TRANSONIC FLOW CALCULATIONS FOR AIRFOILS AND BODIES OF REVOLUTION

by

Antony Jameson

Aerodynamics Report

390-71-1

Grumman Aerospace Corporation

1. Introduction

This report presents results of transonic flow calculations by the relaxation method for a selection of airfoils and bodies of revolution. The flow is assumed to be determined by the potential flow equation

$$(a^2-u^2)\,\varphi_{xx} - 2uv\,\varphi_{xy} + (a^2-v^2)\,\varphi_{yy} = 0 \tag{1.1}$$

where φ is the potential, u and v are the velocity components

$$u = \varphi_x,\; v = \varphi_y \tag{1.2}$$

and a is the local speed of sound. This may be determined from the stagnation speed of sound a_o by the energy equation

$$a^2 = a_o^2 - .2\,(u^2+v^2) \tag{1.3}$$

in which the ratio of the specific heats has been taken as 1.4. If the velocities are normalized with unit free stream velocity, then

$$a_o^2 = \frac{1}{M^2} + .2 \tag{1.4}$$

where M is the free stream Mach number.

There is no assumption of small disturbances and the solution would be exact for subsonic flow in the absence of viscosity. In transonic flow the principal approximation is the neglect of rotation introduced by shock waves. In the absence of rotation Crocco's theorem [1] indicates that the flow is homentropic. The solution must allow discontinuities to approximate to approximate shock waves, and it has been shown by Morawetz [2] that in general a continuous solution does not exist in transonic flow. Since in isentropic flow there is a unique relation between velocity and area along a stream tube, the interpretation of a solution with a jump in velocity is that it is a solution in the presence of a fictitious screen just the correct throat area to permit an isentropic compression. Since there is a pressure differential on either side of the throat a discontinuity in the solution represents a line carrying a force. The total drag is zero in potential flow, so this force is just balanced by a drag on the airfoil, and it turns out that the solution can be used to obtain an estimate of the pressure drag on the airfoil in the presence of a shock wave (see the results).

It should also be noted that in the absence of a directional condition corresponding to the condition that entropy can only increase, the solution of the potential equation is not unique. Given a solution, the equation continues to be satisfied when the velocity is reversed. Thus, for example, a body with for and aft symmetry always admits a solution with a reverse shock as well as one with a forward shock. In order to obtain a unique solution, it is necessary to introduce directionality into the difference scheme used to approximate the equation. Following Murman [3], this is accomplished by using a central difference scheme in the subsonic region together with a backward difference scheme in the supersonic region.

2. Formulation of the equations in the circle domain.

In order to obtain a finite domain of calculation the flow field is first mapped to the interior of a unit circle by a conformal transformation, following Sells [4]. Using polar coordinates r and θ, the potential for a uniform at angle α would then be $\frac{\cos(\theta+\alpha)}{r}$. Also, in the presence of circulation the potential would be multiple valued. It is convenient therefore to set

$$\phi = X + \frac{\cos(\theta+\alpha)}{r} - E\theta \qquad (2.1)$$

where $2\pi E$ is the circulation, and solve for the reduced potential functions X which is finite and everywhere continuous. Let the modulus of the transformation to the interior of the circle be $\frac{H}{r^2}$. H is the modulus of the transformation to the exterior of the circle, and is a smooth function suitable for differencing. The equation for X in polar coordinates then becomes

$$\begin{aligned}(a^2-u^2)\, X_{\theta\theta} - 2uv\,(rX_{\theta r} + X_\theta - E) + (a^2-v^2)(r^2X_{rr} + rX_r)\\ + (u^2-v^2)rX_r + (u^2+v^2)(\tfrac{u}{r}H_\theta + vH_r) = 0\end{aligned} \qquad (2.2)$$

where

$$u = \frac{r(X_\theta - E) - \sin(\theta+\alpha)}{H}, \quad v = \frac{r^2X_r - \cos(\theta+\alpha)}{H} \qquad (2.3)$$

and

$$a^2 = a_o^{\,2} - .2(u^2 + v^2) \qquad (2.4)$$

The boundary condition at the surface r =1 is

$$v = 0, \; X_r = \cos\,(\theta + \alpha) \qquad (2.5)$$

If the airfoil has a sharp trailing edge, it is convenient to orient the circle so that the corresponding point is at θ = 0. At this point H = 0, and the Kutta condition that the velocity is finite at the trailing edge then requires that

$$X_\theta = E + \sin\alpha \qquad (2.6)$$

Also the equation for X can be written as

$$a^2(r^2 X_{rr} + rX_r + X_{\theta\theta}) = Hu \frac{1}{r}\frac{\partial}{\partial\theta}\left(\frac{u^2+v^2}{2}\right) + Hv \frac{\partial}{\partial r}\left(\frac{u^2+v^2}{2}\right)$$

At the trailing edge, the right side vanishes and thus X locally satisfies Laplace's equation.

At the center of the circle $H \rightarrow 1 + O(r^2)$, $u \rightarrow \sin(\theta+\alpha)$, $v \rightarrow \cos(\theta+\alpha)$, $a \rightarrow \frac{1}{M}$. It can be seen therefore that the potential equation reduces to

$$[1-M^2 \sin^2(\theta+\alpha)]\, X_{\theta\theta} - 2M^2 \sin(\theta+\alpha) \cos(\theta+\alpha)(X_\theta - E) = 0 \tag{2.7}$$

This can be integrated to give the far field boundary condition at $r = 0$

$$X = E\left\{\theta - \tan^{-1}\left[(1-M^2)^{\frac{1}{2}} \tan(\theta+\alpha)\right]\right\} \tag{2.8}$$

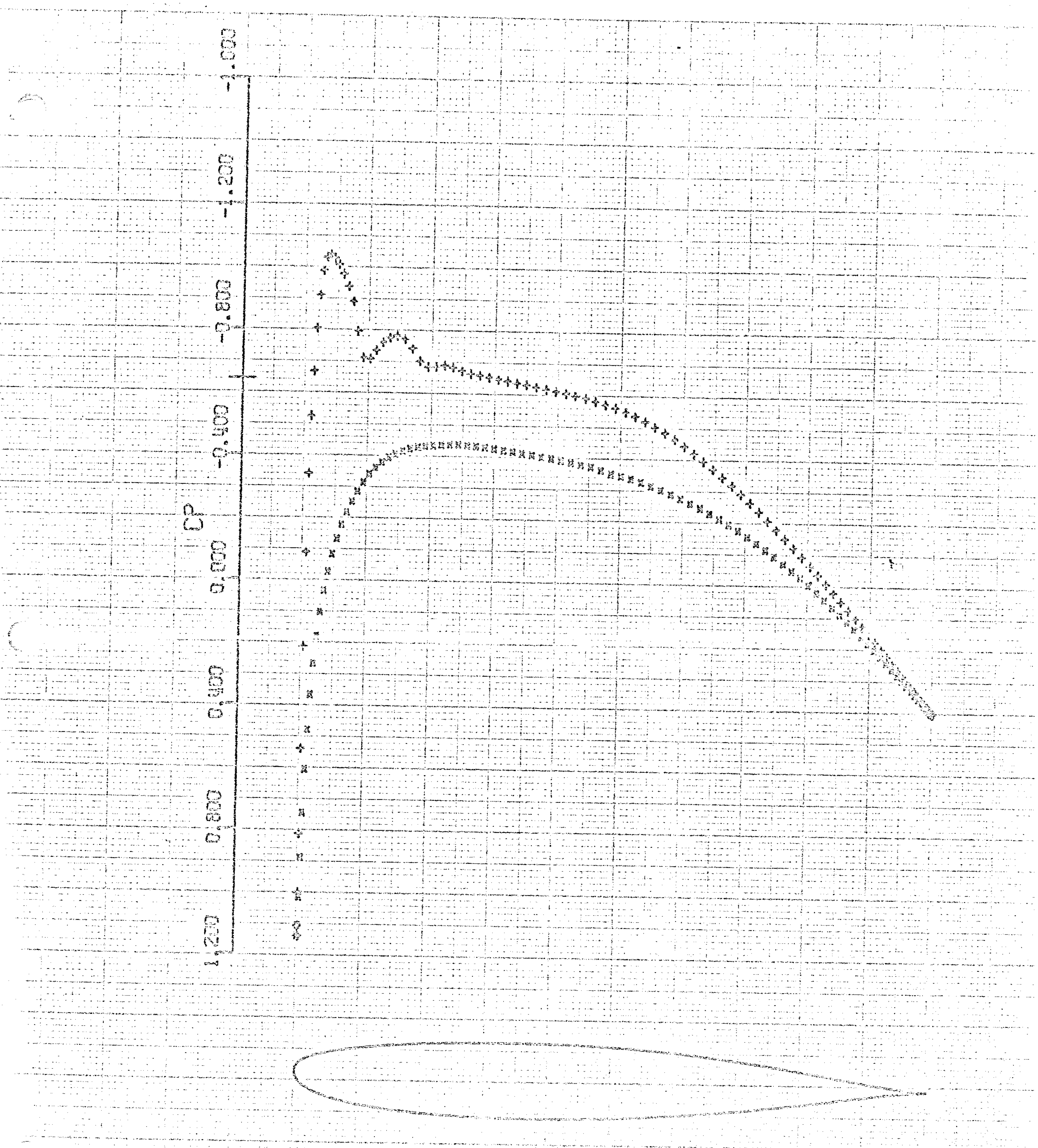
in agreement with the results of Ludford [5], and Imai [6].

References:

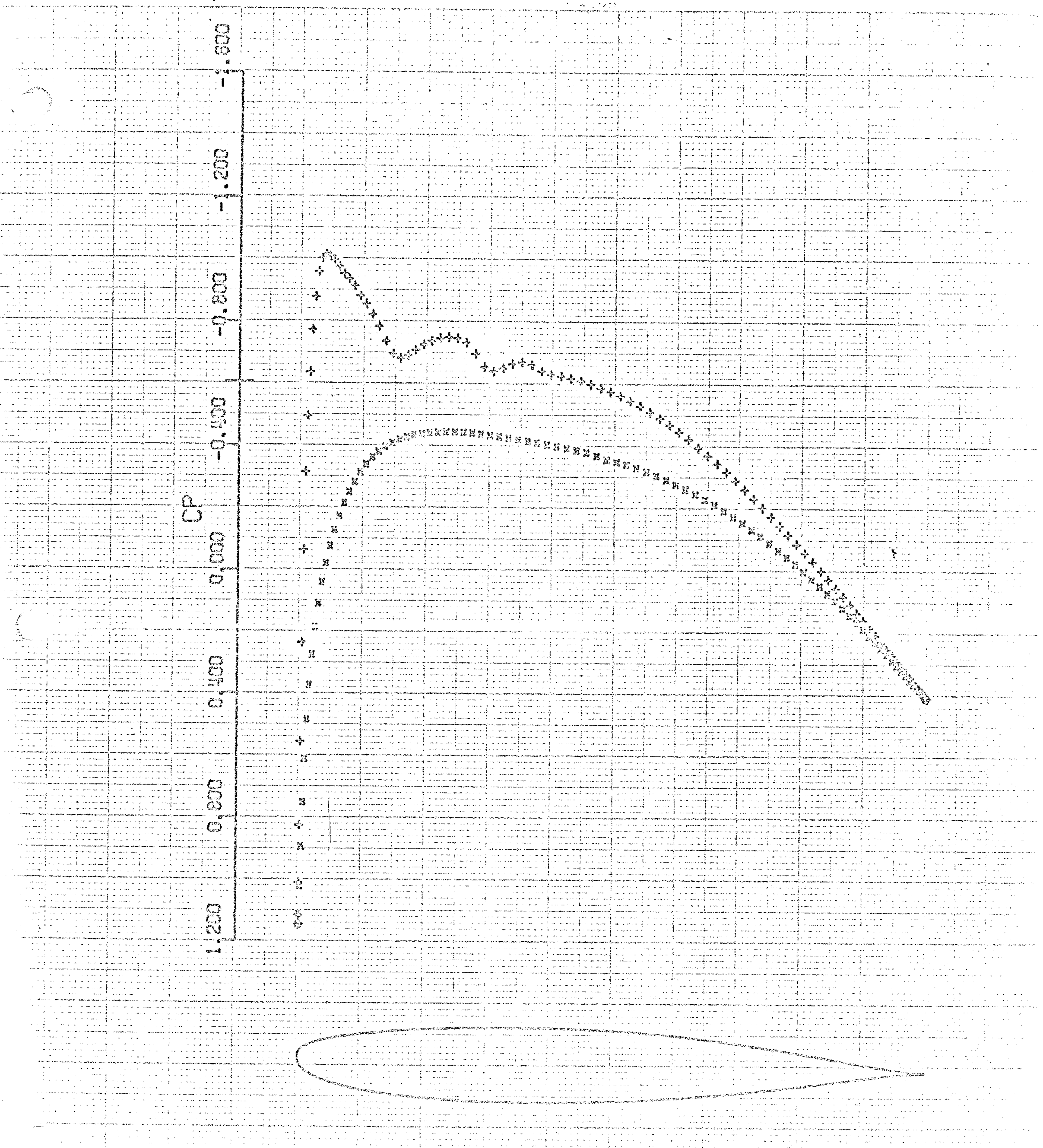
1. H. W. Liepman and A. Roshko, Elements of Gas Dynamics, Wiley, New York, 1957, p. 193
2. C. S. Moraw etz, On the Non-Existence of Continuous Transonic Flows Past Profiles, Comm. Pure Appl. Math, Vol. 9, 1956, pp 45 - 68
3. E. M. Murman and I. D.Cole, Calculation of Plane Steady Transonic Flows, AIAA J. Vol. 9, 1971, pp 114 - 121
4. C. C. L.Sells, Plane Subcritical Flow Past a Lifting Airfoil, Proc. Roy. Soc. Series A, Vol. 308, 1967, pp 377 - 401
5. G. S. S. Ludford, The Behaviour at Infinity of the Potential Function of a Two Dimensional Subsonic Compressible Flow J. Math Phys. 1951, Vol. 30, 1951 pp 131 - 139
6. I. Imai, Approximation Methods in Compressible Fluid Dynamics, University of Maryland Technical Note BN-95, 1957

NLR LIFTING QUASI-ELLIPTICAL AIRFOIL

Designed to be shock free at M = .7557, $\alpha = 0°$ (1.32° when the coordinates are referred to the line from the maximum forward point to the trailing edge). Coordinates and slopes from AGARD Report 575, Test Cases for Numerical Methods in Two Dimensional Transonic Flow, by R. C. Lock, 1970.



NLR LIFTING QUASI-ELLIPTICAL AIRFOIL
M = 0.736 ALF = 0.0 CL = 0.232
CL = 0.2322 CD = 0.0001 CM = 0.1011



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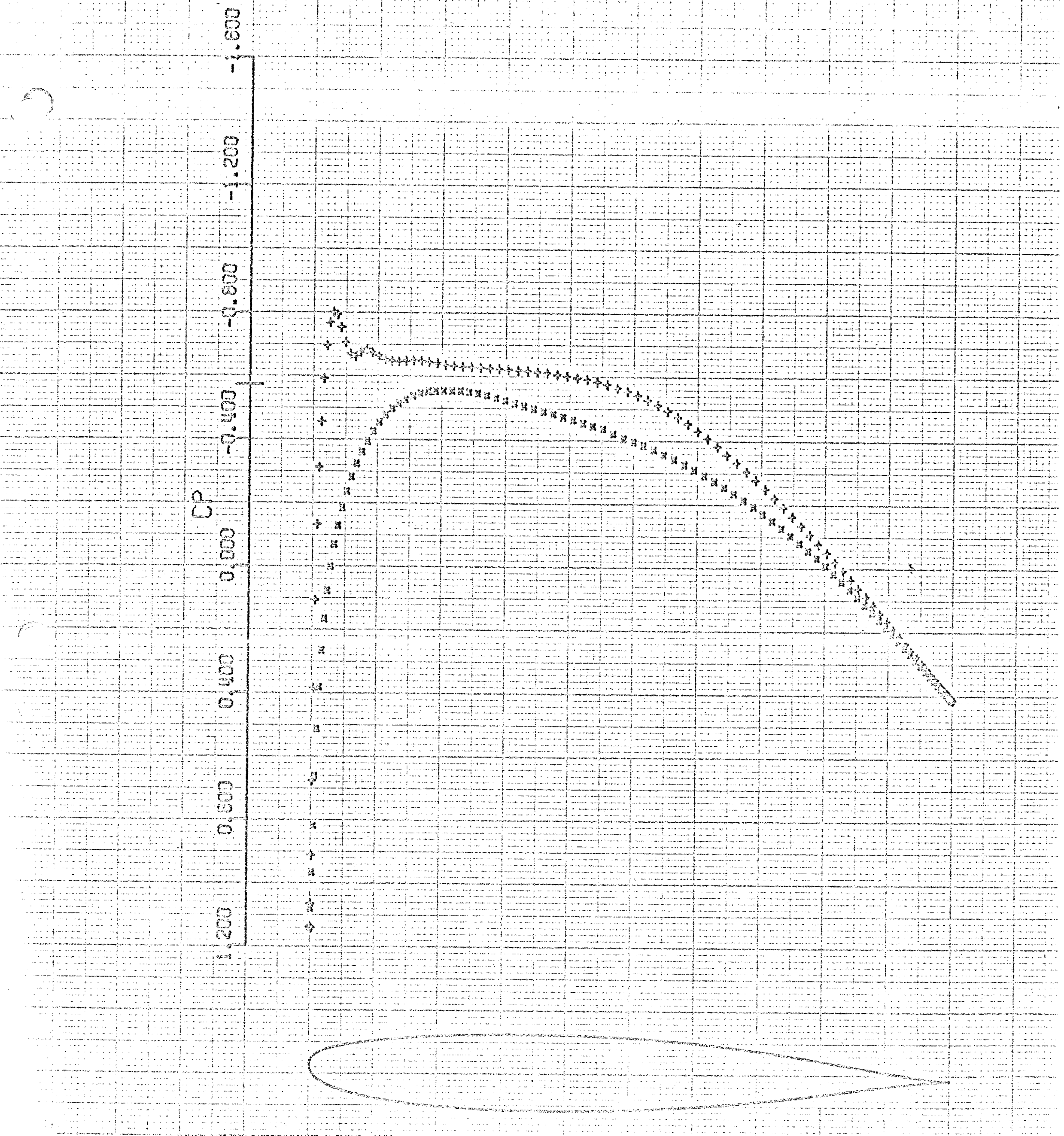
M = 0.746 ALF = 0.0 CL = 0.241
CL = 0.2413 CD = 0.0000 CM = 0.1152



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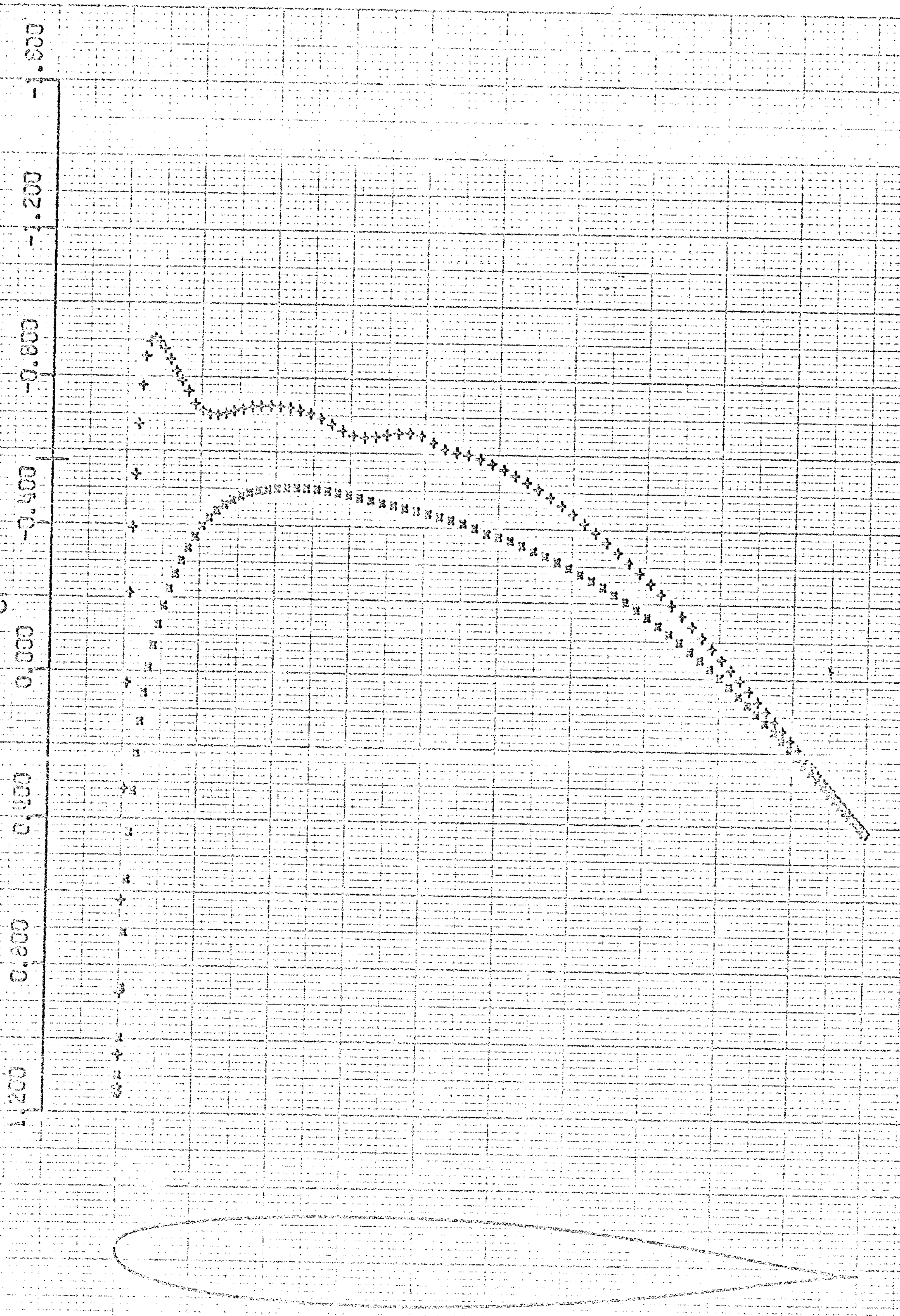
CL = 0.1343   CD = -.0000   CM = 0.0820



NLR LIFTING QUASI-ELLIPTICAL AIRFOIL
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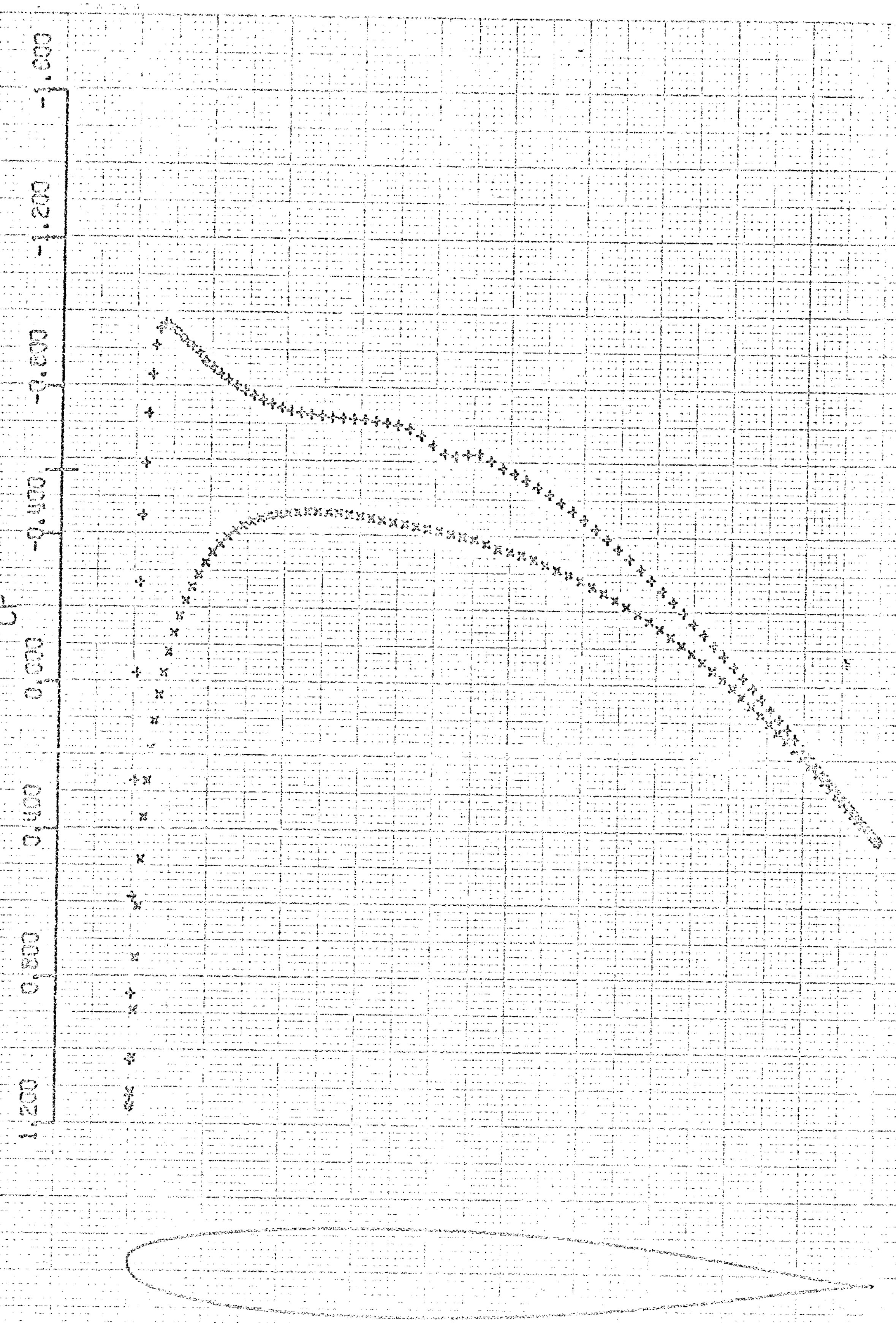
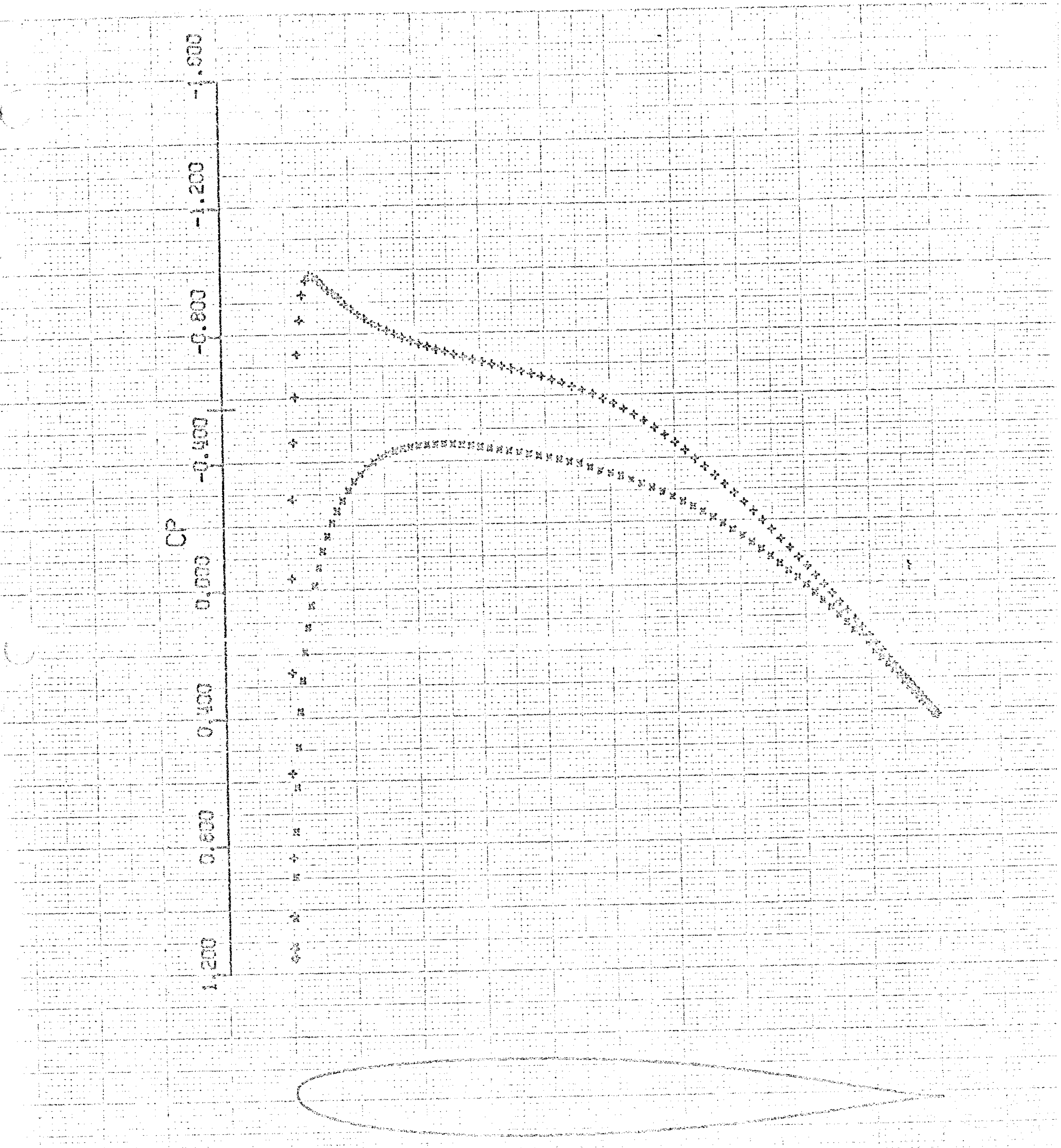
CL = 0.1927   CD = 0.0300   CM = 0.0[illegible]



NLR LIFTING QUASI-ELLIPTICAL AIRFOIL
M = 0.756 NLF = -0.030 CL = 0.240
CL = 0.2401 CD = 0.0000 CM = -0.1153

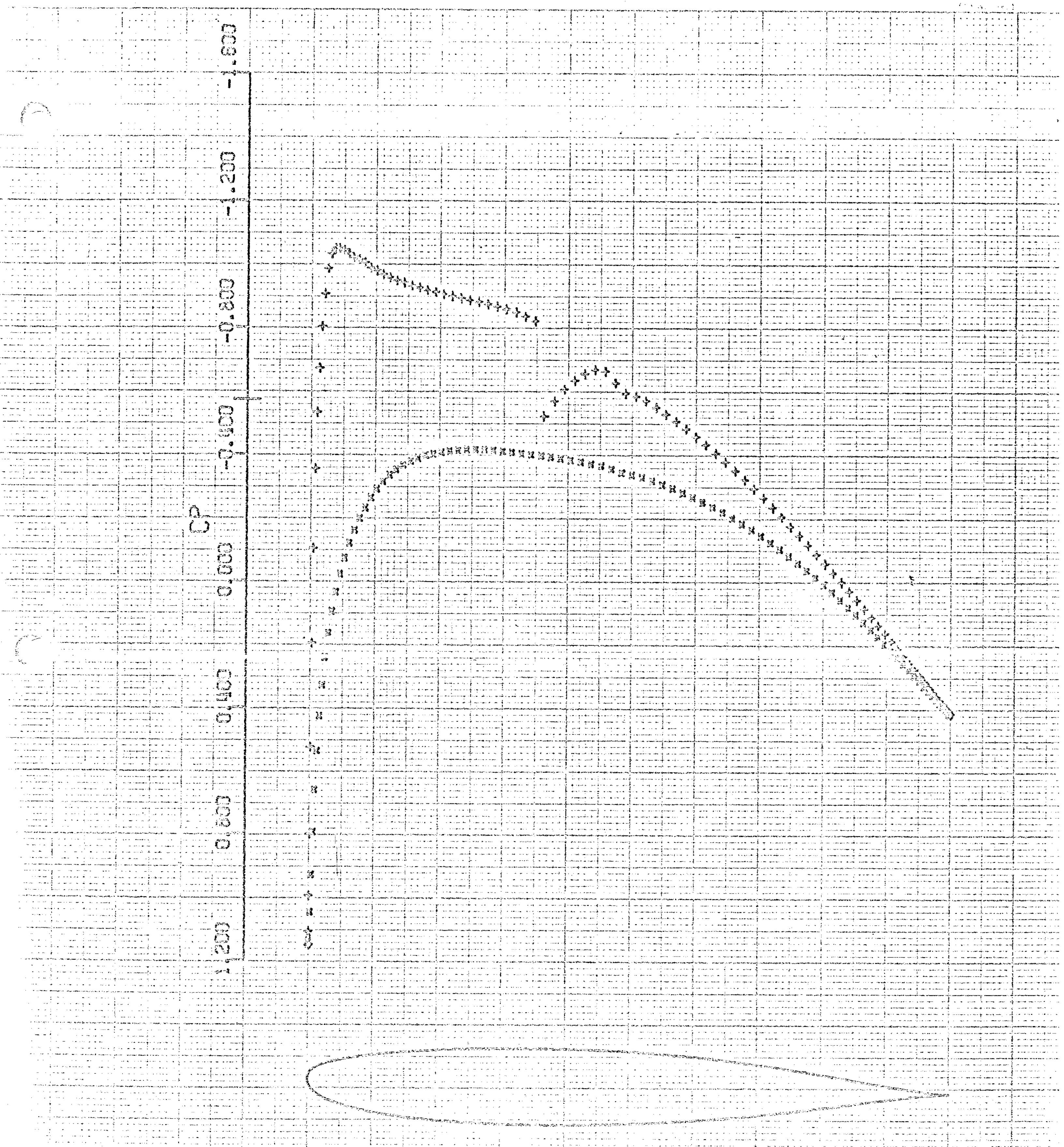


NLR LIFTING QUASI-ELLIPTICAL AIRFOIL

M = 0.755 ALF = 0.0 CL = 0.253

CL = 0.2525 CD = 0.0000 CM = -0.1216

NLR LIFTING QUASI-ELLIPTICAL AIRFOIL
M = 0.755   ALF = 0.060   CL = 0.265
CL = 0.2642   CD = 0.0001   CM = -0.1029



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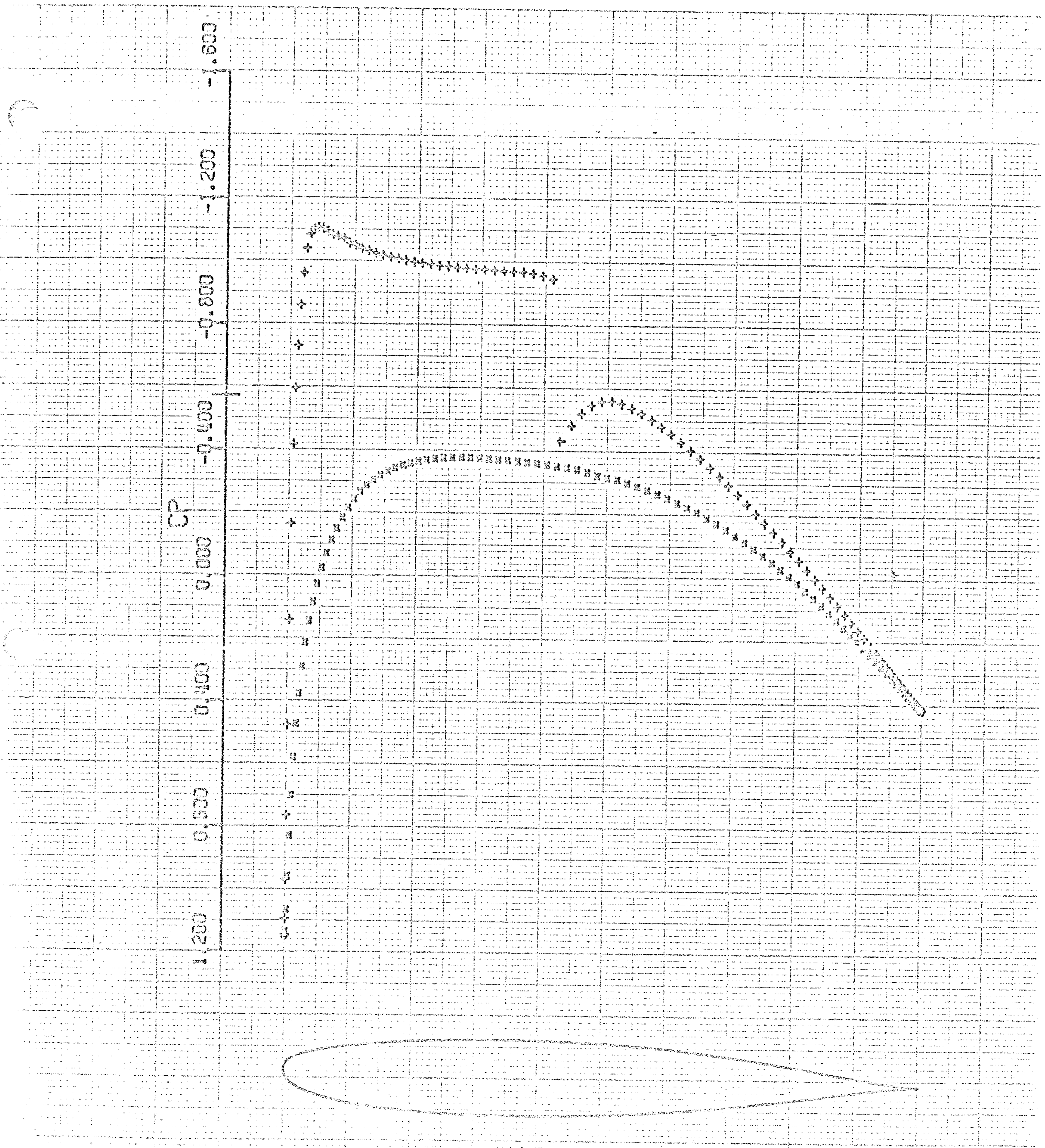
CL = 0.3156   CD = 0.0005   CM = 0.1535



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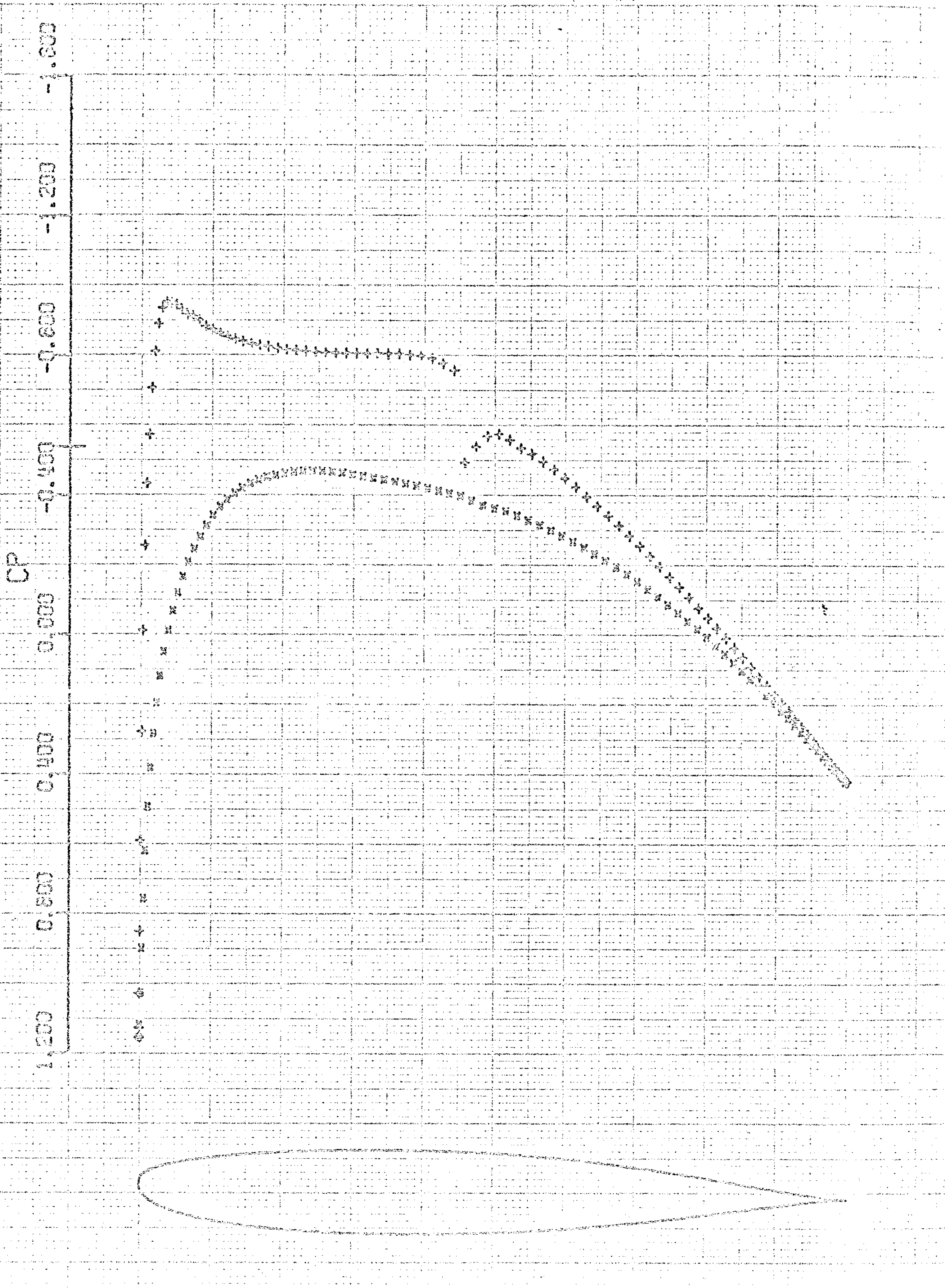
CL = 0.3826 CD = 0.0019 CM = 0.1851



NLR LIFTING QUASI-ELLIPTICAL AIRFOIL
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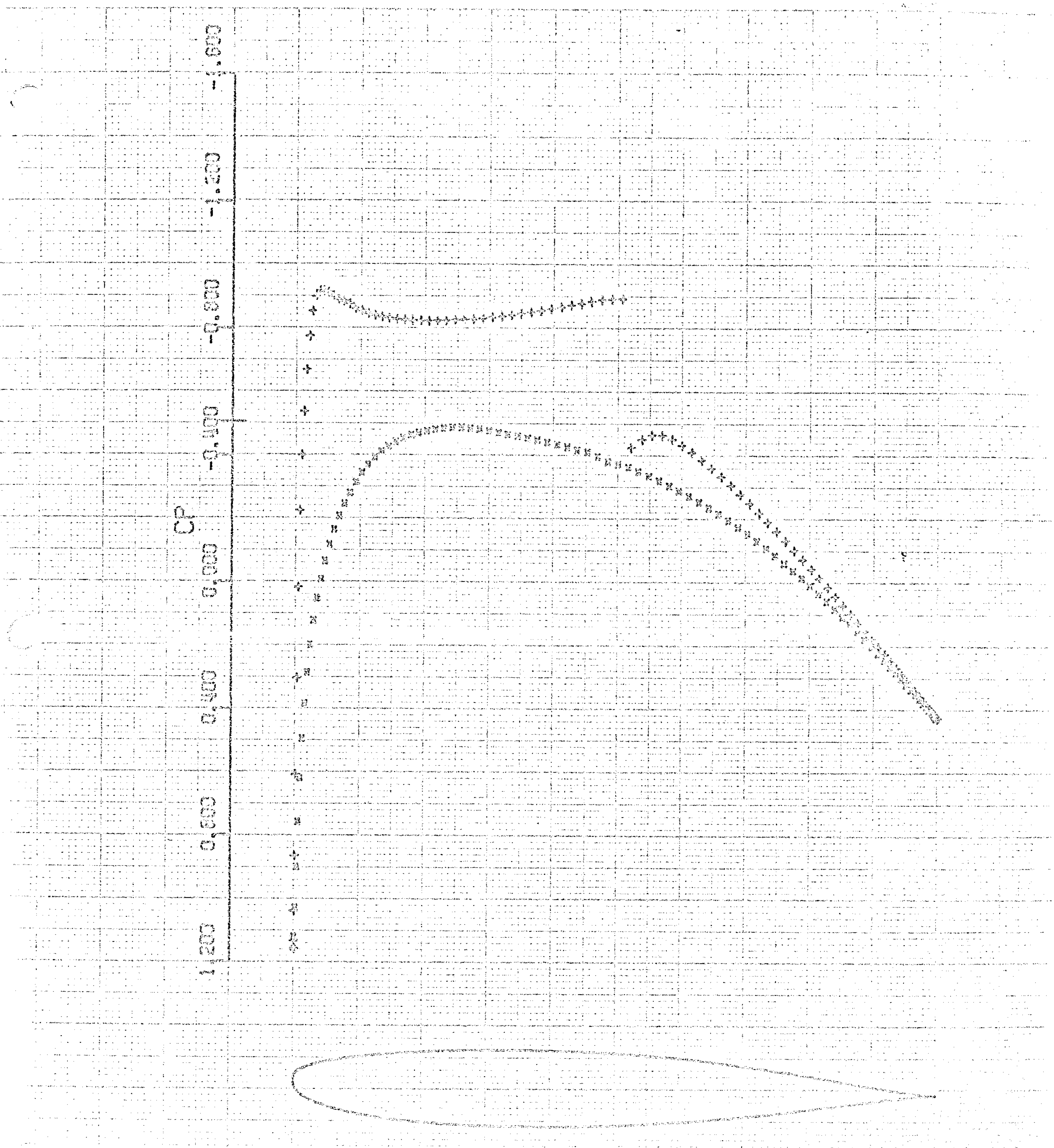
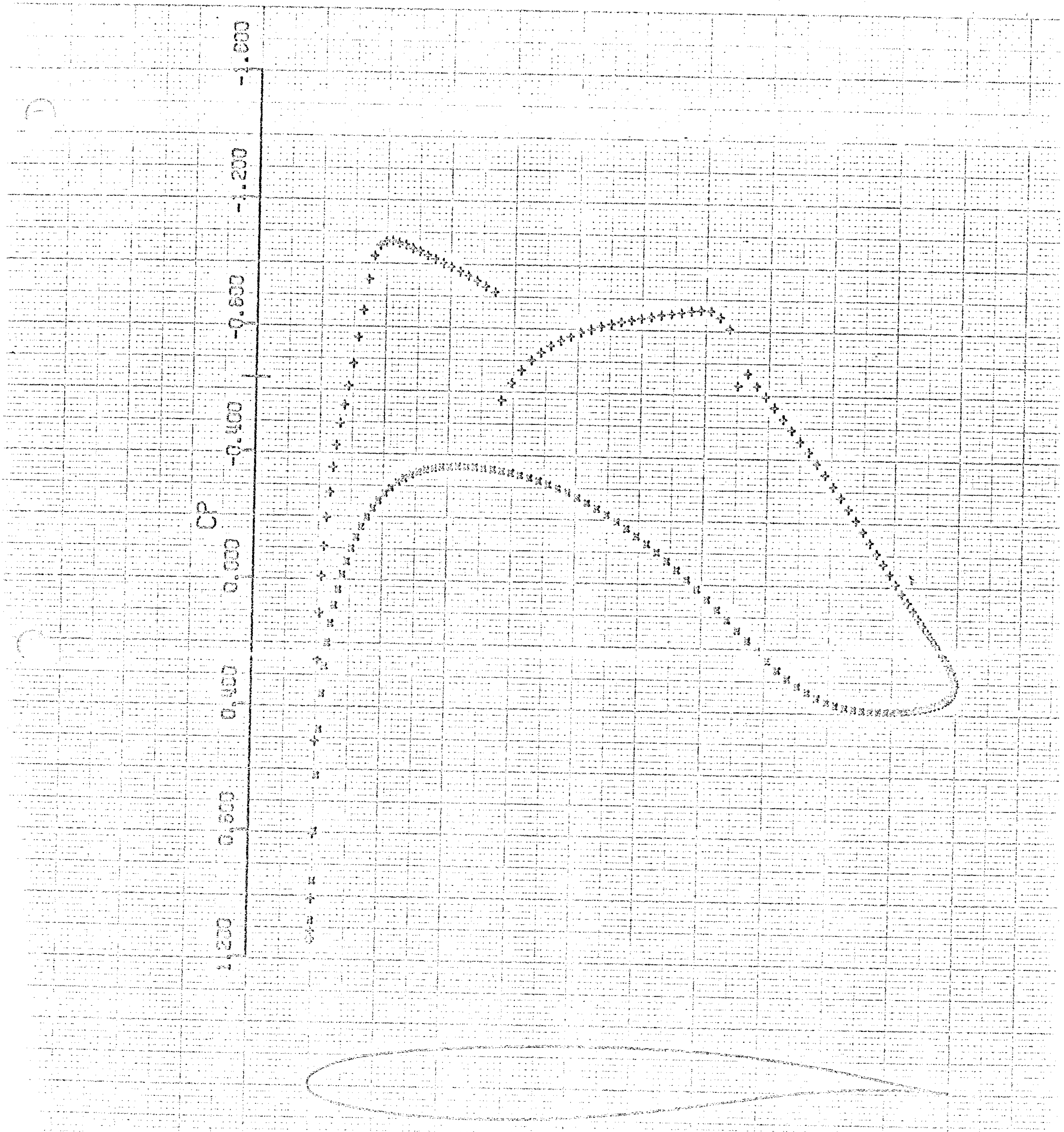
CL = 0.2085 CD = 0.0006 CM = 0.1265



NLR LIFTING QUASI-ELLIPTICAL AIRFOIL
M = 0.776 ALF = 0.0 CL = 0.[illegible]
CL = 0.2[illegible]4 CD = 0.0019 CM = -0.1[illegible]

KORN AIRFOIL

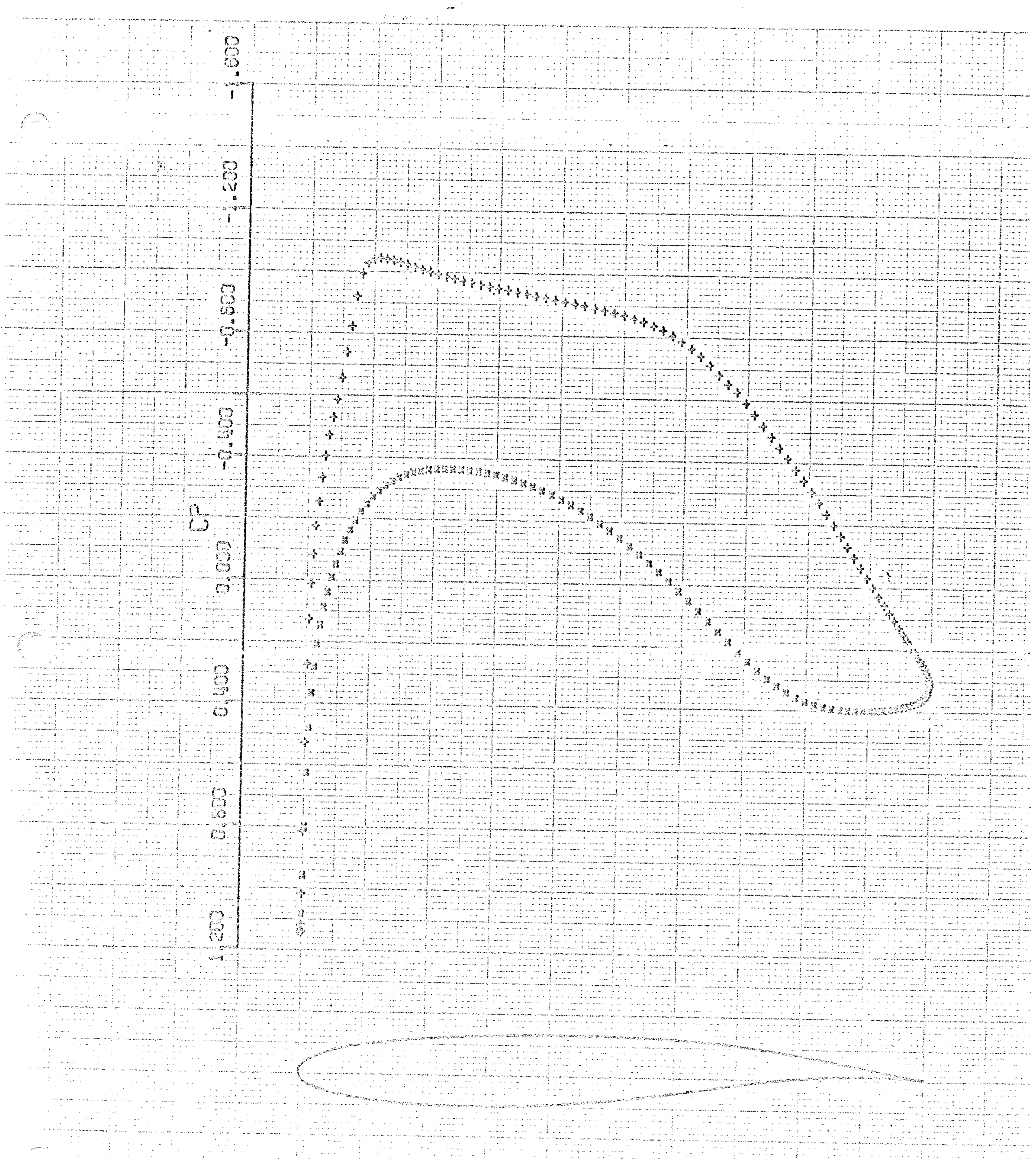
Designed to be shock free at $M = .75$, $\alpha = .118°$. Coordinates and slopes generated by Grumman's version of the Korn synthesis program.



KORN .75 AIRFOIL (S37303)

M = 0.738 ALF = 0.198 CL = 0.605

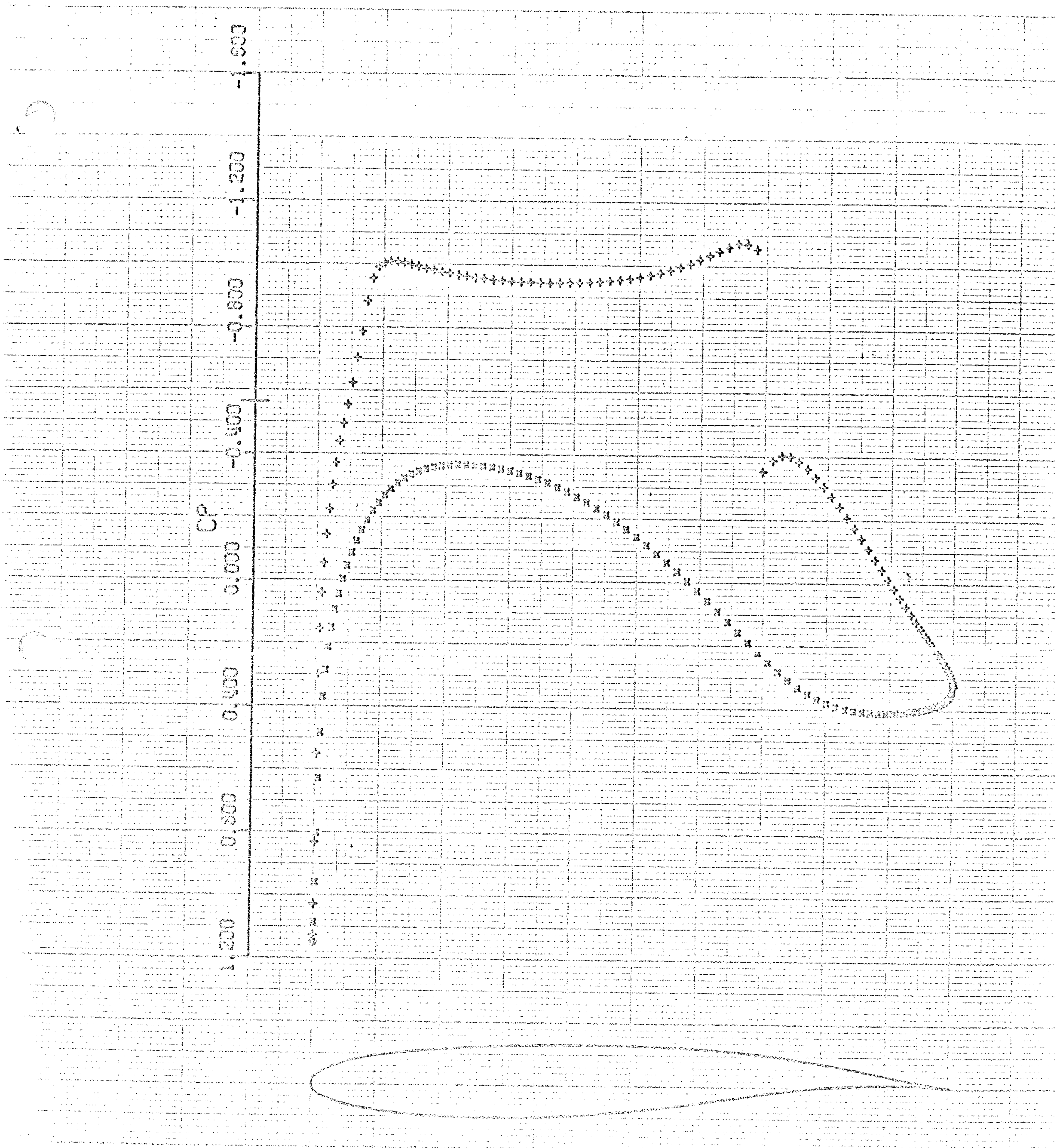
CD = 0.0000 CD = 0.0012 CM = -0.1115



KORN .75 AIRFOIL (537308)

M = 0.748 ALF = 0.198 CL = 0.636

CL = 0.6375 CD = 0.0083 CM = -0.1463



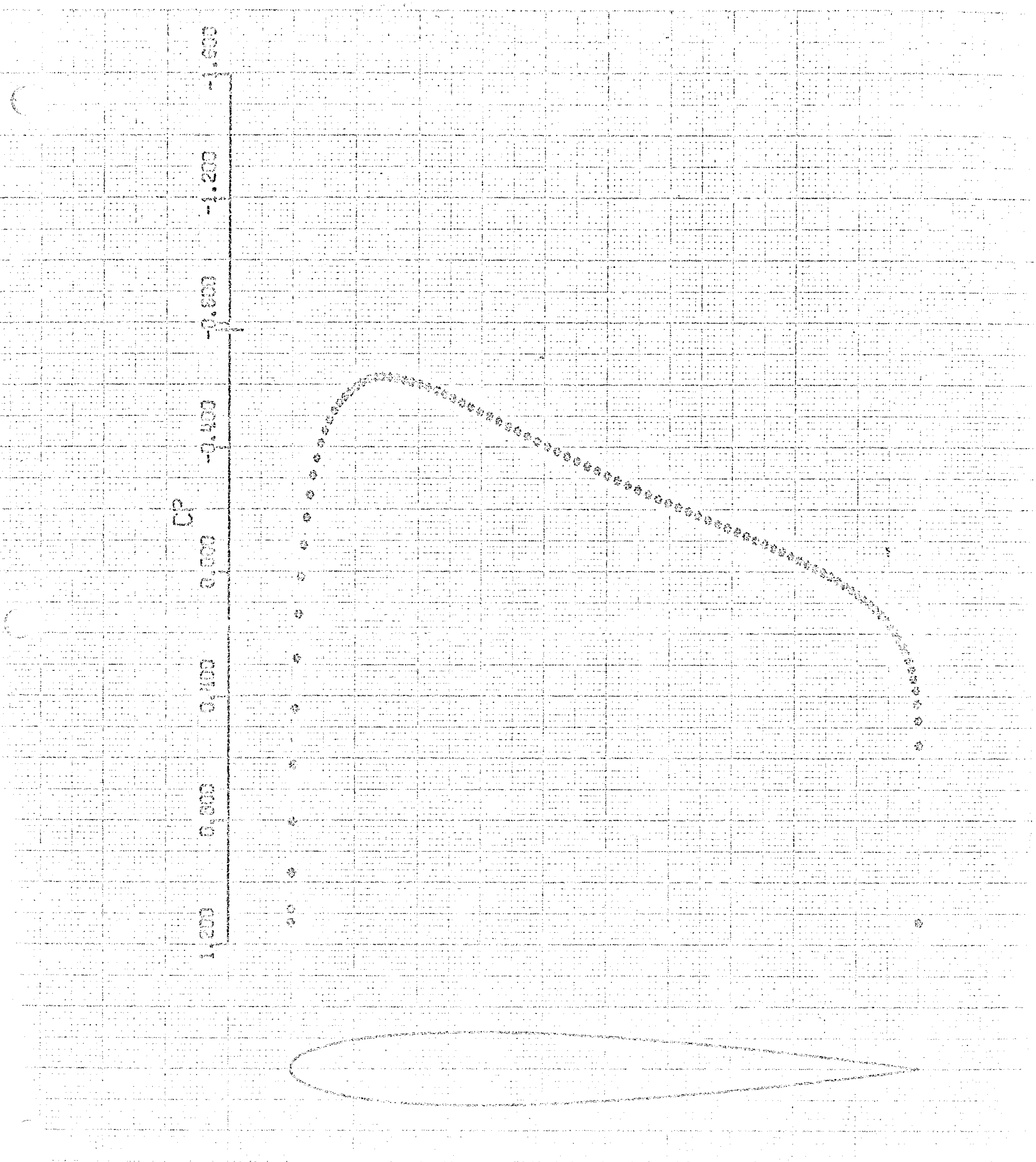
KORN .75 AIRFOIL (537308)

M = 0.758 ALF = 0.198 CL = 0.672

CL = 0.6751 CD = 0.0066 CM = -0.1611

NACA 0012

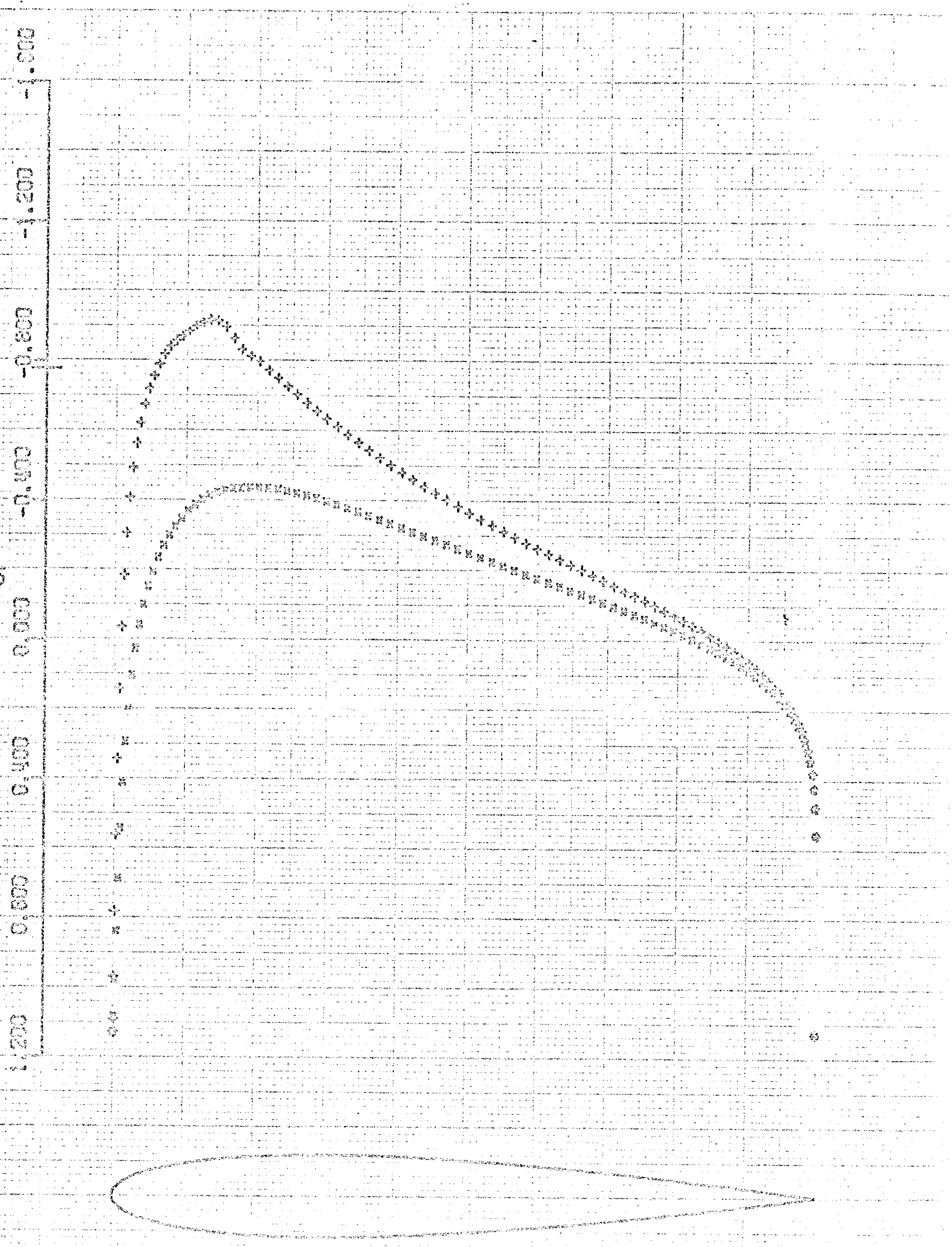
Coordinates from NAE Report LTR-HA-2, Fortran IV Program for the Catherall-Foster-Sells Method for Calculation of the Plane Inviscid Compressible Flow Past a Lifting Airfoil, by J. J. Kacprzynski, 1970.



NACA 0012

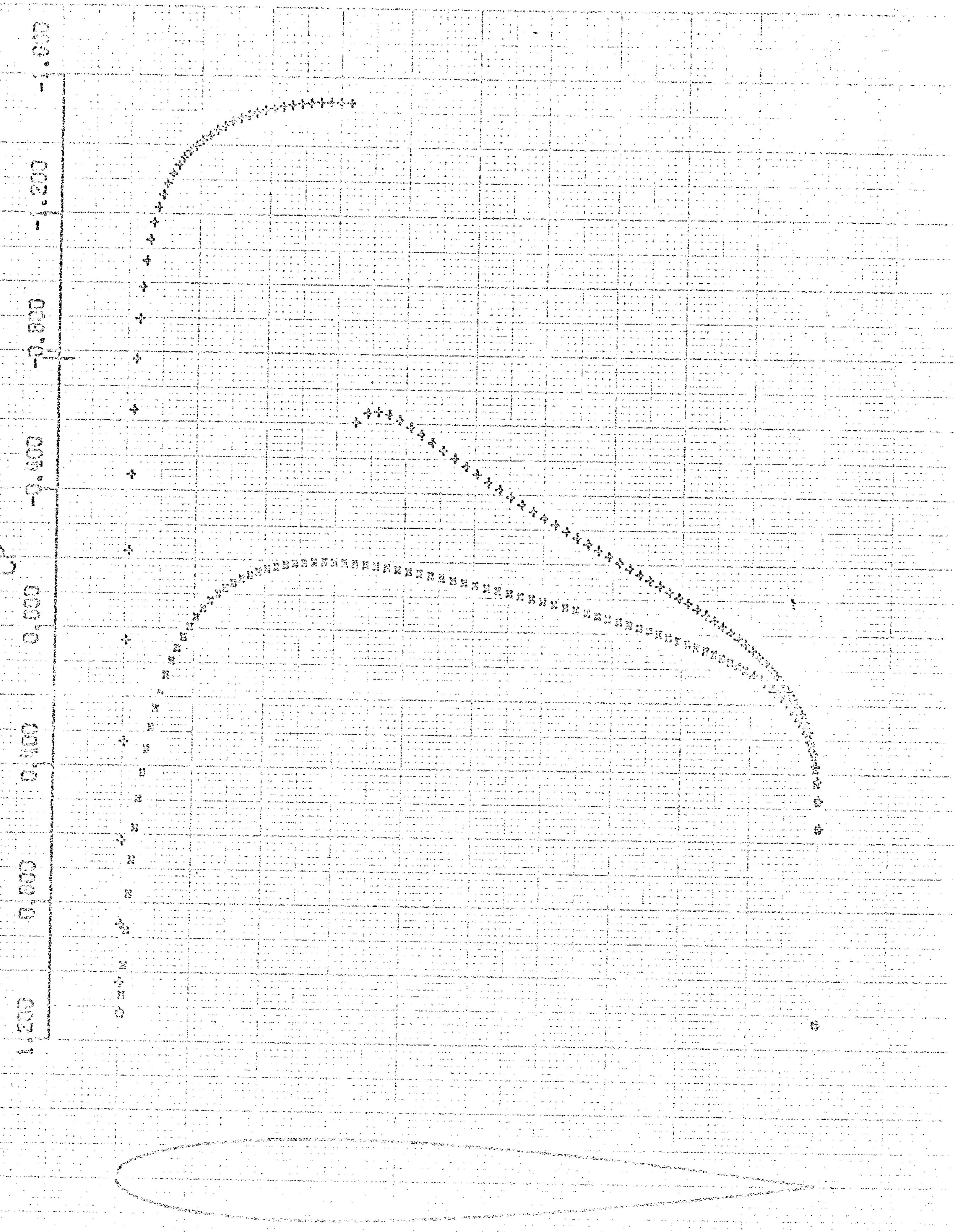
M = 0.700 ALF = 0.0 CL = 0.0

CL = -.0000 CD = 0.0000 CM = 0.0000

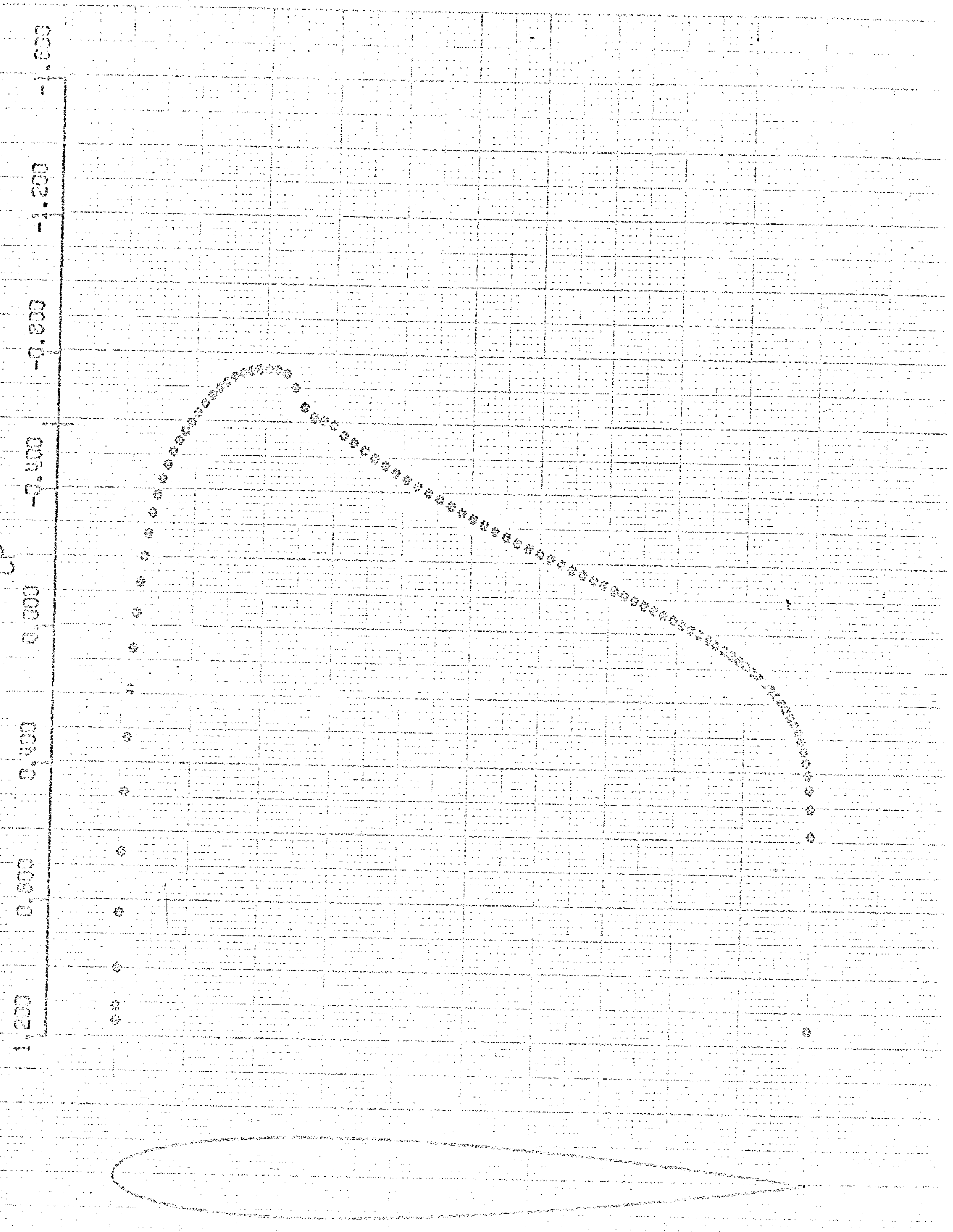


NACA 0012
M = 0.700 ALF = 1.000 CL = 0.191
CL = 0.1903 CD = -.0000 CM = -0.0007

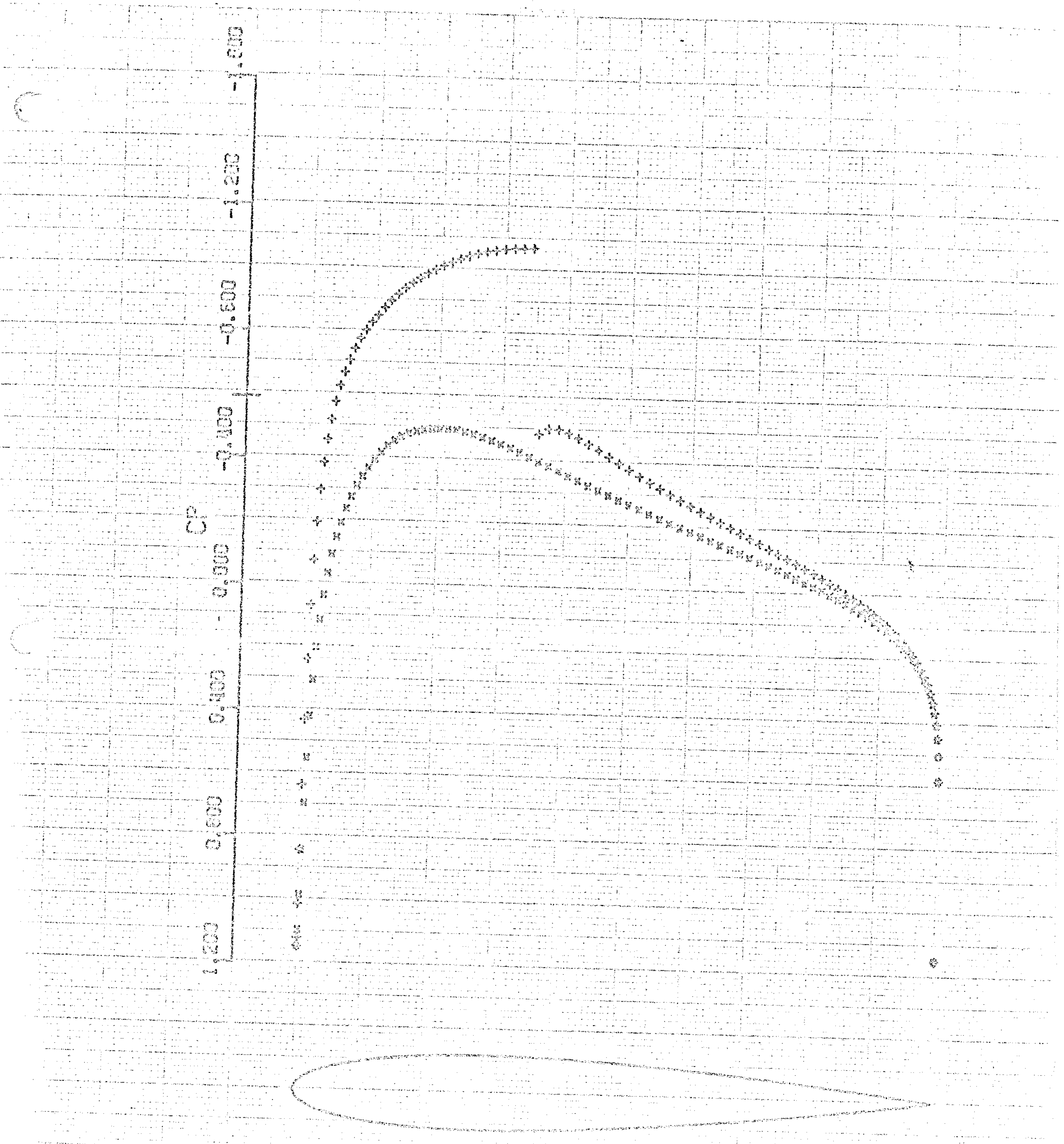
NACA 0012
M = 0.700 ALF = 2.000 CL = 0.368
CL = 0.3[illegible] CD = 0.0016 CM = -0.0[illegible]



NACA 0012
M = 0.700   ALF = [illegible]   CL = [illegible]
CL = 0.5509   CD = 0.0012   CM = -0.0097

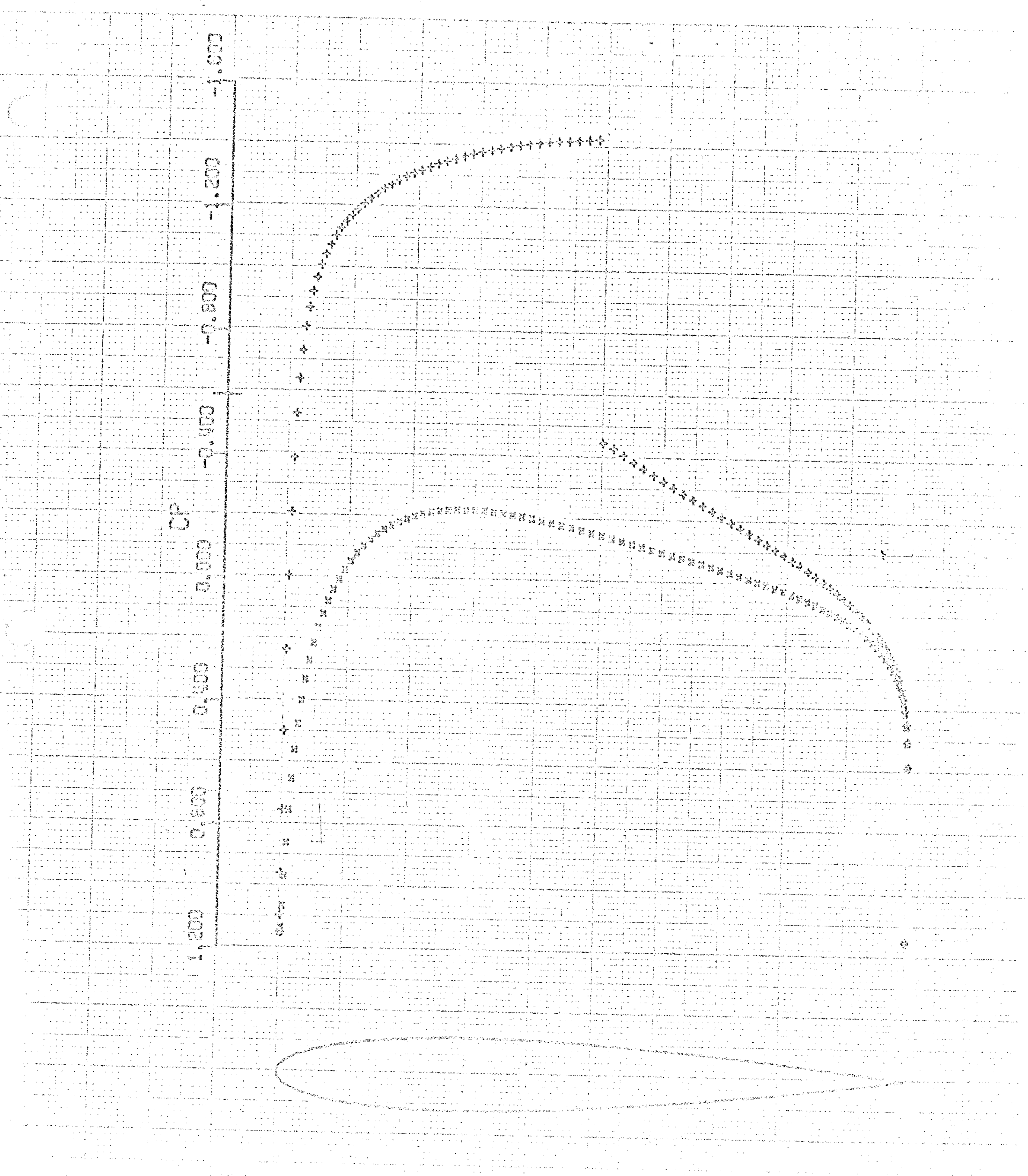


NACA 0012
M = 0.750   ALF = 0.0   CL = 0.0
CL = -.0000   CD = 0.0000   CM = 0.0000

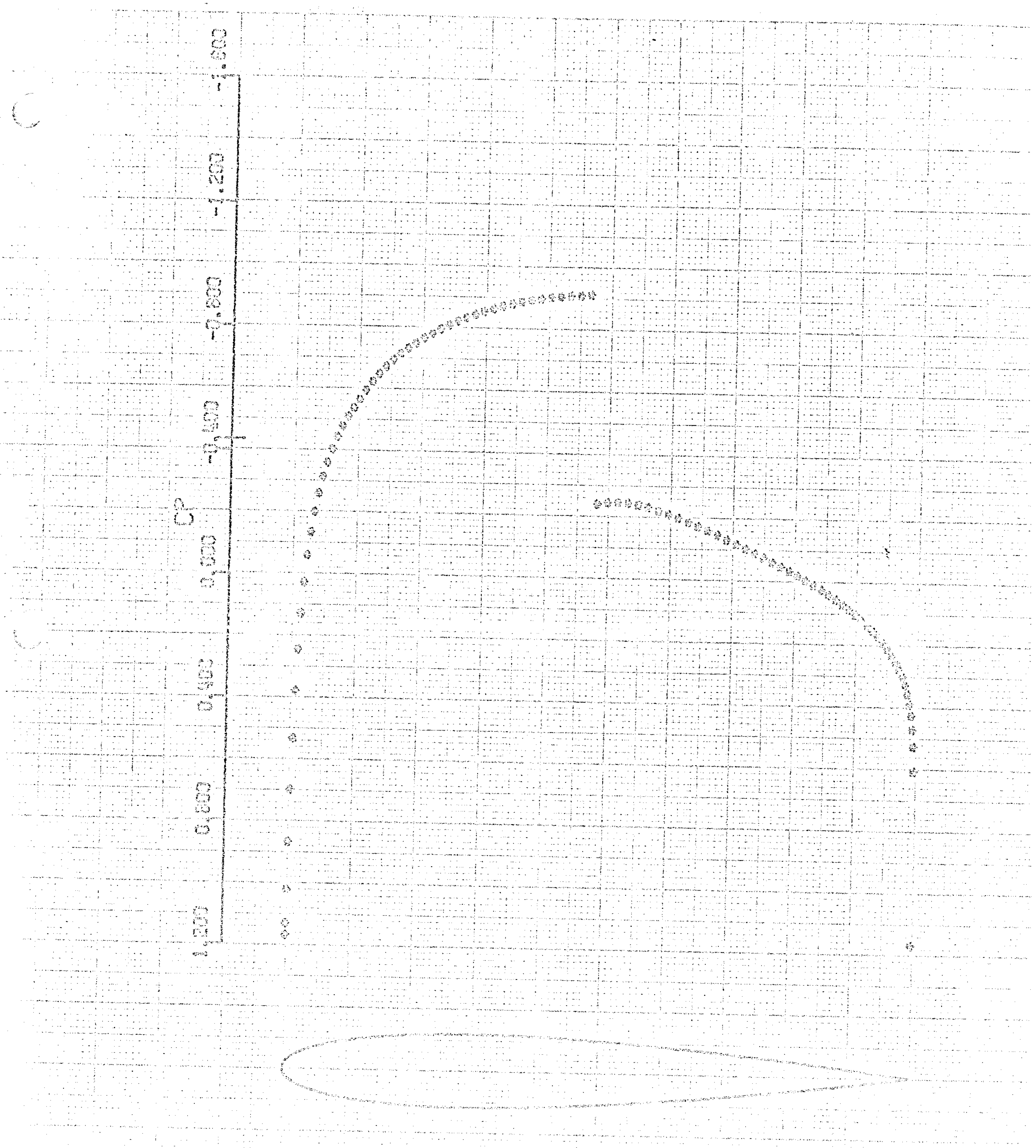


NACA 0012
M = 0.750   ALF = 1.000   CL = 0.223
CL = 0.2272   CD = 0.0053   CM = -0.0061

NACA 0012
M = 0.750 ALF = 2.000 C[illegible] = 0.[illegible]
CL = 0.[illegible] CD = 0.0[illegible] CM = -0.[illegible]



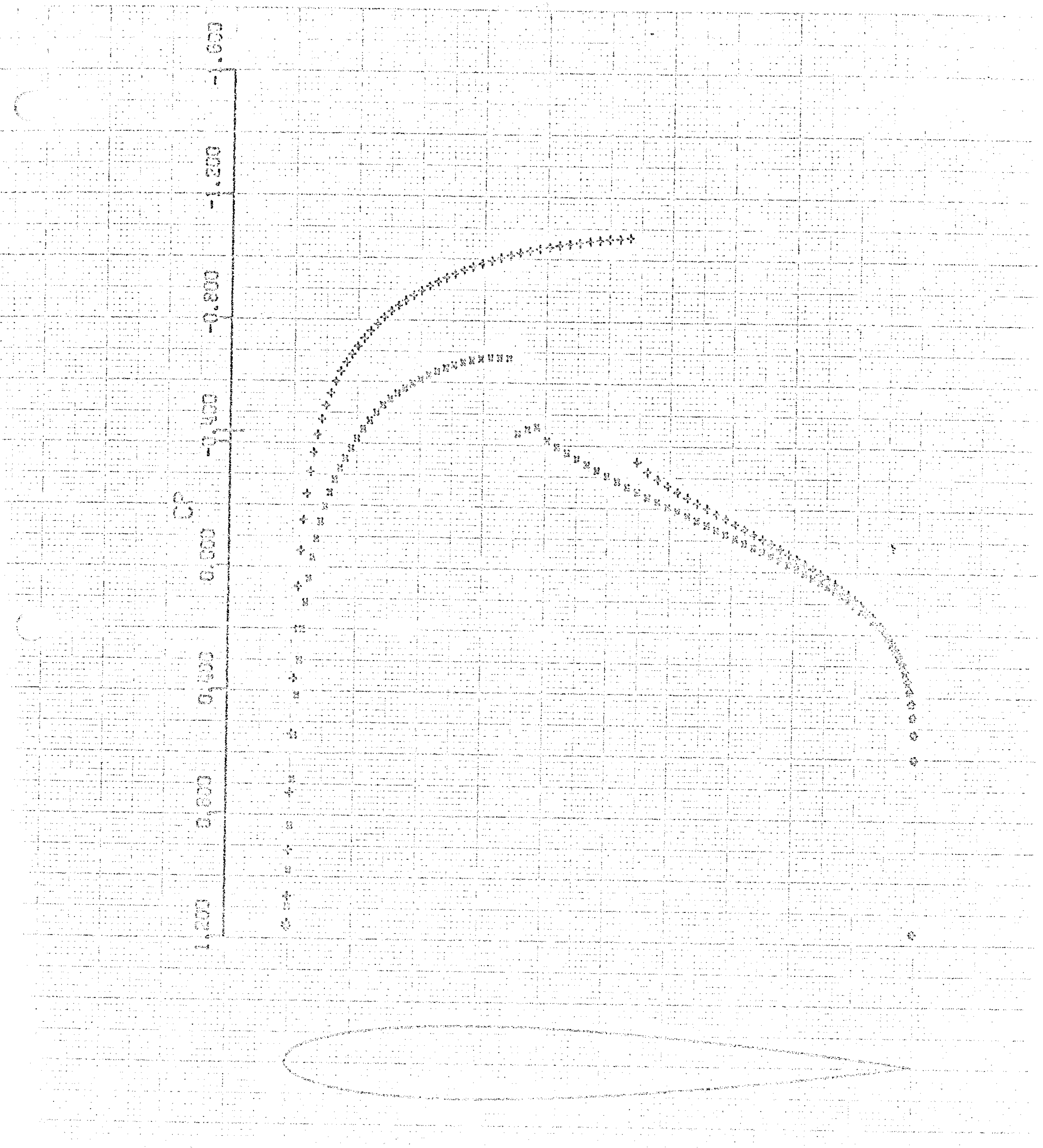
NACA 0012
M = 0.750 NLF = 3.000 CL = 0.6[illegible]
CL = 0.6[illegible]37 CD = 0.0[illegible]21 CM = -[illegible]



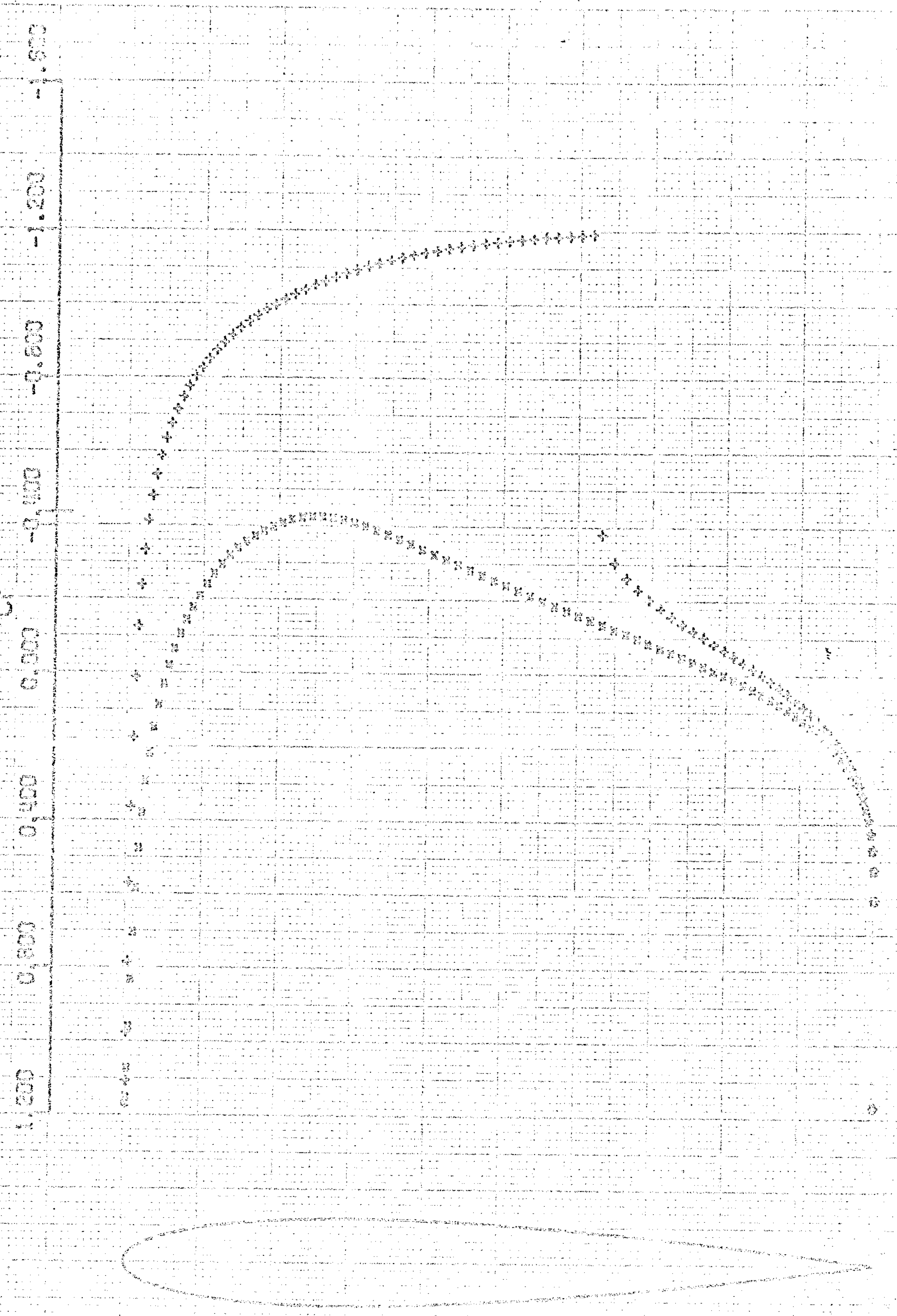
NACA 0012

M = 0.800   ALF = 0.0   CL = 0.0

CL = -.0000   CD = 0.0082   CM = 0.0000



NACA 0012
M = 0.800 AL = 1.000 CL = 0.253
CL = 0.2551 CD = 0.0001 CM = -0.0032



NACA 0012

M = 0.800 ALF = 2.000 CL = 0.5[illegible]

CL = 0.5014 CD = 0.0[illegible] CM = -0.0[illegible]

NACA 0012

H = 0.400   ALF = 3.000   CL = 0.775

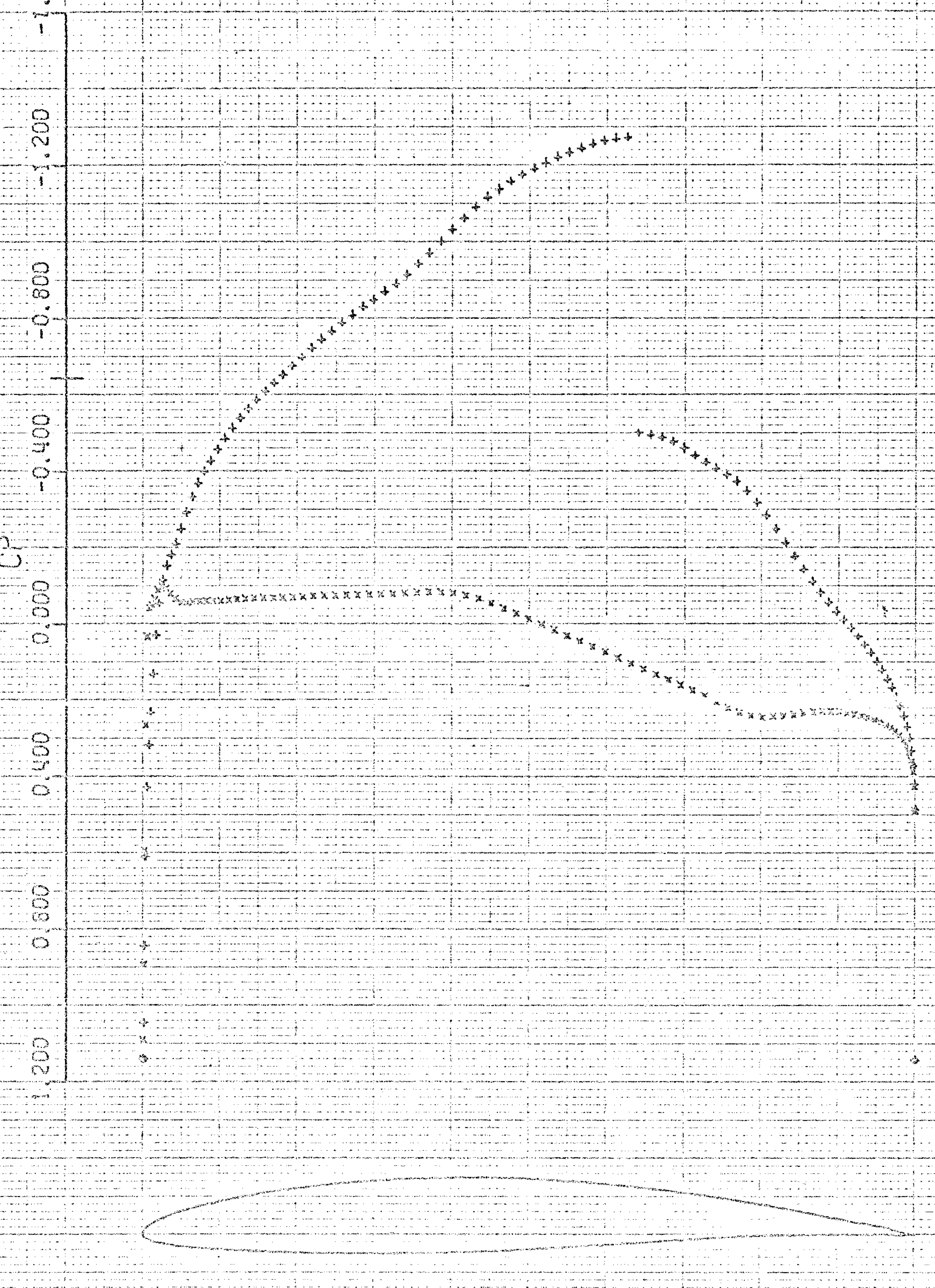
CL = 0.7570   CD = 0.0558   CM = -0.1069

NACA 64A410

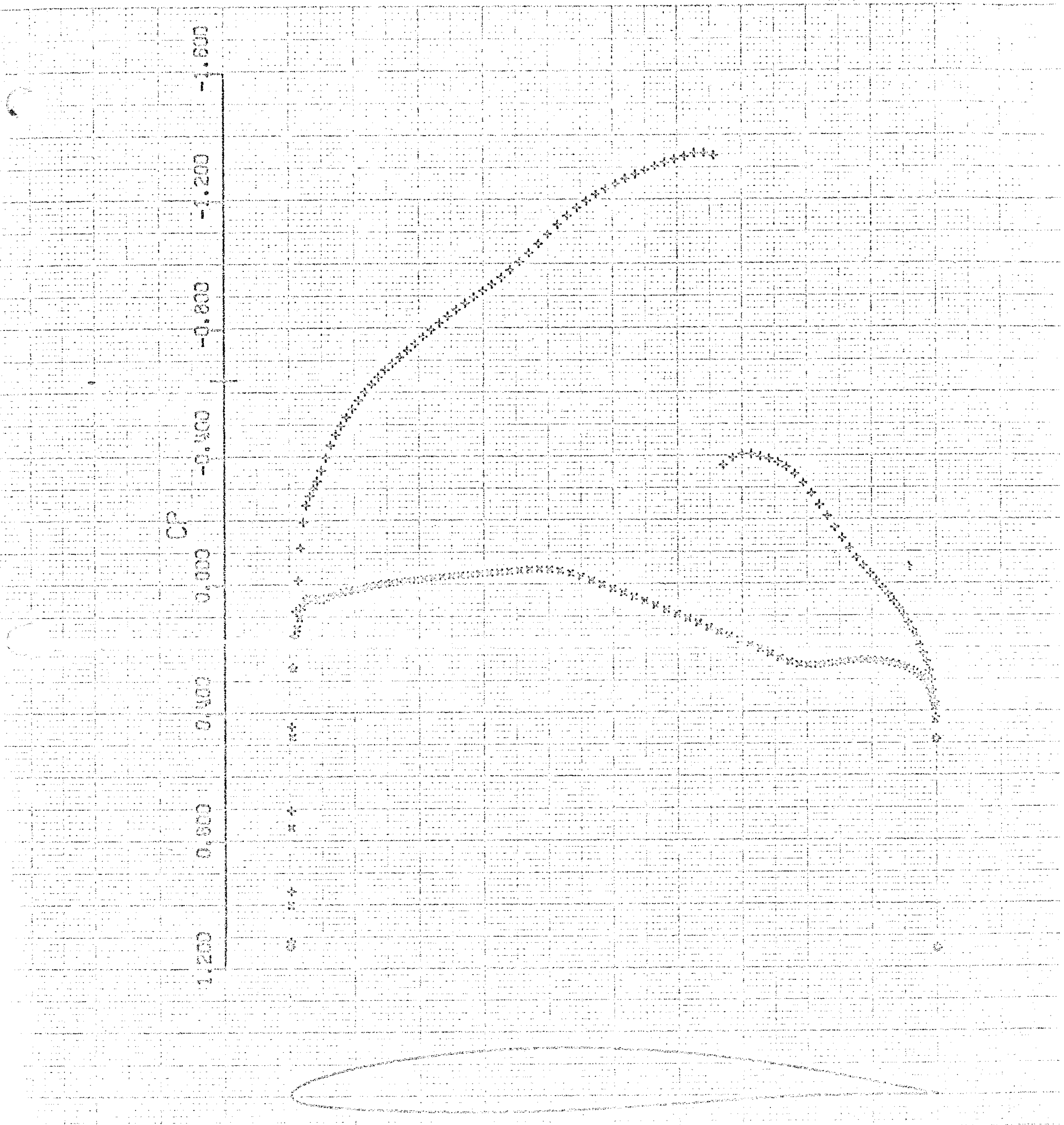
Coordinates from NACA TN 3162,
Effects of Subsonic Mach Number on the
Forces and Pressure Distributions on
Four NACA 64A - Series Airfoil Sections
at Angles of Attack as High as 28°, by
Louis S. Stivers, 1954.

NACA 64A410
M = 0.735 ALF = -0.500 CL = 0.551
CL = 0.5541 CD = 0.0056 CM = -0.1504

24 4 3 μ-1-0.0



NACA 64A410
M = 0.735 ALF = 0.0 CL = 0.653
CL = 0.6555 CD = 0.0089 CM = -0.1583



NACA 64A110

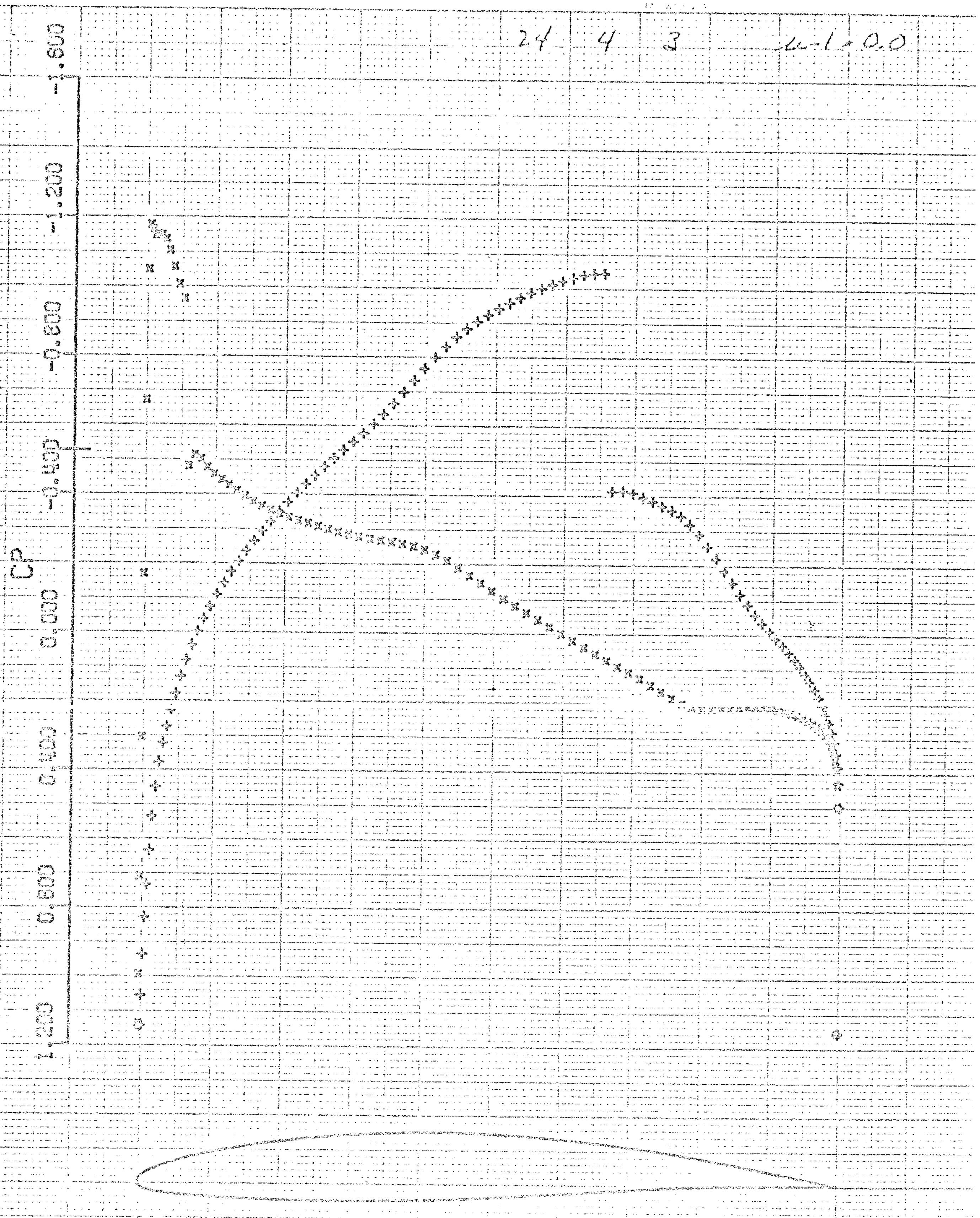
M = 0.735 ALF 0.500 CL = 0.738

CL = 0.7214 CD = 0.0133 CM = -0.1752

NACA [illegible]

M = 0.735 ALF = 1.000 CL = [illegible]

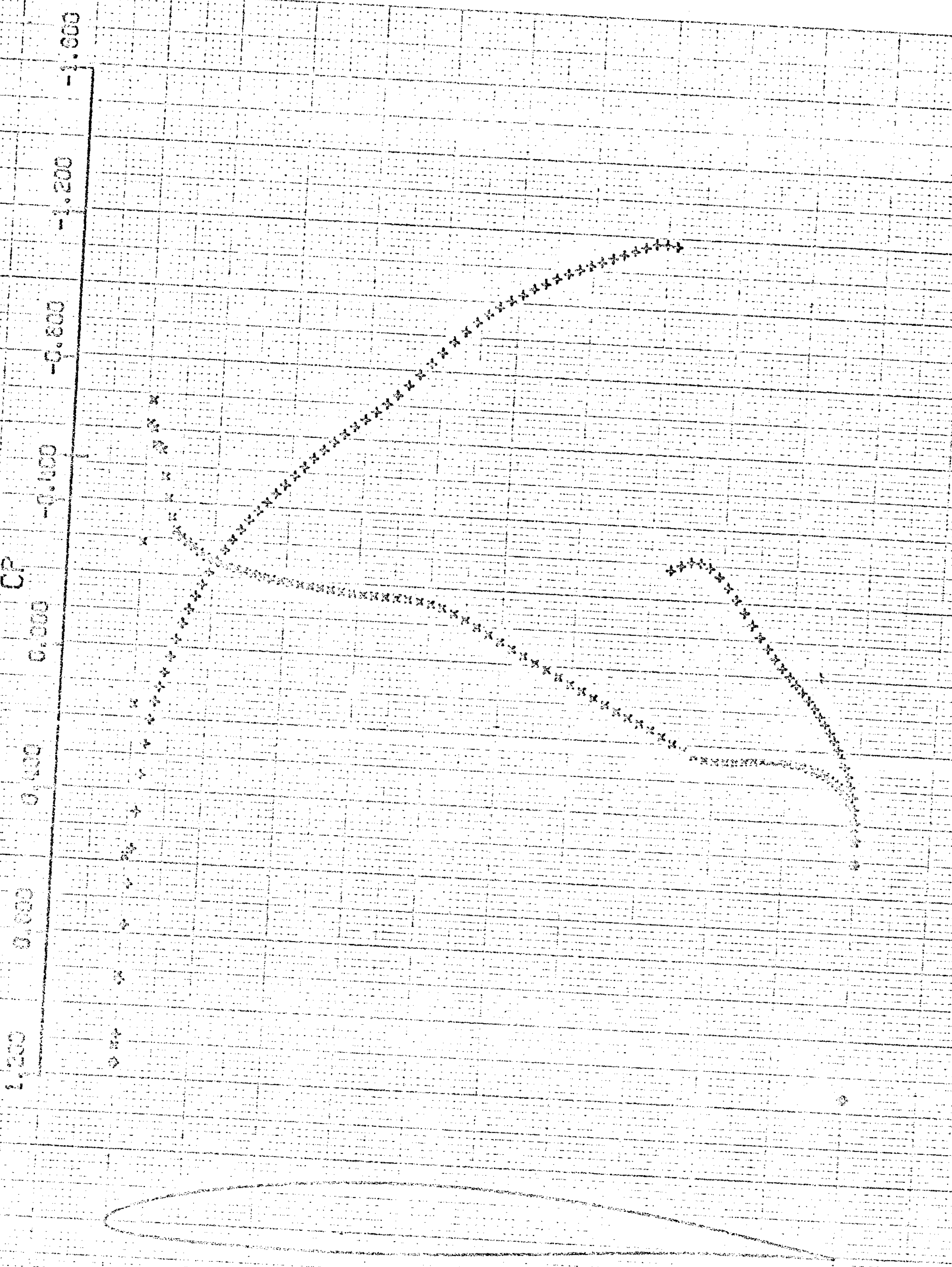
[illegible] = [illegible] CD = [illegible] CM = [illegible]



NACA 64A410
M = 0.770 ALF = -2.000 CL = 0.279
CL = 0.2790 CD = 0.0061 CM = -0.1585

24 4 3 -1 -0.0

NACA 64A410
M = 0.770 ALF = -1.500 CL = 0.385
CL = 0.3902 CD = 0.0069 CM = -0.1663



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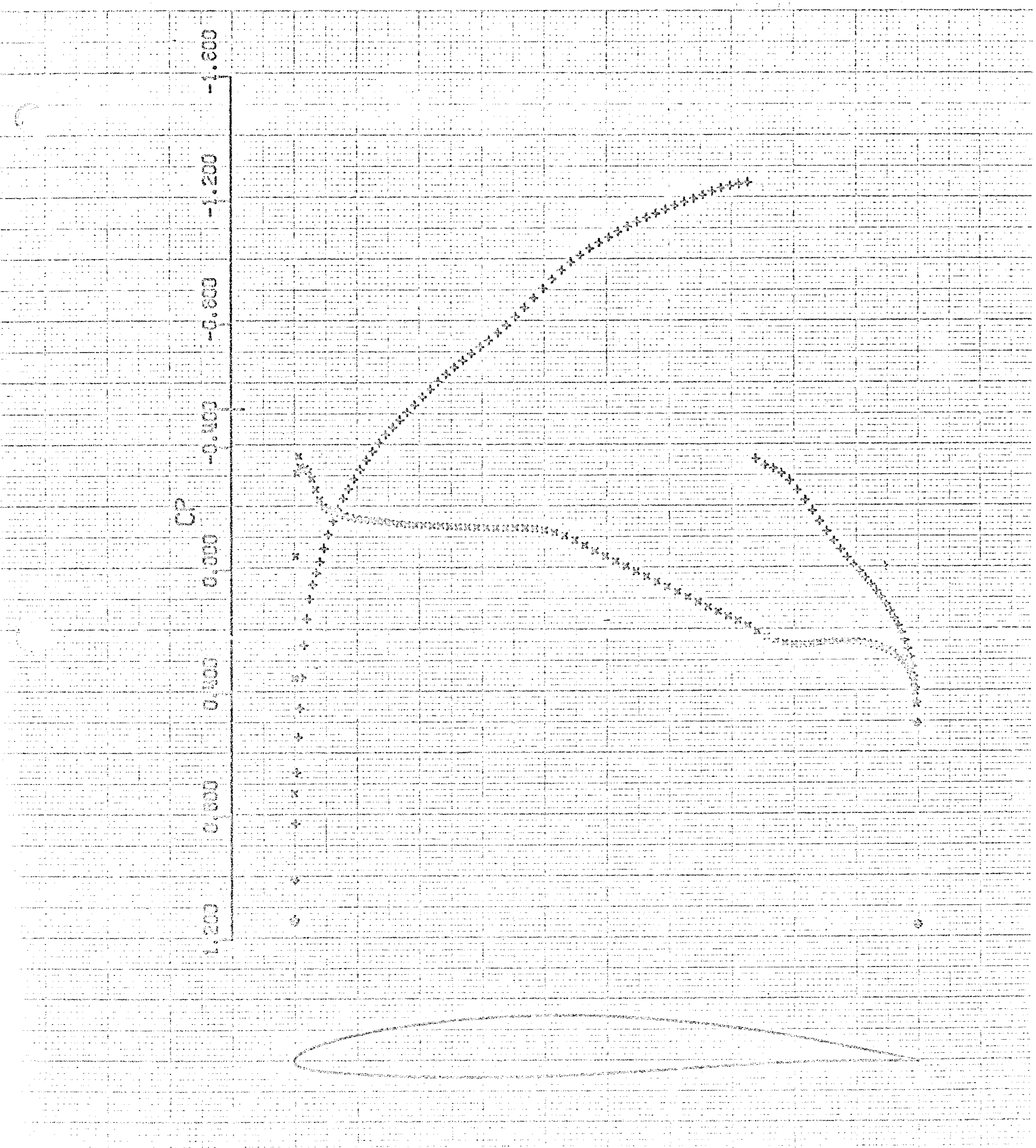
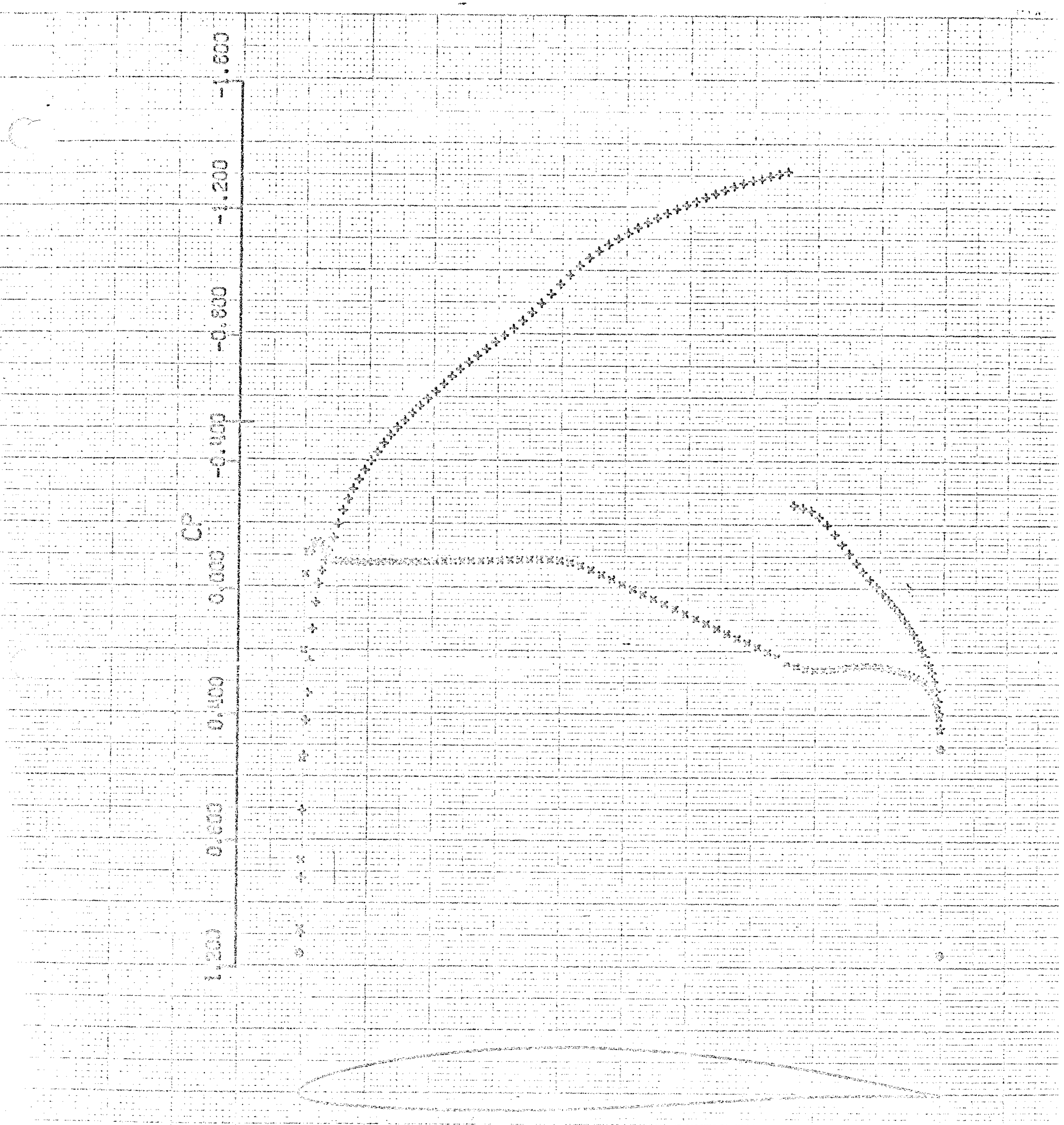
M = 0.770 ALF = -1.000 CL = 0.408
CLI = 0.5000 CD = 0.0131 CM = -0.1723



NACA 64A410

M = 0.770 ALF = -0.500 CL = 0.599

CL = 0.5968 CD = 0.0186 CM = -0.1872



NACA 64A410

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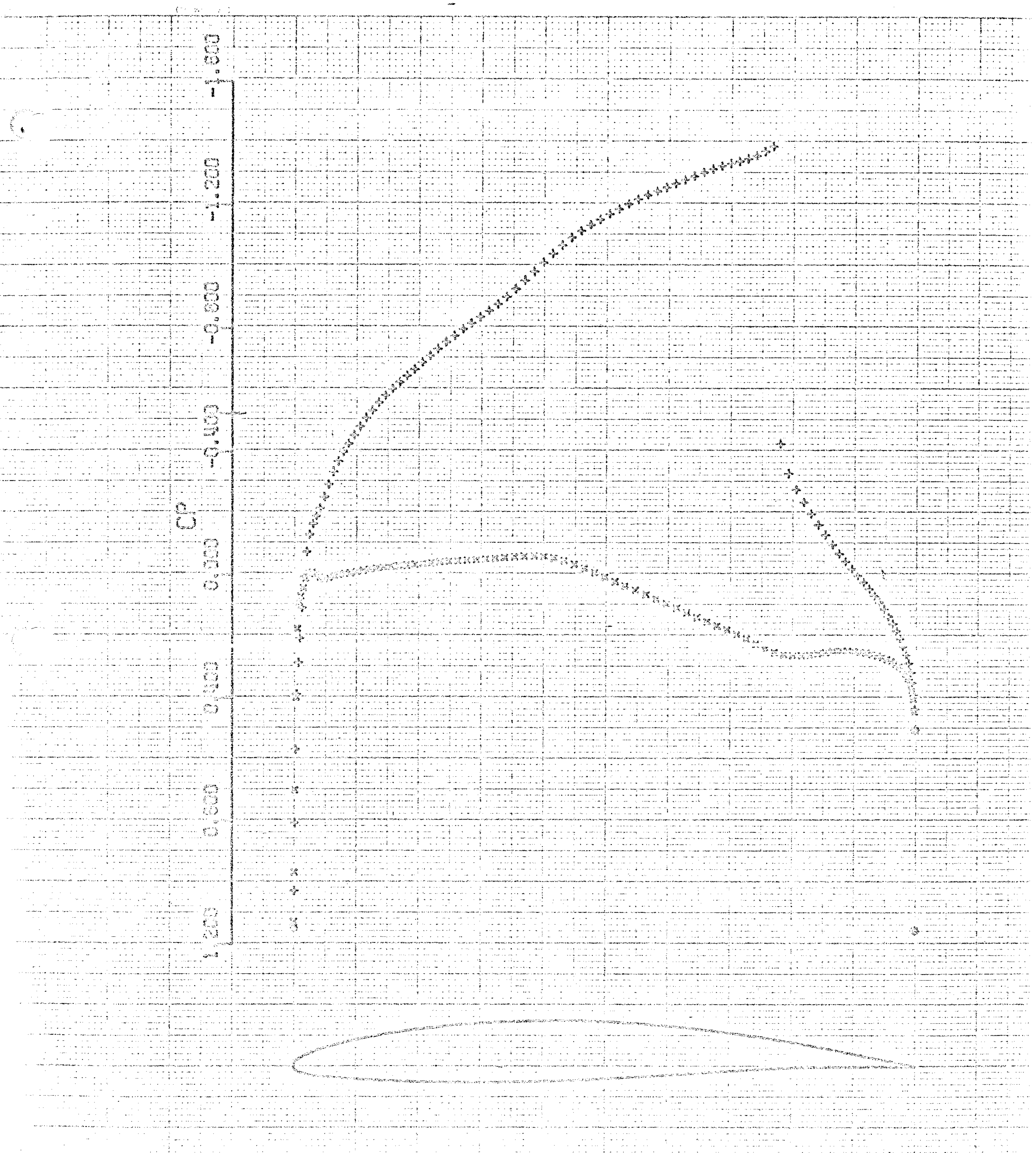
CL = 0.7016 CD = 0.0253 CM = -0.2010



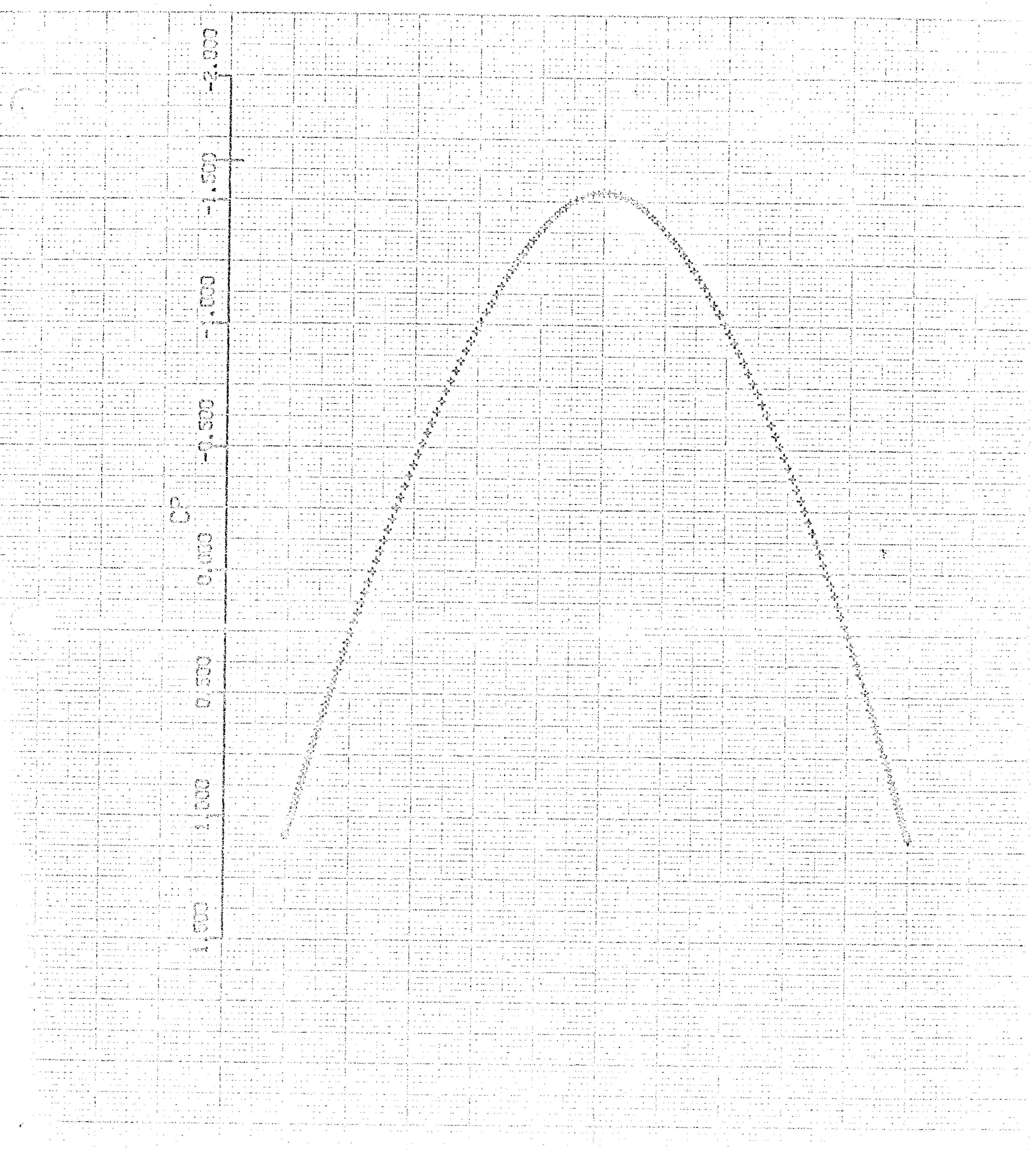
NACA 64A410

M = 0.770 ALF = 0.500 CL = 0.611

CL = 0.6048 CD = 0.0337 CM = -0.2158

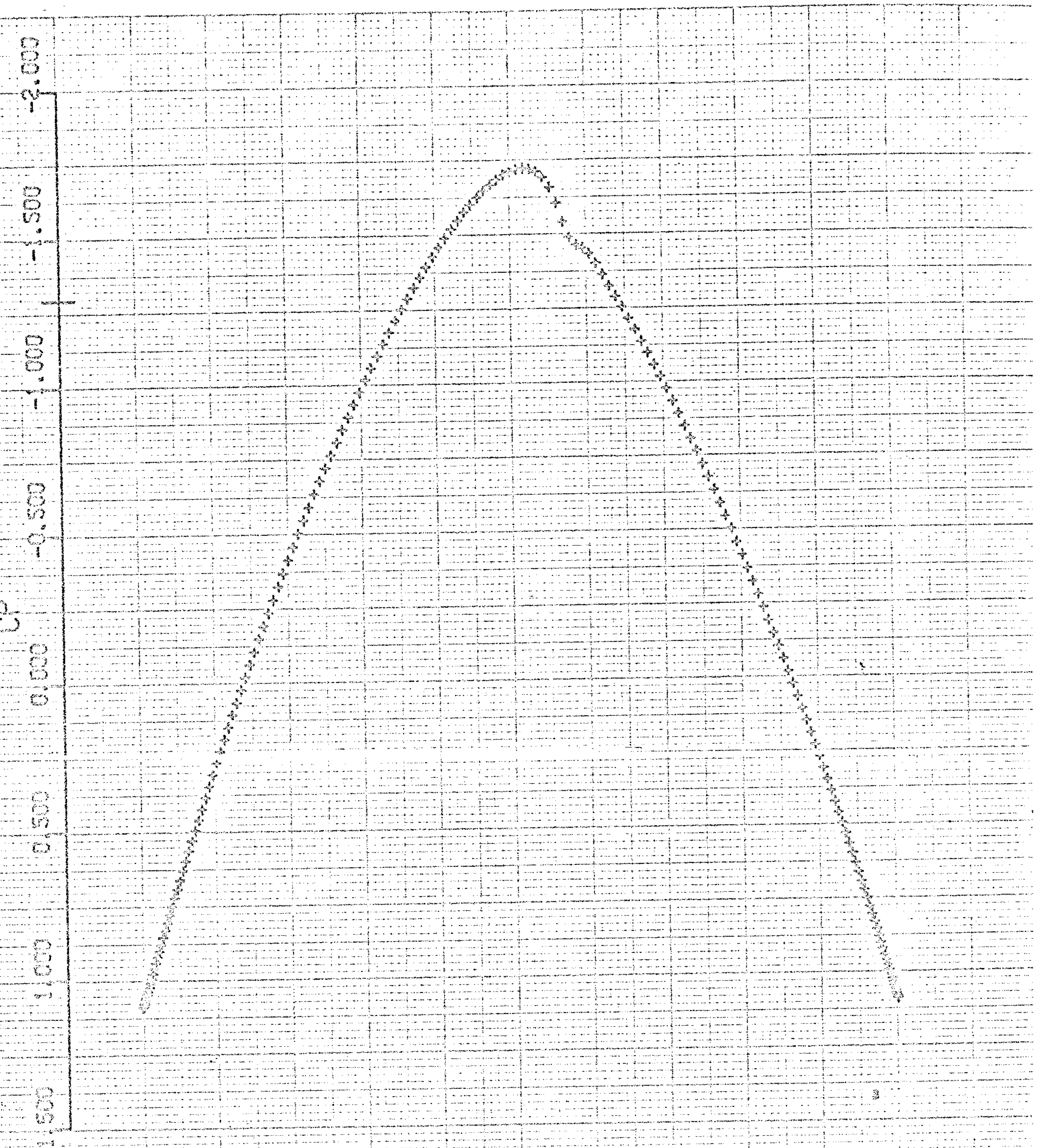
[illegible]
M = 0.770 [illegible] = 1.000 CL = 0.910
CL = 0.[illegible] [illegible]

SPHERE



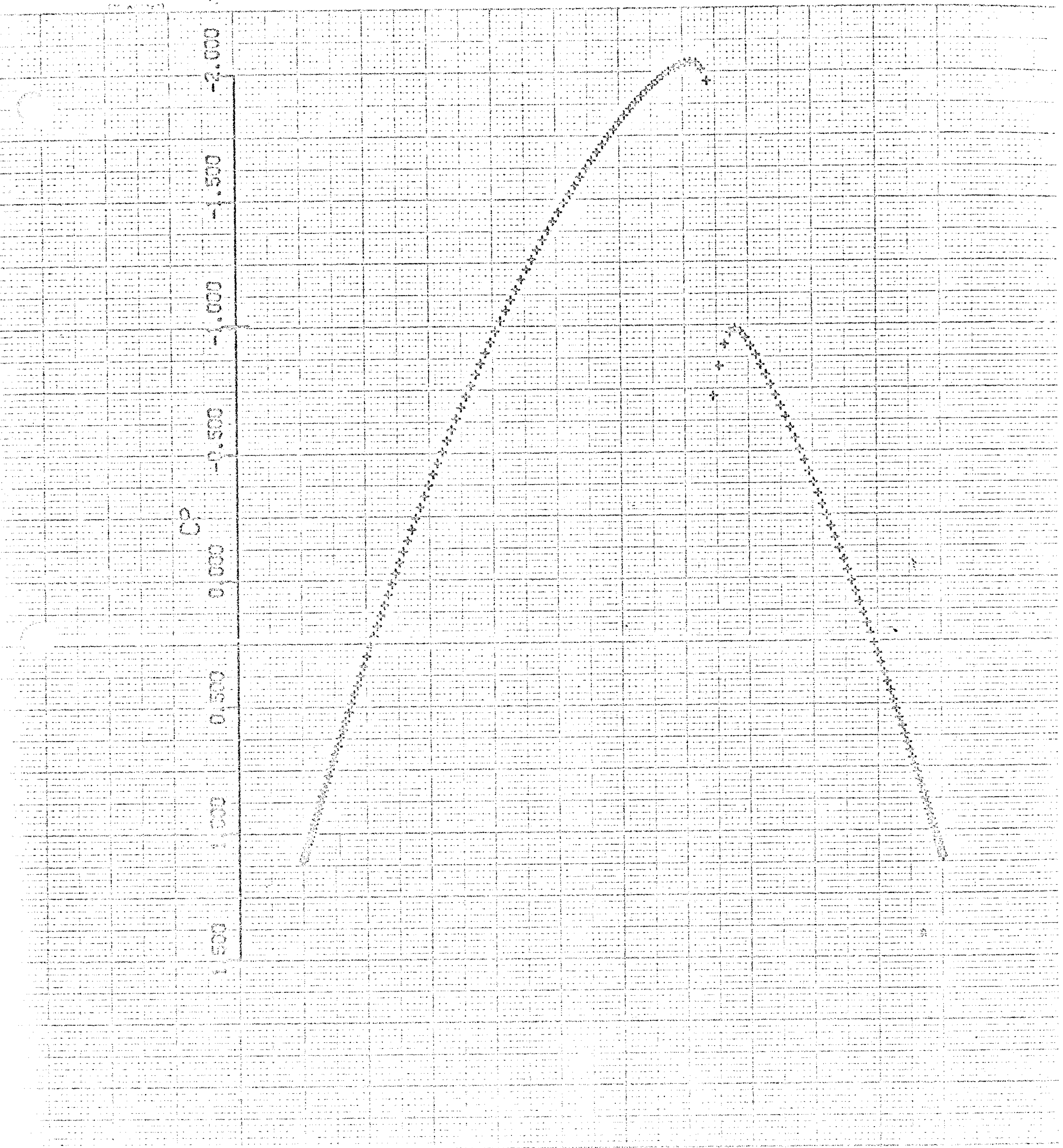
SPHERE

M = 0.850



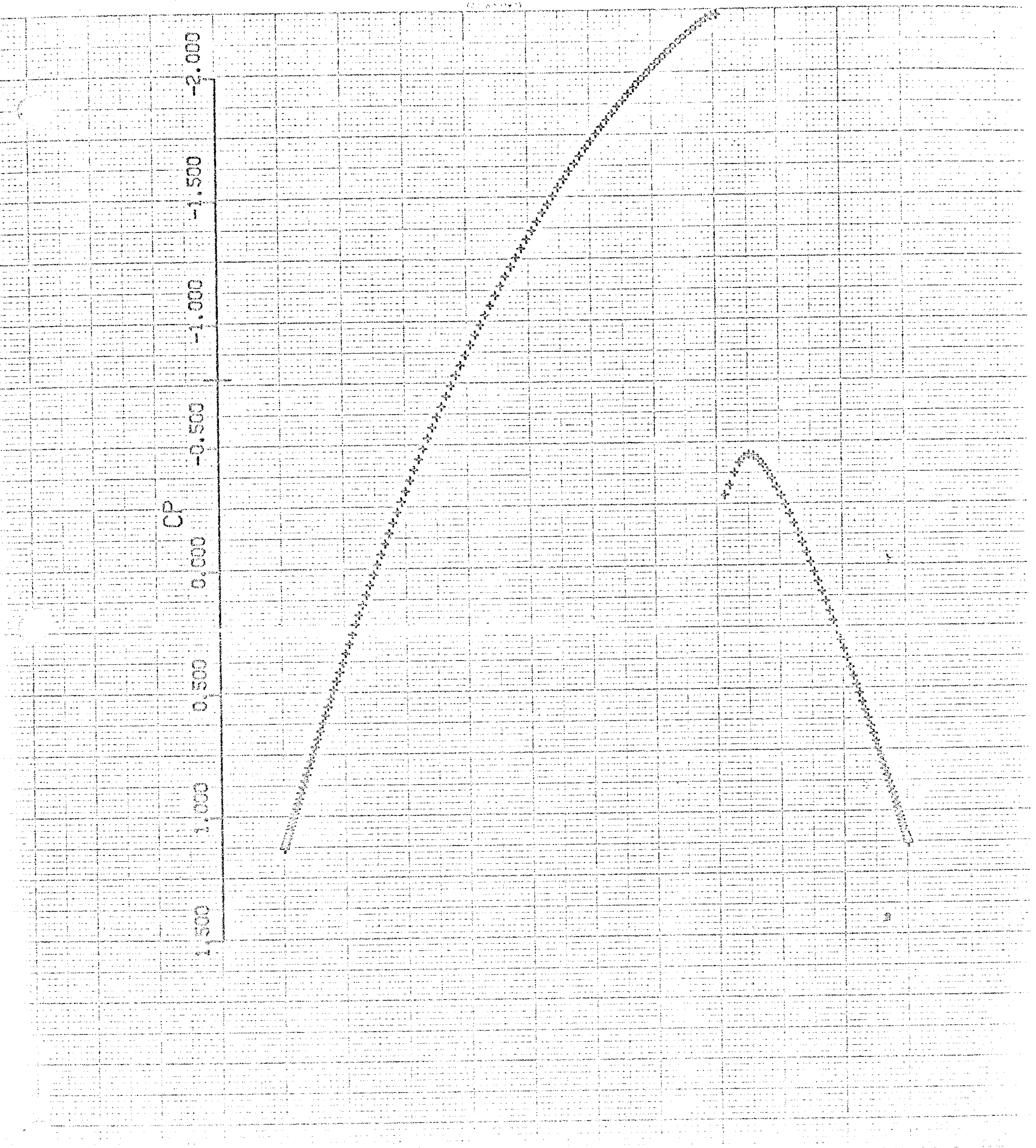
SPHERE

M = 0.500



SPHERE

M = 0.650

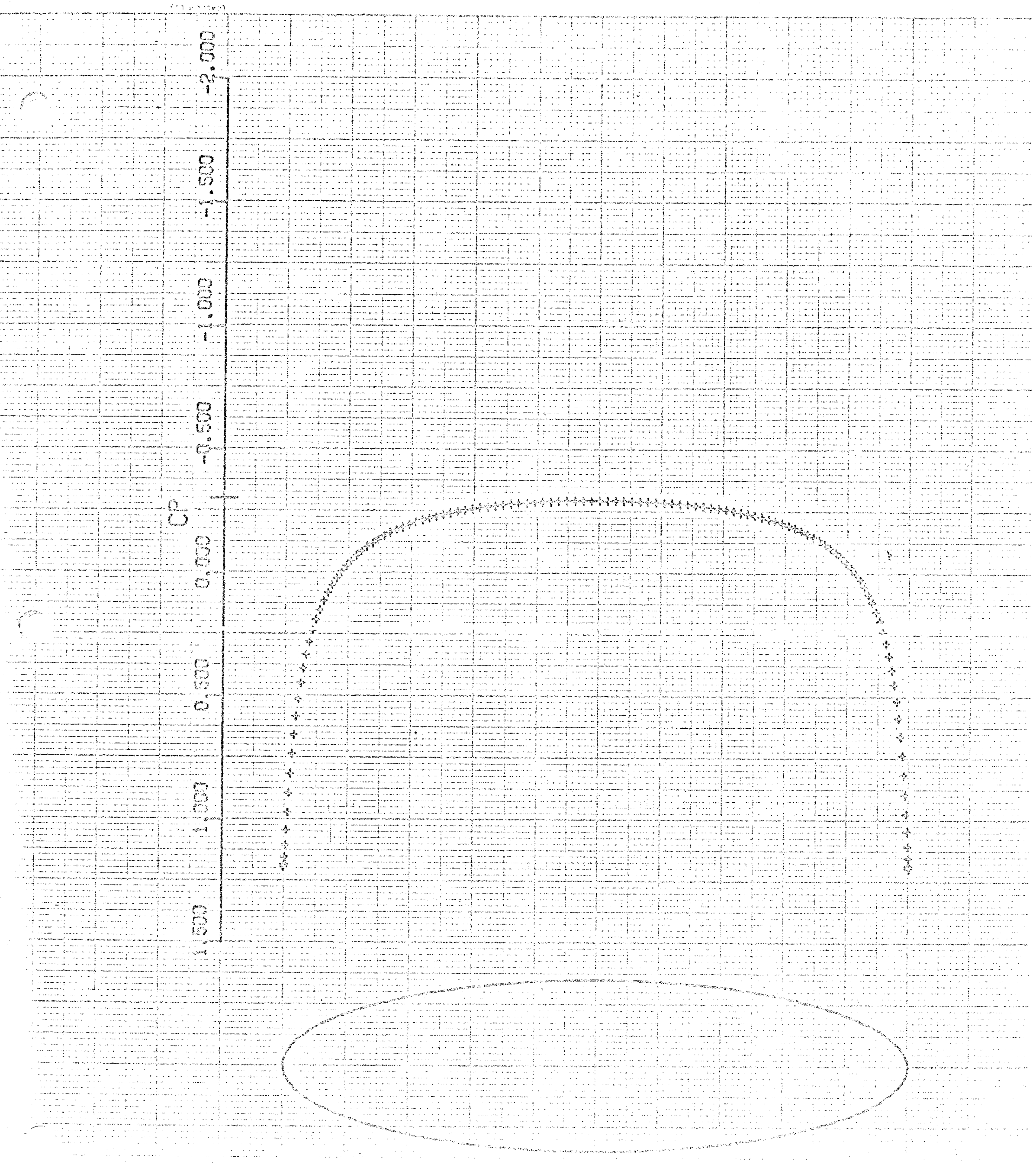


SPHERE

M = 0.700

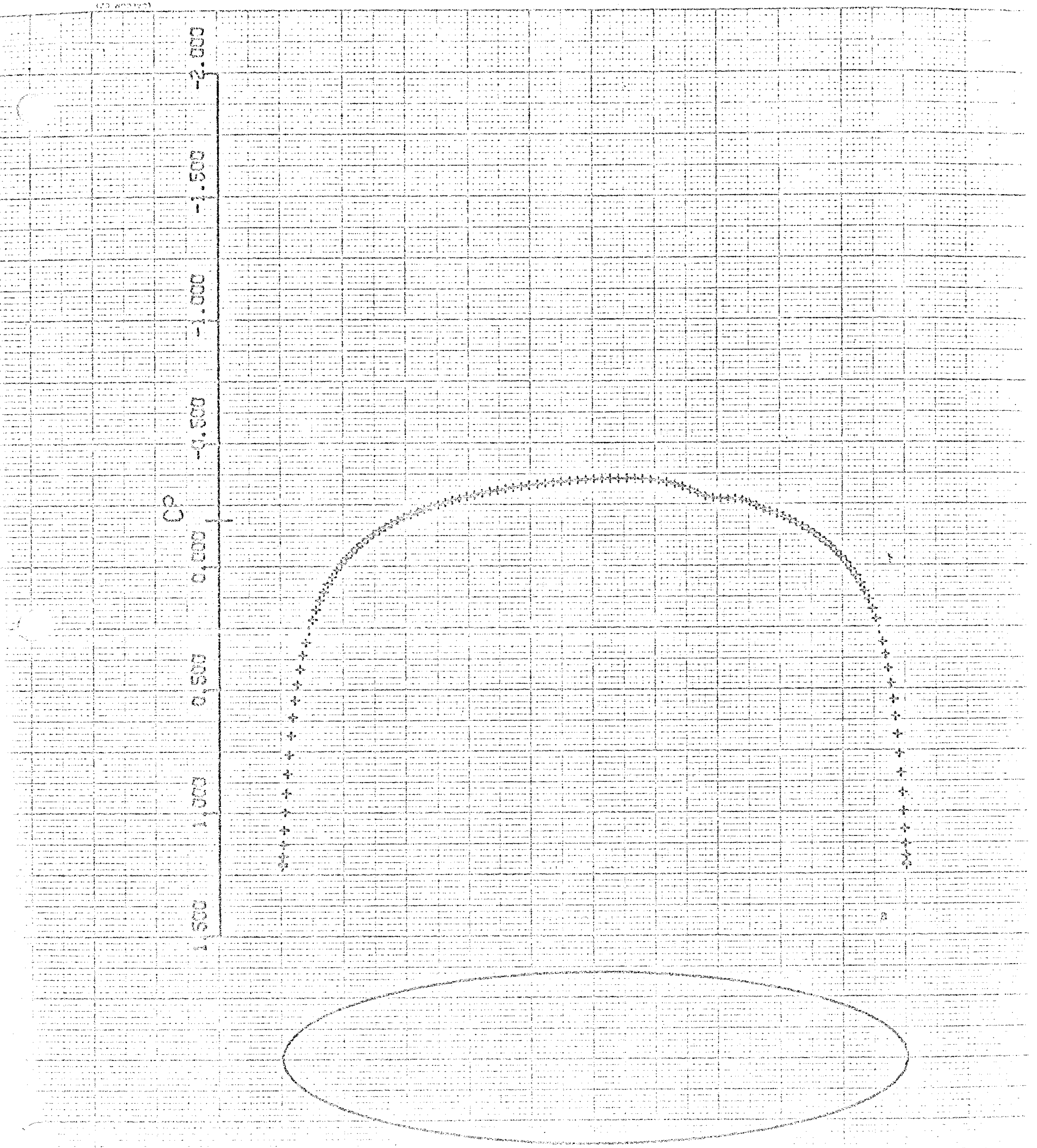
ELLIPTIC BODY OF REVOLUTION

Generated by Joukowsky mapping from unit circle with singular points at ± .75



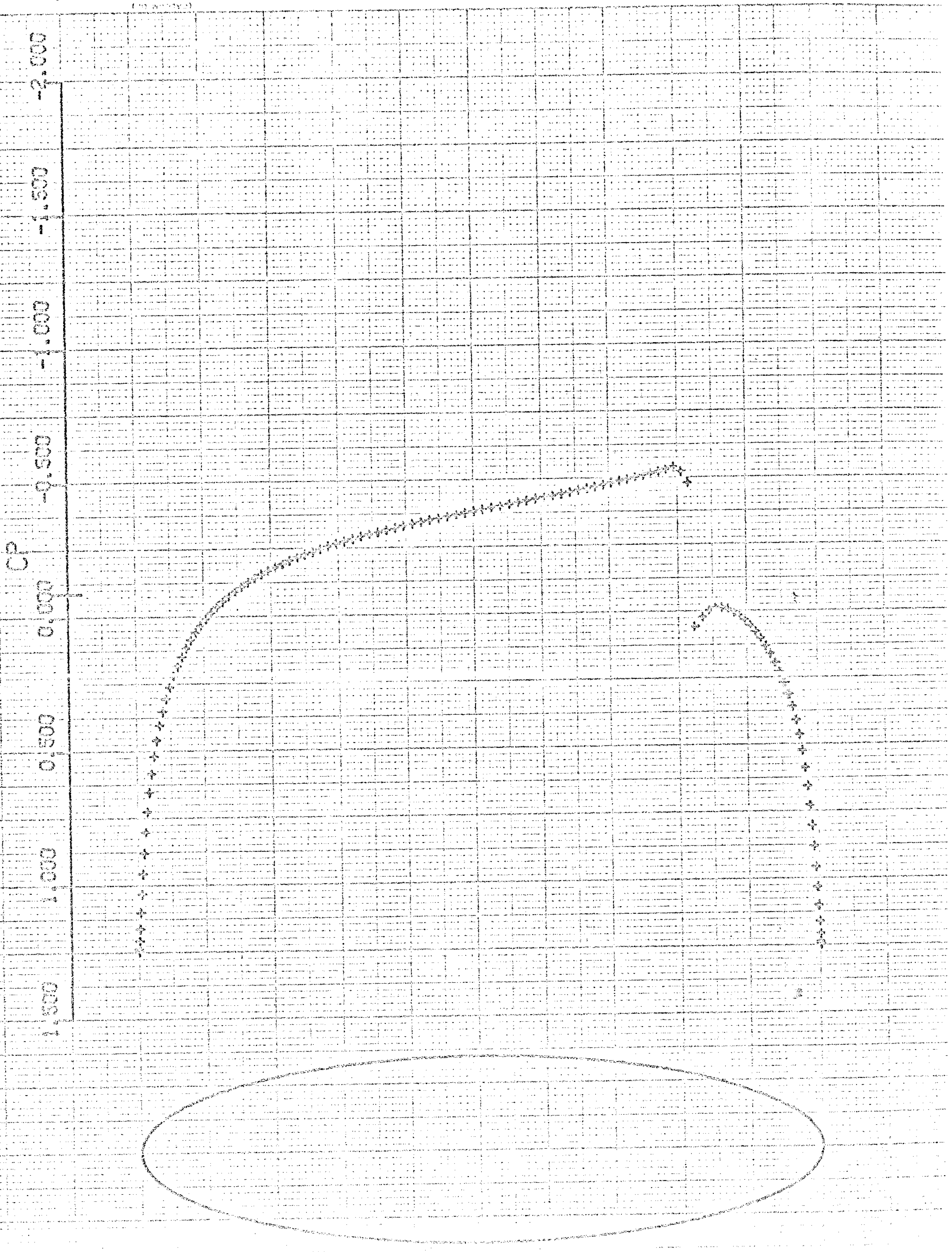
ELLIPTIC BODY OF REVOLUTION

B = [illegible]



ELLIPTIC BODY OF REVOLUTION

M = 0.900



ELLIPTIC BODY OF REVOLUTION

A = 0.950