TRANSONIC FLOW CALCULATIONS
FOR AIRFOILS
AND BODIES OF REVOLUTION
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1. Introduction

This report presents results of transonic flow calculations by the relaxation method for a selection of airfoils and bodies of revolution. The flow is assumed to be determined by the potential flow equation

\[(a^2-u^2) \phi_{xx} - 2uv \phi_{xy} + (a^2-v^2) \phi_{yy} = 0\] (1.1)

where \(\phi\) is the potential, \(u\) and \(v\) are the velocity components

\[u = \phi_x, \ v = \phi_y\] (1.2)

and \(a\) is the local speed of sound. This may be determined from the stagnation speed of sound \(a_o\) by the energy equation

\[a^2 = a_o^2 - .2 (u^2+v^2)\] (1.3)

in which the ratio of the specific heats has been taken as 1.4. If the velocities are normalized with unit free stream velocity, then

\[a_o^2 = \frac{1}{M^2} + .2\] (1.4)

where \(M\) is the free stream Mach number.

There is no assumption of small disturbances and the solution would be exact for subsonic flow in the absence of viscosity. In transonic flow the principal approximation is the neglect of rotation introduced by shock waves. In the absence of rotation Crocco's theorem [1] indicates that the flow is homentropic. The solution must allow discontinuities to approximate to approximate shock waves, and it has been shown by Morawetz [2] that in general a continuous solution does not exist in transonic flow. Since in isentropic flow there is a unique relation between velocity and area along a stream tube, the interpretation of a solution with a jump in velocity is that it is a solution in the presence of a fictitious screen just the correct throat area to permit an isentropic compression. Since there is a pressure differential on either side of the throat a discontinuity in the solution
represents a line carrying a force. The total drag is zero in potential flow, so this force is just balanced by a drag on the airfoil, and it turns out that the solution can be used to obtain an estimate of the pressure drag on the airfoil in the presence of a shock wave (see the results).

It should also be noted that in the absence of a directional condition corresponding to the condition that entropy can only increase, the solution of the potential equation is not unique. Given a solution, the equation continues to be satisfied when the velocity is reversed. Thus, for example, a body with for and aft symmetry always admits a solution with a reverse shock as well as one with a forward shock. In order to obtain a unique solution, it is necessary to introduce directionality into the difference scheme used to approximate the equation. Following Murman [3], this is accomplished by using a central difference scheme in the subsonic region together with a backward difference scheme in the supersonic region.
In order to obtain a finite domain of calculation the flow field is first
mapped to the interior of a unit circle by a conformal transformation, following
Sells [4]. Using polar coordinates \( r \) and \( \theta \), the potential for a uniform at
angle \( \alpha \) would then be \( \frac{\cos(\theta+\alpha)}{r} \). Also, in the presence of circulation the potential
would be multiple valued. It is convenient therefore to set
\[
\phi = x + \frac{\cos(\theta+\alpha)}{r} - E\theta
\]  \hspace{1cm} (2.1)
where \( 2\pi E \) is the circulation, and solve for the reduced potential functions \( X \) which
is finite and everywhere continuous. Let the modulus of the transformation to the
interior of the circle be \( \frac{H}{r^2} \). \( H \) is the modulus of the transformation to the
exterior of the circle, and is a smooth function suitable for differencing. The
equation for \( X \) in polar coordinates then becomes
\[
(a^2 - u^2) X_{\theta\theta} - 2uv (rX_{\theta r} + X_{\theta} - E) + (a^2 - v^2)(r^2 X_{rr} + rX_r)
+ (u^2 - v^2) rX_r + (u^2 + v^2) (\frac{uH_{\theta}}{r} + vH_r) = 0
\]  \hspace{1cm} (2.2)
where
\[
u = r(X_{\theta} - E) - \sin(\theta+\alpha), \quad \nu = \frac{r^2 X_r - \cos(\theta+\alpha)}{H}
\]  \hspace{1cm} (2.3)
and
\[a^2 = a_0^2 - \frac{3}{2}(u^2 + v^2)
\]  \hspace{1cm} (2.4)
The boundary condition at the surface \( r = 1 \) is
\[
u = 0, \quad X_r = \cos(\theta + \alpha)
\]  \hspace{1cm} (2.5)
If the airfoil has a sharp trailing edge, it is convenient to orient the circle
so that the corresponding point is at \( \theta = 0 \). At this point \( H = 0 \), and the
Kutta condition that the velocity is finite at the trailing edge then requires
that
\[
X_\theta = E + \sin \alpha
\]  \hspace{1cm} (2.6)
Also the equation for $X$ can be written as
\[ a^2 (r^2 X_{rr} + r X_r + X_{\theta\theta}) = H u \frac{1}{r} \frac{\partial}{\partial \theta} \left( \frac{u^2 + v^2}{2} \right) + H v \frac{\partial}{\partial r} \left( \frac{u^2 + v^2}{2} \right) \]

At the trailing edge, the right side vanishes and thus $X$ locally satisfies Laplace's equation.

At the center of the circle $H = 1 + O(r^2)$, $u \to \sin(\theta + \alpha)$, $v \to \cos(\theta + \alpha)$, $a \to \frac{1}{M}$. It can be seen therefore that the potential equation reduces to
\[ [1-M^2 \sin^2(\theta+\alpha)] X_{\theta\theta} - 2M^2 \sin(\theta+\alpha) \cos(\theta+\alpha)(X_{\theta}-E) = 0 \quad (2.7) \]

This can be integrated to give the far field boundary condition at $r = 0$
\[ X = E \left\{ \theta - \tan^{-1} \left( \frac{1}{(1-M^2)^{\frac{1}{2}}} \tan(\theta + \alpha) \right) \right\} \quad (2.8) \]
in agreement with the results of Ludford [5], and Imai [6].
References:

1. H. W. Liepmann and A. Roshko, Elements of Gas Dynamics,
   Wiley, New York, 1957, p. 193

2. C. S. Moraw etz, On the Non-Existence of Continuous Transonic Flows

3. E. M. Murman and I. D. Cole, Calculation of Plane Steady Transonic Flows,
   AIAA J. Vol. 9, 1971, pp. 114 - 121


5. G. S. S. Ludford, The Behaviour at Infinity of the Potential Function
   of a Two Dimensional Subsonic Compressible Flow, J. Math Phys. 1951,
   Vol. 30, 1951 pp. 131 - 139

6. I. Imai, Approximation Methods in Compressible Fluid Dynamics,
   University of Maryland Technical Note BN-95, 1957
NLR LIFTING QUASI-ELLiptical AIRFOIL

Designed to be shock free at $M = .7557$, $\alpha = 0^\circ$
(1.32° when the coordinates are referred to the line from the maximum forward point to the trailing edge).
NLR LIFTING QUILTED-ELLIPOTICAL AIRCRAFT

M = 0.755  W/E = -0.200  CL = 0.123

CL = 0.1927  CD = 0.030  CL = 0.0311
MLR LIFTING QUASI-ELLIPSE. AIRW.07L
M = 0.750; DLF = -0.030; CL = 0.240
CL = 0.2300; CD = 0.0900; C1 = 0.1150
PLS LIFTING QUASI-ELLPTICAL AIRFOIL

$M = 0.758$, $MLF = 0.000$, $CL = 0.245$

$CL = 0.250$, $CD = 0.0001$, $CL = 0.1279$
NLR LIFTING QUASI-ELLIPICAL AIRFOIL

M = 0.750  OLFA = 0.250  CL = 0.313
CL = 0.3150  CD = 0.0005  Cn = 0.1935
PLA LIFTING QUINT-ELLIPICAL AIRFOIL

M = 0.758  RF = 0.500  CL = 0.376
CL = 0.3825  CD = 0.0010  CM = 0.188
MLB LIFTING QUASI-ELLiptical Airfoil

Re = 0.785  TLF = 0.0  CL = 0.100
CL = 0.0025  CD = 0.0252  CD = 0.1268
KORN AIRFOIL

Designed to be shock free at $M = .75, \alpha = .113^\circ$. Coordinates and slopes generated by Grumman's version of the Korn synthesis program.
KORN .75 AIRFOIL (S37009)

\[ R = 0.756 \quad Cl_F = 0.103 \quad Cl = 0.806 \]

\[ Cl = 0.9932 \quad Cl_F = 0.0012 \quad Cl = -0.1093 \]
KORN 75 AIRFOIL (537868)

$M = 0.738 \quad CL' = 0.192 \quad CL = 0.672$

$CL = 0.8731 \quad CD = 0.0003 \quad CD = -0.1611$
Coordinates from NAE Report LTR-HA-2,
Fortran IV Program for the Catherall-
Foster-Sells Method for Calculation of
the Plane Inviscid Compressible Flow
Past a Lifting Airfoil, by
$U = 0.750 \quad RLE = 1.000 \quad Cl = 0.320$

$Cl = 0.2222 \quad CD = 0.0003 \quad Cm = -0.0001$
Coordinates from NACA TN 3162,
Effects of Subsonic Mach Number on the
Forces and Pressure Distributions on
Four NACA 64A - Series Airfoil Sections
at Angles of Attack as High as 28°, by
Louis S. Stivers, 1954.
NACA 648110

M = 0.735  ALF = -0.600  CL = 0.551

CL = 0.5541  CD = 0.0036  CR = 0.1030
NACA 0011D

M = 0.766  REL  0.500  CL = 0.728
CL = 0.7699  CD = 0.0135  CM = -0.0171
\[ H = 0.770 \quad MLP = -2.000 \quad CL = 0.270 \]

\[ CL = 0.270 \quad CD = 0.000 \quad CH = -0.1565 \]
ELLIPTIC BODY OF REVOLUTION

Generated by Joukowsky mapping from unit circle with singular points at ± .75
ELLIPSTIC BODY OF REVOLUTION

\[ h = 0.250 \]