. The progress towards the achievement of this goal has been truly remarkable, to the point that high-fidelity Computational Fluid Dynamics (CFD) has secured its place in the toolbox of the preliminary design engineer.

In the main body of the paper we summarize the algorithmic concepts that have enabled fast and reliable computation of compressible flows with shock waves. The ability to analyze complex flows with rapid turnaround motivated efforts to develop an automatic design capability. In the next part of the paper we outline the use of concepts drawn from control theory that have enabled the design of optimum aerodynamic shapes.

Once the ability to analyze complex flows has become routine the emphasis will shift towards the development of multidisciplinary analysis and design capabilities that will result in a new paradigm of computational engineering. This new paradigm constitutes our vision for the next 25 years. A relatively small team of competent engineers (somewhere in the range of $15-25$) should be able to turn around the preliminary design of a new aerospace systems, from an integrated, multidisciplinary point of view, in a period of one to two weeks (not the current months required by a team an order of magnitude larger). In the conclusion we discuss some of the remaining steps that are needed to realize this vision.

2 Non-oscillatory Shock Capturing Schemes

2.1 Local Extremum Diminishing (LED) Schemes

Early efforts to solve the equations of compressible flow produced disappointing results. Consequently, the development of non-oscillatory schemes has been a prime focus of algorithm research for compressible flow. Some of the underlying ideas are outlined in the following paragraphs. Consider a general semi-discrete scheme of the form

$$\frac{d}{dt}v_j = \sum_{k\neq j} c_{jk}(v_k - v_j). \tag{2.1}$$

A maximum cannot increase and a minimum cannot decrease if the coefficients c_{jk} are non-negative, since at a maximum $v_k - v_j \leq 0$, and at a minimum $v_k - v_j \geq 0$. Thus the condition

$$c_{jk} \geq 0, \quad k \neq j \tag{2.2}$$

is sufficient to ensure stability in the maximum norm. Moreover, if the scheme has a compact stencil, so that $c_{jk} = 0$ when j and k are not nearest neighbors, a local maximum cannot increase and local minimum cannot decrease. This local extremum diminishing (LED) property prevents the birth and growth of oscillations. The one-dimensional conservation law

$$\frac{\partial u}{\partial t} + \frac{\partial}{\partial x} f(u) = 0$$

provides a useful model for analysis. In this case waves are propagated with a speed $a(u) = \frac{\partial f}{\partial u}$, and the solution is constant along the characteristics $\frac{dx}{dt} = a(u)$. Thus the LED property is satisfied. In fact the total variation

$$TV(u) = \int_{-\infty}^{\infty} \left| \frac{\partial u}{\partial x} \right| dx$$

of a solution of this equation does not increase, provided that any discontinuity appearing in the solution satisfies an entropy condition [2]. Harten proposed that difference schemes ought to be designed so that the discrete total variation cannot increase [3]. If the end values are fixed, the total variation can be expressed as

$$TV(u) = 2 \left(\sum \text{ maxima } - \sum \text{ minima } \right).$$

Thus a LED scheme is also total variation diminishing (TVD). The converse is not necessarily true, since it is possible for adjacent maxima and minima to be shifted an equal amount, say upwards, so that the total variation is unchanged, while the local maximum is increased. Positivity conditions of the type expressed in equations (2.1) and (2.2) lead to diagonally dominant schemes, and are the key to the elimination of improper oscillations. The positivity conditions may be realized by the introduction of diffusive terms or by the use of upwind biasing in the discrete scheme.

This can be conveniently illustrated for the case of the one-dimensional scalar conservation law

$$\frac{\partial v}{\partial t} + \frac{\partial}{\partial x} f(v) = 0. \tag{2.3}$$

When using a three point scheme

$$\frac{dv_j}{dt} = c^+_{j+\frac{1}{2}} \left(v_{j+1} - v_j \right) + c^-_{j-\frac{1}{2}} \left(v_{j-1} - v_j \right),$$

the scheme is LED if

$$c^+_{j+\frac{1}{2}} \geq 0, \quad c^-_{j-\frac{1}{2}} \geq 0. \tag{2.4}$$

A conservative semi-discrete approximation to the one-dimensional conservation law can be derived by subdividing the line into cells. Then the evolution of the value v_j in the jth cell is given by

$$\Delta x \frac{dv_j}{dt} + h_{j+\frac{1}{2}} - h_{j-\frac{1}{2}} = 0, \tag{2.5}$$

where $h_{j+\frac{1}{2}}$ is an estimate of the flux between cells j and $j+1$. The simplest estimate is the arithmetic average $(f_{j+1}+f_j)/2$, but this leads to a scheme that does not satisfy the positivity conditions. To correct this, one may add a dissipative term and set

$$h_{j+\frac{1}{2}} = \frac{1}{2}\left(f_{j+1}+f_j\right) - \alpha_{j+\frac{1}{2}}\left(v_{j+1}-v_j\right). \tag{2.6}$$

In order to estimate the required value of the coefficient $\alpha_{j+\frac{1}{2}}$, let $a_{j+\frac{1}{2}}$ be a numerical estimate of the wave speed $\frac{\partial f}{\partial u}$,

$$a_{j+\frac{1}{2}} = \begin{cases} \frac{f_{j+1}-f_j}{v_{j+1}-v_j} & \text{if} \quad v_{j+1} \neq v_j \\ \left.\frac{\partial f}{\partial v}\right|_{v=v_j} & \text{if} \quad v_{j+1} = v_j \end{cases}. \tag{2.7}$$

Then it is easily verified that the LED condition (2.4) is satisfied if

$$\alpha_{j+\frac{1}{2}} \geq \frac{1}{2}\left|a_{j+\frac{1}{2}}\right|. \tag{2.8}$$

If one takes

$$\alpha_{j+\frac{1}{2}} = \frac{1}{2}\left|a_{j+\frac{1}{2}}\right|,$$

one obtains the first order upwind scheme

$$h_{j+\frac{1}{2}} = \begin{cases} f_j & \text{if } a_{j+\frac{1}{2}} > 0 \\ f_{j+1} & \text{if } a_{j+\frac{1}{2}} < 0 \end{cases}.$$

This is the least diffusive first order scheme which satisfies the LED condition. In this sense upwinding is a natural approach to the construction of non-oscillatory schemes. It may be noted that the successful treatment of transonic potential flow also involved the use of upwind biasing. This was first introduced by Murman and Cole to treat the transonic small disturbance equation [4].

Higher order non-oscillatory schemes can be derived by introducing anti-diffusive terms in a controlled manner. Following the pioneering work of Godunov [5], a variety of dissipative and upwind schemes designed to have good shock capturing properties have been developed during the past two decades [6; 7; 8; 9; 10; 3; 11; 12; 13; 14; 15; 16; 17].

A comparatively simple high resolution scheme which has proved successful in practice is the Jameson-Schmidt-Turkel (JST) scheme [18]. Suppose that anti-diffusive terms are introduced by subtracting neighboring differences to produce a third order diffusive flux

$$d_{j+\frac{1}{2}} = \alpha_{j+\frac{1}{2}}\left\{\Delta v_{j+\frac{1}{2}} - \frac{1}{2}\left(\Delta v_{j+\frac{3}{2}} + \Delta v_{j-\frac{1}{2}}\right)\right\}, \tag{2.9}$$

which is an approximation to $\frac{1}{2}\alpha\Delta x^3\frac{\partial^3 v}{\partial x^3}$. The positivity condition (2.2) is violated by this scheme. It proves that it generates substantial oscillations in the vicinity of shock waves, which can be eliminated by switching locally to the first order scheme. The JST scheme therefore introduces blended diffusion of the form

$$\begin{aligned} d_{j+\frac{1}{2}} &= \epsilon^{(2)}_{j+\frac{1}{2}} \Delta v_{j+\frac{1}{2}} \\ &- \epsilon^{(4)}_{j+\frac{1}{2}} \left(\Delta v_{j+\frac{3}{2}} - 2\Delta v_{j+\frac{1}{2}} + \Delta v_{j-\frac{1}{2}} \right), \end{aligned} \tag{2.10}$$

The idea is to use variable coefficients $\epsilon^{(2)}_{j+\frac{1}{2}}$ and $\epsilon^{(4)}_{j+\frac{1}{2}}$ which produce a low level of diffusion in regions where the solution is smooth, but prevent oscillations near discontinuities. If $\epsilon^{(2)}_{j+\frac{1}{2}}$ is constructed so that it is of order Δx^2 where the solution is smooth, while $\epsilon^{(4)}_{j+\frac{1}{2}}$ is of order unity, both terms in $d_{j+\frac{1}{2}}$ will be of order Δx^3.

The JST scheme has proved very effective in practice in numerous calculations of complex steady flows. A statement of sufficient conditions on the coefficients $\epsilon^{(2)}_{j+\frac{1}{2}}$ and $\epsilon^{(4)}_{j+\frac{1}{2}}$ for the JST scheme to be LED is as follows:

Theorem 2.1. (Positivity of the JST scheme) *Suppose that whenever either v_{j+1} or v_j is an extremum the coefficients of the JST scheme satisfy*

$$\epsilon^{(2)}_{j+\frac{1}{2}} \geq \frac{1}{2} \left| \alpha_{j+\frac{1}{2}} \right|, \; \epsilon^{(4)}_{j+\frac{1}{2}} = 0. \tag{2.11}$$

Then the JST scheme is local extremum diminishing (LED).

Proof: *We need only consider the rate of change of v at extremal points. Suppose that v_j is an extremum. Then*

$$\epsilon^{(4)}_{j+\frac{1}{2}} = \epsilon^{(4)}_{j-\frac{1}{2}} = 0,$$

and the semi-discrete scheme (2.5) reduces to

$$\begin{aligned} \Delta x \frac{dv_j}{dt} &= \left(\epsilon^{(2)}_{j+\frac{1}{2}} - \frac{1}{2} a_{j+\frac{1}{2}} \right) \Delta v_{j+\frac{1}{2}} \\ &- \left(\epsilon^{(2)}_{j-\frac{1}{2}} + \frac{1}{2} a_{j-\frac{1}{2}} \right) \Delta v_{j-\frac{1}{2}}, \end{aligned}$$

and each coefficient has the required sign. □

In order to construct $\epsilon^{(2)}_{j-\frac{1}{2}}$ and $\epsilon^{(4)}_{j-\frac{1}{2}}$ with the desired properties define

$$R(u,v) = \begin{cases} \left| \frac{u-v}{|u|+|v|} \right|^q & \text{if } u \neq 0 \text{ or } v \neq 0 \\ 0 & \text{if } u = v = 0, \end{cases} \tag{2.12}$$

where q is a positive integer. Then $R(u,v) = 1$ if u and v have opposite signs. Otherwise $R(u,v) < 1$. Now set

$$Q_j = R(\Delta v_{j+\frac{1}{2}}, \Delta v_{j-\frac{1}{2}}), \;\; Q_{j+\frac{1}{2}} = \max(Q_j, Q_{j+1}).$$

and

$$\epsilon^{(2)}_{j+\frac{1}{2}} = \alpha_{j+\frac{1}{2}} Q_{j+\frac{1}{2}}, \;\; \epsilon^{(4)}_{j+\frac{1}{2}} = \frac{1}{2}\alpha_{j+\frac{1}{2}}(1 - Q_{j+\frac{1}{2}}). \tag{2.13}$$

This scheme can be easily extended to systems of equations, such as the gasdynamics equations, by introducing characteristic variables and applying the same formula to each characteristic field. For this purpose, the linearization proposed by Roe is particularly useful [9].

Conditions under which a one-point numerical shock structure can be attained have been derived by Jameson, and yield a class of alternative schemes where the diffusive flux is constructed from a blend of state and flux differences

$$d_{j+\frac{1}{2}} = \frac{1}{2}\alpha^*_{j+\frac{1}{2}} c\,(w_{j+1} - w_j) + \frac{1}{2}\beta_{j+\frac{1}{2}}\,(f_{j+1} - f_j)\,, \tag{2.14}$$

where $\alpha^*_{j+\frac{1}{2}}$ and $\beta_{j+\frac{1}{2}}$ satisfy a relation that is a function of the local Mach number [19].

3 Industrial CFD

In order to carry out the inner loop of the aerodynamic design process the main requirements for effective CFD software are:

(1) Sufficient and known level of accuracy
(2) Acceptable computational and manpower costs
(3) Fast turn around time

Performance estimation in the cruise condition is crucial to the design of transport aircraft. The error should be in the range of $\pm\frac{1}{2}$ percent. The drag coefficient of a long range transport aircraft such as the Boeing 747 is in the range of .0275 (275 counts), depending on the lift coefficient, which is in approximately .5. The drag coefficient of proposed supersonic transport designs is in the range of .0120 to .0150 at much lower lift coefficients in the range of .1 - .12. Thus one should aim to predict drag with an accuracy of the order of $\pm$ .0001 ($\pm$1 count). Manufacturers have to guarantee performance, and errors can be very expensive through the costs of redesign, penalty payments and lost orders.

A first consideration is the choice of appropriate mathematical models of fluid flow which are adequate for trustworthy flow predictions. Many critical phenomena of fluid flow, such as shock waves and turbulence, are essentially non-linear. They also exhibit extreme disparities of scales. While the actual thickness of a shock wave is of the order of a mean free path of the gas particles, on a macroscopic scale its thickness is essentially zero. In turbulent flow energy is transferred from large scale motions to progressively smaller eddies until the scale becomes so small that the motion is dissipated by viscosity. The ratio of the length scale of the global flow to that of the smallest persisting eddies is of the order $\mathrm{Re}^{\frac{3}{4}}$, where Re is the Reynolds number, typically in the range of 30 - 50 million for an aircraft. In order to resolve such scales in all three space directions a computational grid with the order of $\mathrm{Re}^{\frac{9}{4}}$ cells would be required.

This is beyond the range of any current or foreseeable computer. Consequently mathematical models with varying degrees of simplification have to be introduced in order to make computational simulation of flow flow feasible and produce viable and cost-effective methods. Figure 1 (supplied by Pradeep Raj) indicates a hierarchy of models at different levels of simplification which have proved useful in practice.

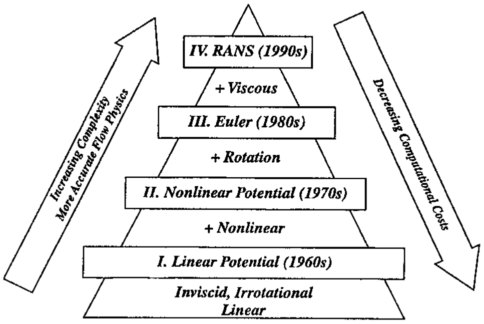

FIG. 1. Hierarchy of Fluid Flow Models

3.1 Aerodynamic Design Problems

Roughly speaking, the objective in wing design is to maximize ML/D where M is the Mach number, and L/D is the lift-to-drag ratio. This is a consequence of the Breguet range formula

$$Range = \frac{V}{c}\frac{L}{D}\log\frac{W_i}{W_f},$$

where V is the flight velocity, c is the specific fuel consumption of the propulsion plant, and W_i and W_f are the initial an final weights of the aircraft. Therefore, optimum range is obtained by increasing the speed until the onset of compressibility drag rise near the speed of sound due to the appearance of shock waves.

Also, according to classical wing theory, the drag is given by

$$C_D = C_{D_0} + \frac{C_L^2}{\pi e AR}$$

where C_D is the total drag coefficient, C_{D_0} is the parasite drag coefficient, C_L is the total lift coefficient, AR is the wing aspect ratio, and e is the airplane efficiency factor. The second term in the previous equation is often called induced drag. Here, L/D is found to be maximized when the induced drag is half of the total drag. This leads to a trade-off between aerodynamic performance and wing structure weight since increasing the span reduces the vortex drag at the expense of an increase in weight. practice, optimization has proved effective as a means to reduce or postpone the onset of shock drag.

An alternate approach to optimization is to specify a desirable pressure distribution and then solve the inverse problem of obtaining the shape that produces that target. Unfortunately, a physically realizable shape may not necessarily exist, unless the pressure distribution satisfies certain constraints. The difficulty that the target pressure may be unattainable may be circumvented by treating the inverse problem as a special case of the optimization problem. For example, if p_d is the desired surface pressure, one may take the cost function to be an integral over the body surface of the pressure error,

$$I = \frac{1}{2} \int_{\mathcal{B}} (p - p_d)^2 d\mathcal{B},$$

or possibly a more general Sobolev norm. This has the advantage of converting a possibly ill-posed problem into a well-posed one.

3.2 Application of Control Theory

In order to reduce the computational costs, it turns out that there are advantages in formulating both the inverse problem and more general aerodynamic problems within the framework of the mathematical theory for the control of systems governed by partial differential equations [20]. A wing, for example, is a device to produce lift by controlling the flow, and its design can be regarded as a problem in the optimal control of the flow equations by variation of the shape of the boundary. If the boundary shape is regarded as arbitrary within some requirements of smoothness one must use the concept of the Frechet derivative of the cost with respect to a function. Using techniques of control theory, the gradient can be determined indirectly by solving an adjoint equation which has coefficients defined by the solution of the flow equations. The cost of solving the adjoint equation is comparable to that of solving the flow equations. Thus the gradient can be determined with roughly the computational costs of two flow solutions, independently of the number of design variables.

For flow about an airfoil or wing, the aerodynamic properties which define the cost function are functions of the flow-field variables (w) and the physical location of the boundary, which may be represented by the function $\mathcal{F}$, say. Then

$$I = I\left(w, \mathcal{F}\right),$$

and a change in $\mathcal{F}$ results in a change

$$\delta I = \frac{\partial I^T}{\partial w} \delta w + \frac{\partial I^T}{\partial \mathcal{F}} \delta \mathcal{F}, \tag{3.1}$$

in the cost function. Using control theory, the governing equations of the flowfield are introduced as a constraint in such a way that the final expression for the gradient does not require reevaluation of the flowfield. In order to achieve this δw must be eliminated from (3.1). Suppose that the governing equation R which expresses the dependence of w and $\mathcal{F}$ within the flowfield domain D can be written as

$$R\left(w, \mathcal{F}\right) = 0. \tag{3.2}$$

Then δw is determined from the equation

$$\delta R = \left[\frac{\partial R}{\partial w}\right] \delta w + \left[\frac{\partial R}{\partial \mathcal{F}}\right] \delta \mathcal{F} = 0. \tag{3.3}$$

Next, introducing a Lagrange Multiplier ψ, we have

$$\begin{aligned} \delta I &= \frac{\partial I^T}{\partial w} \delta w + \frac{\partial I^T}{\partial \mathcal{F}} \delta \mathcal{F} - \psi^T \left(\left[\frac{\partial R}{\partial w}\right] \delta w + \left[\frac{\partial R}{\partial \mathcal{F}}\right] \delta \mathcal{F} \right) \\ &= \left\{ \frac{\partial I^T}{\partial w} - \psi^T \left[\frac{\partial R}{\partial w}\right] \right\} \delta w + \left\{ \frac{\partial I^T}{\partial \mathcal{F}} - \psi^T \left[\frac{\partial R}{\partial \mathcal{F}}\right] \right\} \delta \mathcal{F}. \end{aligned}$$

Choosing ψ to satisfy the adjoint equation

$$\left[\frac{\partial R}{\partial w}\right]^T \psi = \frac{\partial I}{\partial w} \tag{3.4}$$

the first term is eliminated, and we find that

$$\delta I = \mathcal{G} \delta \mathcal{F}, \tag{3.5}$$

where

$$\mathcal{G} = \frac{\partial I^T}{\partial \mathcal{F}} - \psi^T \left[\frac{\partial R}{\partial \mathcal{F}}\right].$$

The advantage is that (3.5) is independent of δw, with the result that the gradient of I with respect to an arbitrary number of design variables can be determined without the need for additional flow-field evaluations. In the case that (3.2) is a partial differential equation, the adjoint equation (3.4) is also a partial differential equation and appropriate boundary conditions must be determined.

After making a step in the negative gradient direction, the gradient can be recalculated and the process repeated to follow a path of steepest descent until a minimum is reached. In order to make sure that each new shape in the optimization sequence remains smooth, it proves essential to smooth the gradient and to replace $\mathcal{G}$ by its smoothed value $\bar{\mathcal{G}}$ in the descent process. This also acts as a preconditioner which allows the use of much larger steps. To apply smoothing in the ξ_1 direction, for example, the smoothed gradient $\bar{\mathcal{G}}$ may be calculated from a discrete approximation to

$$\bar{\mathcal{G}} - \frac{\partial}{\partial \xi_1} \epsilon \frac{\partial}{\partial \xi_1} \bar{\mathcal{G}} = \mathcal{G} \tag{3.6}$$

where ϵ is the smoothing parameter. If one sets $\delta \mathcal{F} = -\lambda \bar{\mathcal{G}}$, then, assuming the modification is applied on the surface ξ_2 = constant, the first order change in the cost function is

$$\delta I = -\iint \mathcal{G} \delta \mathcal{F} \, d\xi_1 d\xi_3$$

$$
\begin{aligned}
&= -\lambda \iint \left(\bar{\mathcal{G}} - \frac{\partial}{\partial \xi_1} \epsilon \frac{\partial \bar{\mathcal{G}}}{\partial \xi_1} \right) \bar{\mathcal{G}} \, d\xi_1 d\xi_3 \\
&= -\lambda \iint \left(\bar{\mathcal{G}}^2 + \epsilon \left(\frac{\partial \bar{\mathcal{G}}}{\partial \xi_1} \right)^2 \right) d\xi_1 d\xi_3 \\
&< 0,
\end{aligned}
$$

assuring an improvement if λ is sufficiently small and positive, unless the process has already reached a stationary point at which $\bar{\mathcal{G}} = 0$ (and therefore, according to Equation 3.6, $\mathcal{G} = 0$).

The smoothing process is equivalent to a redefinition of the inner product used to measure the gradient as

$$
< g, \delta f >= \int \left(g \delta f + \epsilon \frac{\partial g}{\partial x} \frac{\partial}{\partial x} \delta f \right) dx,
$$

which, on integration by parts reduces to

$$
\int \bar{g} \delta f dx.
$$

It turns out that this approach is tolerant to the use of approximate values of the gradient, so that neither the flow solution nor the adjoint solution need be fully converged before making a shape change. This results in very large savings in the computational cost so that a full optimization can be obtained with a cost equivalent to that of 2-10 flow solutions.

4 Industrial Experience and Results

The methods described in this paper have been quite thoroughly tested in industrial applications in which they were used as a tool for aerodynamic design. They have proved useful both in inverse mode to find shapes that would produce desired pressure distributions, and for direct minimization of the drag. They have been applied both to well understood configurations that have gradually evolved through incremental improvements guided by wind tunnel tests and computational simulation, and to new concepts for which there is a limited knowledge base. In either case they have enabled engineers to produce improved designs.

Similar considerations apply to three-dimensional wing design. Since the vortex drag can be reduced simply by reducing the lift, the lift coefficient must be fixed for a meaningful drag minimization. In order to do this the angle of attack α is adjusted during the flow solution. It has proved most effective to make a small change $\delta\alpha$ proportional to the difference between the actual and the desired lift coefficient every few iterations in the flow calculation. A typical wing of a transport aircraft is designed for a lift coefficient in the range of 0.4 to 0.6. The total wing drag may be broken down into vortex drag, drag due to viscous effects, and shock drag. The vortex drag coefficient is typically in the range of 0.0100 (100 counts), while the friction drag coefficient is in the range of 45 counts, and the shock drag at a Mach number just before the onset of severe drag rise is of the order of 15 counts. With a fixed span, typically dictated by structural limits or a constraint imposed by airport gates, the vortex drag is entirely a function of span loading, and is minimized by an elliptic loading unless winglets are added. Transport aircraft usually have highly tapered wings with very large root chords to accommodate retraction of the undercarriage. An elliptic loading may lead to excessively large section lift coefficients on the outboard wing, leading to premature shock stall or buffet when the load is increased. The structure weight is also reduced by a more inboard loading which reduces the root bending moment. The skin friction of transport aircraft is typically very close to flat plate skin friction in turbulent flow, and is very insensitive to section variations. This leaves the shock drag as the primary target for wing section optimization. This is reduced to zero if the wing is shock-free, leaving no room for further improvement. Thus the attainment of a shock-free flow is a demonstration of a successful drag minimization. One may try to find the largest wing thickness for which the shock drag can be eliminated at a given Mach number. This can yield both savings in structure weight and increased fuel volume. If there is no fixed limit for the wing span, such as a gate constraint, increased thickness can be used to allow an increase in aspect ratio for a wing of equal weight, in turn leading to a reduction in vortex drag. Since the vortex drag is usually the largest component of the total wing drag, this is probably the most effective design strategy, and it may pay to increase the wing thickness to the point where the optimized section produces a weak shock wave rather than a shock-free flow [21].

The first major industrial application of an adjoint based aerodynamic optimization method was the wing design of the Beech Premier I [22] in 1995. The method was successfully used in inverse mode as a tool to obtain pressure distributions favorable to the maintenance of natural laminar flow over a range of cruise Mach numbers. Wing contours were obtained which yielded the desired pressure distribution in the presence of closely coupled engine nacelles on the fuselage above the wing trailing edge.

During 1996 some preliminary studies indicated that the wings of both the McDonnell Douglas MD-11 and the Boeing 747-200 could be made shock-free in a representative cruise condition by using very small shape modifications, with consequent drag savings which could amount to several percent of the total drag. This led to a decision to evaluate adjoint-based design methods in the design of the McDonnell Douglas MDXX during the summer and fall of 1996. Promising results were obtained [23], and the McDonnell Douglas corporation planned to test the wing resulting from the optimization. However, these tests were not performed due to the cancellation of the entire MDXX project.

In the next subsection we present a typical example of the use of the adjoint method for the drag minimization of the Boeing 747 wing-body combination modeled by the Euler equations.

4.1 Three point inviscid redesign of the Boeing 747 wing

Our example shows a redesign of the wing of the Boeing 747 to reduce its drag in a typical cruise condition. It has been our experience that drag minimization at a single point tends to produce a wing which is shock free at its design point, but tends to display undesirable characteristics off its design point. Typically, a double shock pattern forms below the design lift coefficient and Mach number, and a single, but fairly strong shock above the design point. To alleviate this tendency the calculation was performed with three design points. In carrying out multipoint designs of this kind a composite gradient is calculated as a weighted average of the gradients calculated for each design point separately. In this case the design points were selected as lift coefficients of .38, .42 and .46 for the exposed wing at Mach .85. Because the fuselage has a significant effect on the flow over the wing, the calculations were performed for the wing-body combination, but the shape modifications were restricted to the wing alone. The fuselage also contributes to the lift, so that the total lift coefficient at the mid design point was estimated to be .50.

The results are displayed in Figures 2 - 3. It can been seen that a drag reduction was obtained over the entire range of lift coefficients, and at the mid design point the redesigned wing is almost shock free. Figure 4 shows the modification in the wing section about half way out the span. It can be seen that a useful drag reduction can be obtained by a very small change in the wing shape. This is because of the extreme sensitivity of the transonic flow. Also, it is clear that without a tool of this kind it would be almost impossible to find an optimum shape.

5 Conclusions and Outlook

A framework for the development of highly accurate and robust numerical algorithms for the computation of high-speed compressible flows has been outlined. Details can be found in our prior publications. These algorithms are at the heart of a series of analysis programs that have been thoroughly tested in an industrial environment. In addition, a procedure for the aerodynamic shape optimization of modern aircraft has been presented.

Our efforts have been largely motivated by the general consensus that the view of an aerospace vehicle as an integrated system is necessary to compete in the current industrial environment. On the one hand, high-fidelity multidisciplinary analysis is being extended to those disciplines that interact closely. On the other hand, there have been some very important initial contributions to the problem of single-discipline design optimization. Our vision for the direction that computational engineering will take in the coming years is one in which the two areas of research mentioned above will merge into one. Knowledge will still be distributed among a host of disciplinary experts, with the essential difference that information from other disciplines will be readily available regardless of format, source, geographical location, or complexity and level of fidelity.

In sum, we envision an engineering design and development environment in which a design is carried out through the various stages largely by computation, and in which trade-offs between participating disciplines are not only enabled but greatly facilitated and required. The final objective is to allow for the possibility of designing aerospace vehicles with suitably defined optimum performance in a short period of time. The following is a list of some of the critical research topics that must be addressed before our long term goal can be achieved.

- Continued development and refinement of fundamental algorithms.
- Refinement and development of optimization techniques for problems involving a single disciplines.
- Accuracy-preserving algorithms for the coupling of disciplines.
- Optimization methods and environments for multidisciplinary optimization (MDO).
- Distributed design environments (Python, Java, CORBA, and others) which are independent of data locality.
- Efficient use of large scale parallel computational resources.
- Reduced order modeling of disciplines participating in the design.

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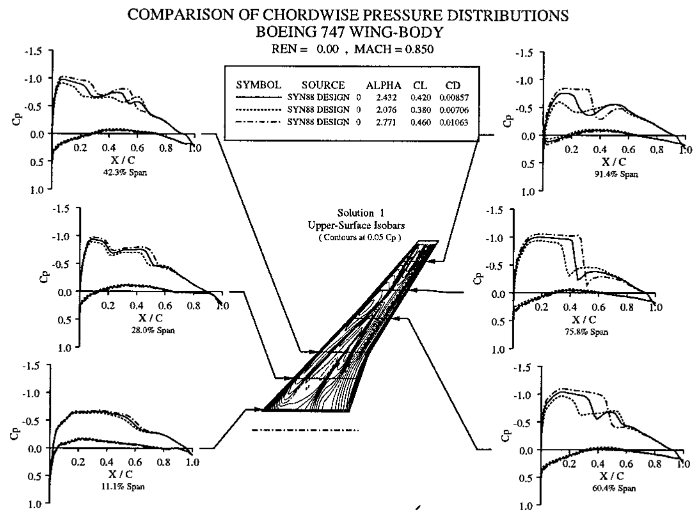

FIG. 2. Pressure distribution of the Boeing 747 Wing-Body before optimization.

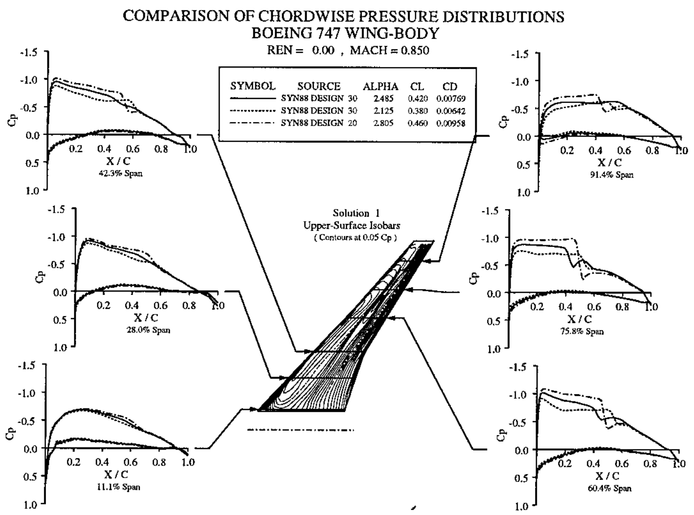

FIG. 3. Pressure distribution of the Boeing 747 Wing-Body after a three point optimization.

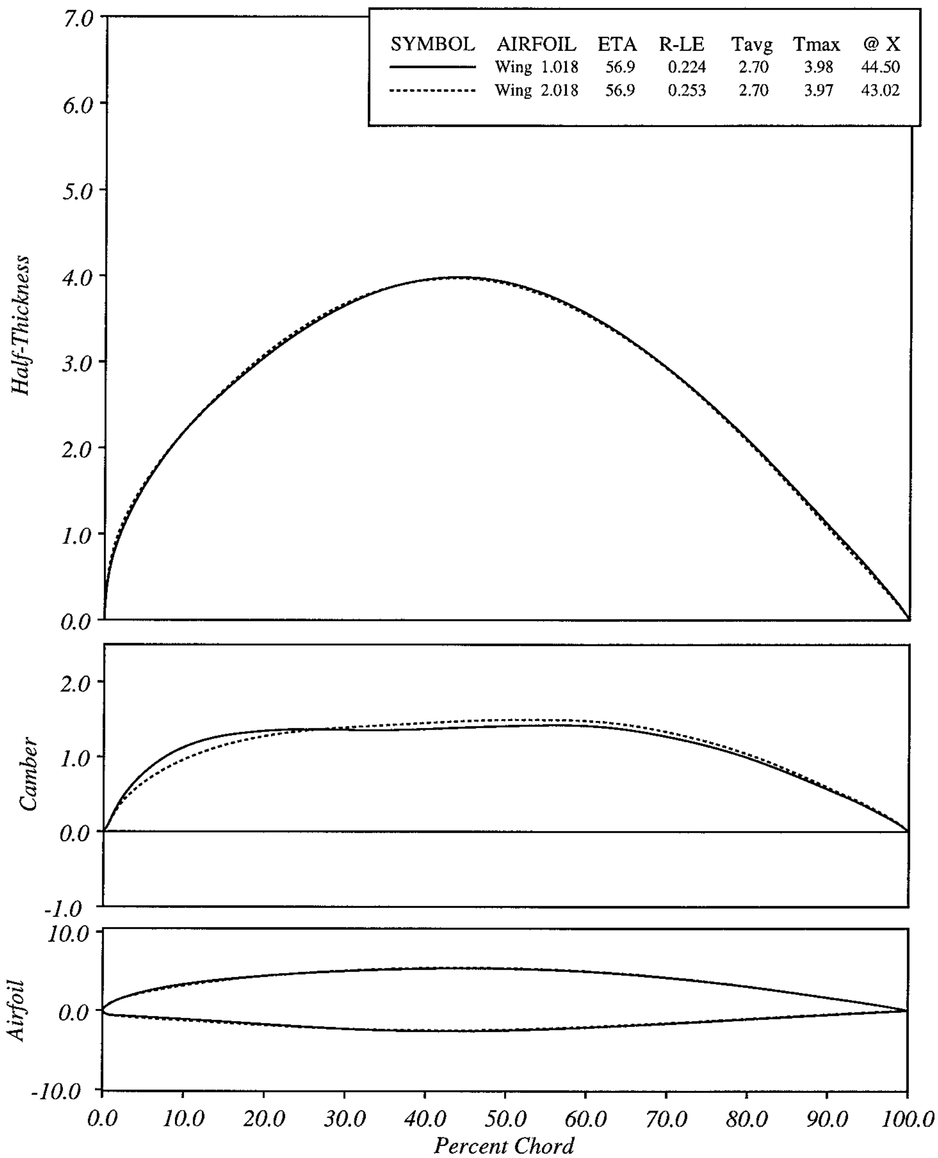

FIG. 4. Original and Re-designed Wing section for the Boeing 747 Wing-Body at mid-span.