Time Spectral Method for Rotorcraft Flow with Vorticity Confinement

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   - Time Spectral Method
2. Time Spectral Method
   - Time Spectral Method Formulation
3. Rotorcraft Simulation Results
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   - Lifting Forward Flight Calculations
4. Vorticity Confinement
   - Introduction
   - Formulation
   - Compressible Euler Calculations
   - Application to Rotorcraft Flows

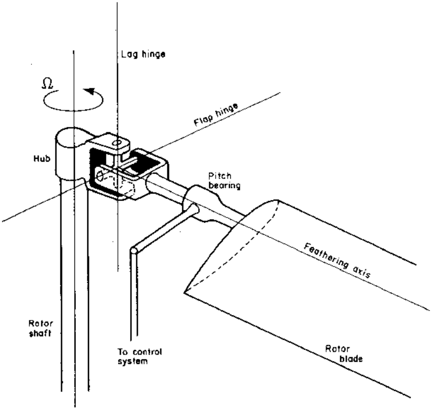
Introduction

Introduction

Helicopter simulation is very complex and computationally expensive:

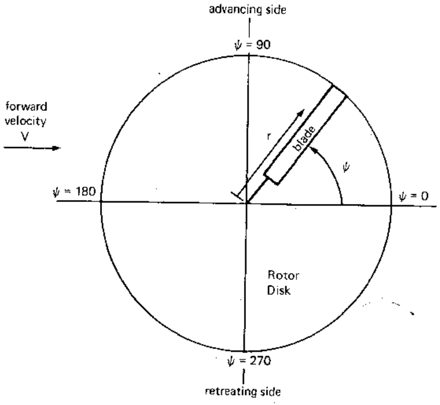
- The flow is highly nonlinear.
- Interactions between the vortices with the blades and fuselage.
- There is a wide range of scales.
- Blades are highly elastic.
- Variety of blade motion:
  - Lead
  - Lag
  - Flapping
  - Collective pitch, cyclic pitch, yaw

Articulated Rotor



★ Johnson, W., "Helicopter Theory", 1980.

Forward Flight



★ Johnson, W., "Helicopter Theory", 1980.

Background

A lot has been done over the past 3 decades.

- Potential flow calculations:
  - Caradonna & Isom (1972, 1976)
  - Caradonna & Philippe (1976)
  - Arieli, Taubert & Caughey (1986): the first three-dimensional, full potential flow based on Jameson & Caughey's FLO22
- Euler and Reynolds averaged Navier–Stokes (RANS) calculations:
  - Agarwal & Deese (1987, 1988)
  - Srinivasan *et al.* (1991, 1992)
  - Pomin & Wagner (2002, 2004)
  - Allen (2003, 2004, 2005, 2006, 2007): 32 million mesh points and 25,000 CPU hours for Euler calculation of a four-bladed rotor!

What is the Time Spectral Method?

Time Spectral Method

- Time integration method based on Fourier representation.
- Efficient and accurate method for periodic problems.
- No need to Fourier transform variables back and forth between time and frequency domains, everything is solved in the time domain.
- Algorithm is easily adapted to the current solvers.
  - Existing convergence acceleration techniques are applicable.
- The method is able to achieve spectral accuracy in theory.

What Has Been Done?

Fully nonlinear methods:

1. Harmonic Balance method of Hall, Thomas & Clark (2002): originally for turbomachinery.
   - Ekici & Hall (2008): Rotorcraft simultion.
2. Nonlinear frequency domain (NLFD) of McMullen, Jameson & Alonso (2001, 2002).
3. Time Spectral method of Gopinath & Jameson (2005).

Time Spectral Method

Time Spectral Method

The discrete Fourier transform of the flow variables $\mathbf{w}$ for a time period T is

$$\widehat{\mathbf{w}}_k = \frac{1}{N} \sum_{n=0}^{N-1} \mathbf{w}^n e^{-ik\frac{2\pi}{T} n\Delta t},$$

and its inverse transform:

$$\mathbf{w}^n = \sum_{k=-\frac{N}{2}}^{\frac{N}{2}-1} \widehat{\mathbf{w}}_k e^{ik\frac{2\pi}{T} n\Delta t}. \quad (1)$$

The spectral derivative of equation (1) with respect to time at the n-th time instance is given by

$$D\mathbf{w}^n = \frac{2\pi}{T} \sum_{k=-\frac{N}{2}+1}^{\frac{N}{2}-1} \mathrm{i}k\widehat{\mathbf{w}}_k e^{\mathrm{i}k\frac{2\pi}{T}n\Delta t}.$$

The right hand side can be written in terms of the flow variables $\mathbf{w}^n$ as follows:

$$D\mathbf{w}^n = \sum_{j=0}^{N-1} d_n^j \mathbf{w}^j$$

where

$$d_n^j = \begin{cases} \frac{2\pi}{T}\frac{1}{2}(-1)^{n-j}\cot\left\{\frac{\pi(n-j)}{N}\right\} & : \quad n \neq j \\ 0 & : \quad n = j \end{cases}.$$

Let $n - j = -m$, one can rewrite the time derivative as

$$D\mathbf{w}^n = \sum_{m=-\frac{N}{2}+1}^{\frac{N}{2}-1} d_m \mathbf{w}^{(n+m)}$$

where d_m is given by

$$d_m = \begin{cases} \frac{2\pi}{T}\frac{1}{2}(-1)^{m+1}\cot\left\{\frac{\pi m}{N}\right\} & : \quad m \neq 0 \\ 0 & : \quad m = 0 \end{cases} .$$

The original flow equations in semi-discrete form:

$$\mathcal{V}\frac{\mathrm{d}\mathbf{w}^n}{\mathrm{d}t} + R(\mathbf{w}^n) = 0,$$

becomes

$$\mathcal{V}D\mathbf{w}^n + R(\mathbf{w}^n) = 0. \tag{2}$$

These comprise a four dimensional coupled space–time set of nonlinear equations, which need to be solved simultaneously. For this purpose we introduce a pseudo time derivative term to equation (2), the equations can now be marched towards a periodic steady state using well known convergence acceleration techniques.

$$\mathcal{V}\frac{\mathrm{d}\mathbf{w}^n}{\mathrm{d}\tau} + \mathcal{V}D\mathbf{w}^n + R(\mathbf{w}^n) = 0.$$

Flow Solver Methodology

1. Convergence Acceleration via
    - Modified 5-stage Runge–Kutta★
    - Local time stepping★
    - Multigrid★
2. Space Discretization:★
    - Jameson–Schmidt–Turkel (JST)
    - Symmetric LImited Positive (SLIP)
    - Convective Upwind and Split Pressure (CUSP)
3. Internal mesh generator via conformal mapping
4. Baldwin–Lomax turbulence model (Baldwin–Lomax, 1978)

★ A. Jameson, *A perspective on computational algorithms for aerodynamics analysis and design*, Progress in Aerospace Sciences, **37**, pp. 197–243, 2001.

★ A. Jameson, *Aerodynamics*, Encyclopedia of Computational Mechanics, Ch. 11, 2004.

Rotorcraft Simulation Results

Hover Calculations were Presented at 46th AIAA Aerospace Sciences Meeting and Exhibit, Reno, NV

AIAA Paper 2008–403

Forward Flight Calculations

Caradonna *et al.* Experiment (1984)

Experimental setup:

- Untapered, untwisted two-bladed rotor
- NACA 0012 section
- Aspect ratio of 7
- Diameter of the rotor is 7 ft
- Chord is 6 in

Nonlifting Rotor in Forward Flight

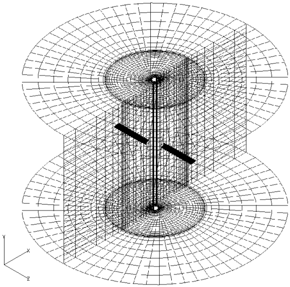
- Flow Condition:

$$
\begin{aligned}
\theta_c &= 0^\circ \\
\mathrm{M_{tip}} &= 0.8 \\
\mu &= 0.2 \\
\mathrm{Re} &= 2.89 \times 10^6
\end{aligned}
$$

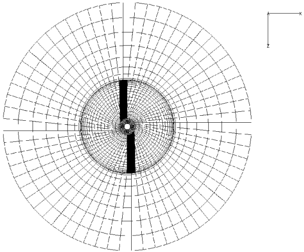
- Twelve time instances were used, $N = 12$

Mesh

- Euler: 128 × 48 × 32 cells per blade, 16 cells on the blade.
- RANS: 192 × 64 × 48 cells per blade, 32 cells on the blade.



(a) Isometric view



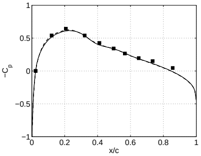
(b) Top view

Comparison with the Experimental Data

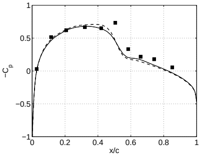
- Dissipation schemes are JST and CUSP
- Results are compared at six azimuthal angles on the advancing side:

$$
\begin{aligned}
&(a) \quad \psi = 30^\circ \\
&(b) \quad \psi = 60^\circ \\
&(c) \quad \psi = 90^\circ \\
&(d) \quad \psi = 120^\circ \\
&(e) \quad \psi = 150^\circ \\
&(f) \quad \psi = 180^\circ
\end{aligned}
$$

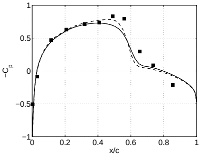
Euler Calculations



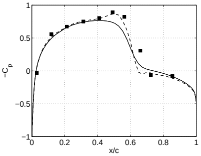
(a) $\psi = 30°$



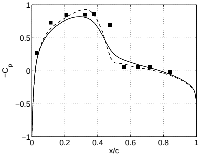
(b) $\psi = 60°$



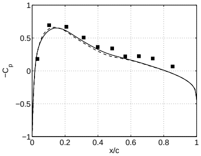
(c) $\psi = 90°$



(d) $\psi = 120°$



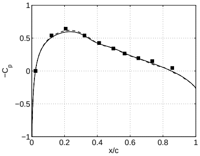
(e) $\psi = 150°$



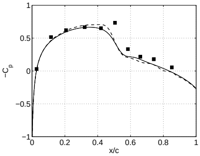
(f) $\psi = 180°$

■ experiment, — JST scheme, – – CUSP scheme

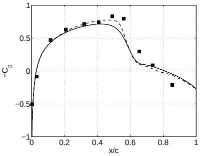
RANS Calculations



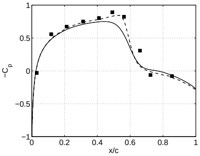
(a) $\psi = 30^\circ$



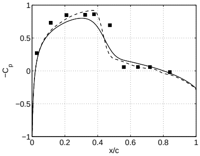
(b) $\psi = 60^\circ$



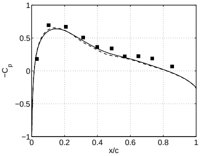
(c) $\psi = 90^\circ$



(d) $\psi = 120^\circ$



(e) $\psi = 150^\circ$



(f) $\psi = 180^\circ$

■ experiment, — JST scheme, – – CUSP scheme

Computational Cost

- 300 multigrid cycles for Euler calculations.
  - Residual reduced by four orders of magnitude.
- 500 multigrid cycles for RANS calculations.
  - 5 hours on four dual-core processors (clock speed is 3.0 GHz).
  - Residual reduced by three orders of magnitude.

Comparison with Backward Difference Formula (BDF)★

$$\mathcal{V}\left\{\frac{3}{2\Delta t}\mathbf{w}^{n+1}-\frac{4}{2\Delta t}\mathbf{w}^{n}+\frac{1}{2\Delta t}\mathbf{w}^{n-1}\right\}+\mathbf{R}\left(\mathbf{w}^{n+1}\right)=0.$$

- Periodicity is established, not enforced.
- Usually requires at least 4 cycles (for pitching airfoil/wing).

★ A. Jameson, "Time Dependent Calculations Using Multigrid, with Applications to Unsteady Flows Past Airfoils and Wings", AIAA Paper 1991–1596.

Comparison with Backward Difference Formula (BDF)

For the same RANS calculations, the BDF would need:

- 180 time steps per revolution
- 40 multigrid cycles per time step
- 6 cycles to convergence
- $\Rightarrow$ 43200 steps
- Time Spectral method used 500 multigrid cycles with 12 time instances
- In terms of the number of multigrid cycles required ...
- Time Spectral method is 87 times faster
- In terms of CPU hours ...
- Time Spectral method is still 7.2 times faster

Time-Lagged Periodic Boundary Condition

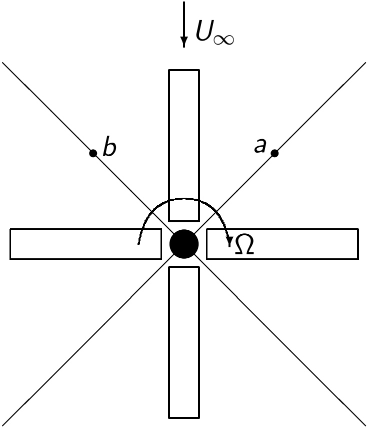
- First proposed by Ekici & Hall★ (2008)
- One blade is required for forward flight simultions
- Further saving of N_b times

where N_b is the number of blades per rotor

$$\mathbf{w}(r, \psi, z, t) = \mathbf{w}\left(r, \psi - \frac{2\pi}{N}, z, t - \frac{T}{N}\right)$$

★ Ekici, Hall & Dowell, "Computationally Fast Harmonic Balance Methods for Unsteady Aerodynamic Predictions of Helicopter Rotors", AIAA Paper 2008–1439.

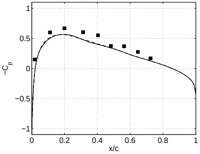
Time-Lagged Periodic Boundary Condition



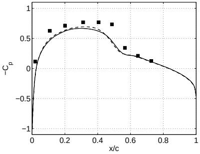
Euler Calculations

- Collective pitch, $\theta_c = 0^\circ$
- Tip Mach number, $\mathrm{M}_{\mathrm{tip}} = 0.7634$
- Advance ratio, $\mu = 0.25$
- $128 \times 48 \times 32$ mesh cells

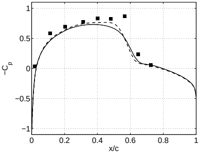
Euler Calculations



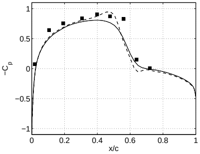
(a) $\psi = 30°$



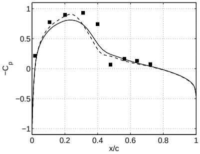
(b) $\psi = 60°$



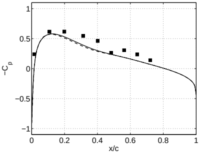
(c) $\psi = 90°$



(d) $\psi = 120°$



(e) $\psi = 150°$

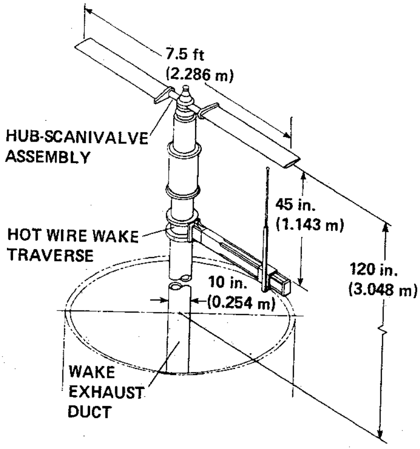


(f) $\psi = 180°$

■ experiment, — JST scheme, – – CUSP scheme

Lifting Rotor in Forward Flight

Caradonna & Tung Experiment (1981)



Test Case

- Caradonna & Tung rotor
- Collective pitch, $\theta_c = 8^\circ$
- Tip Mach number, $\mathrm{M}_{\mathrm{tip}} = 0.7$
- Advance ratio, $\mu = 0.2857$

Numerical Data Provided by C. B. Allen

- Over 2 million mesh points around the two blades and hub (not including the other blocks that cover the far-fields)
- BDF time stepping scheme
- 180 steps per revolution
- 6 revolutions
- Periodicity is established after the second revolution
- JST dissipation scheme
- 70 3-level V-cycle multigrid cycles per time step

★ C.B. Allen, "An Unsteady Multiblock Multigrid Scheme for Lifting Forward Flight Rotor Simulation", International Journal for Numerical Methods in Fluids, 2004.

Lift Comparison

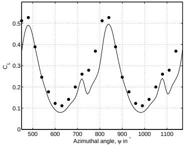
- Load variation on each blade around the azimuth

$$C_L = \frac{F_y}{\frac{1}{2}\rho\,(\Omega R)^2\,c\,R}$$

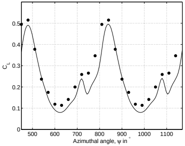
where

$$\begin{aligned} F_y &= \text{force in the } y \text{ direction} \\ \Omega &= \text{angular velocity} \\ c &= \text{chord} \\ R &= \text{rotor radius} \\ \rho &= \text{density} \end{aligned}$$

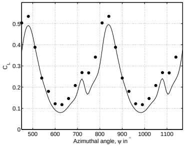
C_L comparison – JST Scheme



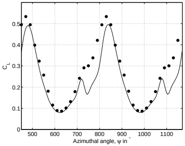
(a) $128 \times 48 \times 32$



(b) $160 \times 48 \times 48$



(c) $192 \times 64 \times 48$

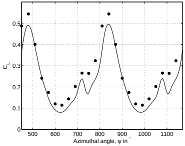


(d) $160 \times 48 \times 48^{\star}$

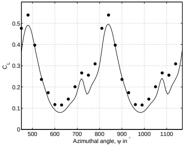
$\star$ with 18 time instances

— Allen, • computed result

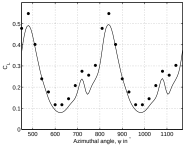
C_L comparison – CUSP Scheme



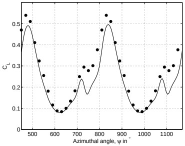
(a) $128 \times 48 \times 32$



(b) $160 \times 48 \times 48$



(c) $192 \times 64 \times 48$



(d) $160 \times 48 \times 48^{\star}$

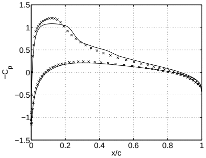
★ with 18 time instances

— Allen, • computed result

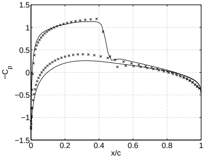
C_p Comparison

- Comparison is made at blade section $r/R = 0.90$
- Strong transonic flow on the advancing side

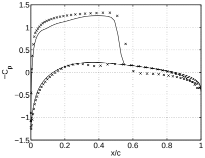
$160 \times 48 \times 48$: JST scheme (Advancing Side)



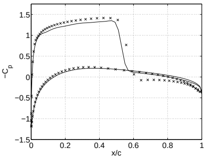
(a) $\psi = 30^\circ$



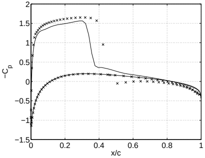
(b) $\psi = 60^\circ$



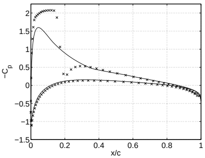
(c) $\psi = 90^\circ$



(d) $\psi = 120^\circ$



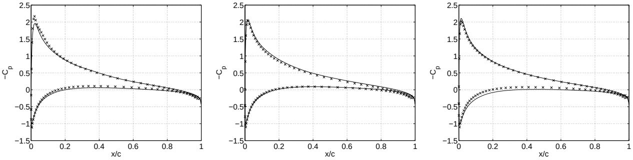
(e) $\psi = 150^\circ$



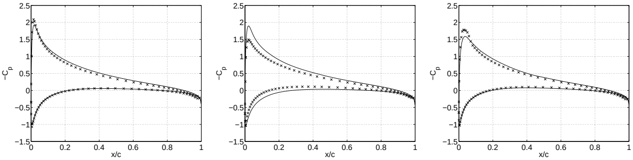
(f) $\psi = 180^\circ$

$\times$ Allen, — computed result

$160 \times 48 \times 48$: JST scheme (Retreating Side)



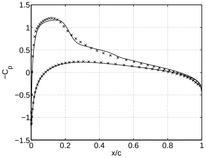
(g) $\psi = 210^\circ$ (h) $\psi = 240^\circ$ (i) $\psi = 270^\circ$



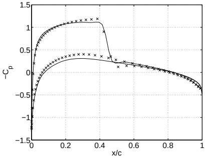
(j) $\psi = 300^\circ$ (k) $\psi = 330^\circ$ (l) $\psi = 360^\circ$

$\times$ Allen, — computed result

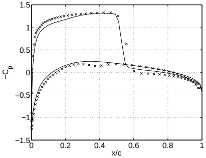
$160 \times 48 \times 48$: CUSP scheme (Advancing Side)



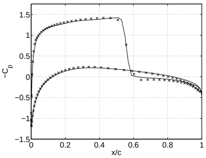
(a) $\psi = 30°$



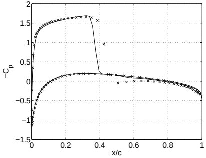
(b) $\psi = 60°$



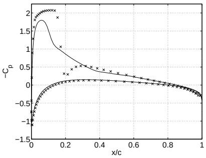
(c) $\psi = 90°$



(d) $\psi = 120°$



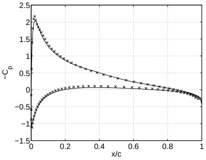
(e) $\psi = 150°$



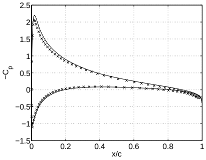
(f) $\psi = 180°$

$\times$ Allen, — computed result

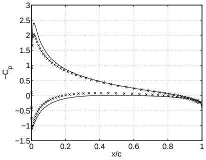
$160 \times 48 \times 48$: CUSP scheme (Retreating Side)



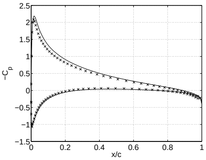
(g) $\psi = 210^\circ$



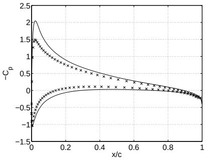
(h) $\psi = 240^\circ$



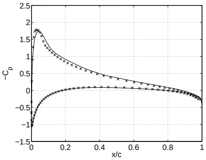
(i) $\psi = 270^\circ$



(j) $\psi = 300^\circ$



(k) $\psi = 330^\circ$



(l) $\psi = 360^\circ$

$\times$ Allen, — computed result

Vorticity Confinement

What is Vorticity Confinement?

- John Steinhoff first suggested the idea in 1994.
- A forcing term added to the momentum equations (inviscid, incompressible), "so that as the vorticity diffuses away from the centroids of vortical regions, it is transported back".
- Vorticity is added in the direction normal to both $\vec{\omega}$ and the gradient $|\vec{\omega}|$.
- Unfortunately momentum is not conserved.

Original Formulation

Steinhoff & Underhill (1994); Steinhoff (1994):

$$\frac{\partial \mathbf{u}}{\partial t} + (\mathbf{u} \cdot \nabla)\,\mathbf{u} = \frac{1}{\rho}\nabla p + \mu \nabla^2 \mathbf{u} - \epsilon \mathbf{s}$$

where the simplest form of $\mathbf{s}$ is

$$\mathbf{s} = \frac{\nabla|\vec{\omega}|}{|\nabla|\vec{\omega}||} \times \vec{\omega}$$

Compressible Formulation

Hu & Grossman (2001); Hu *et al.* (2001) and Dadone *et al.* (2001) introduced a body force per unit mass term to the total energy equation:

$$\int_{\Omega} \frac{\partial \mathbf{w}}{\partial t} \, \mathrm{d}\mathcal{V} + \oint_{\partial\Omega} \mathbf{f}_j \cdot \mathbf{n} \, \mathrm{d}\mathcal{S} = -\int_{\Omega} \epsilon \mathbf{s} \, \mathrm{d}\mathcal{V}$$

where $\vec{s}$ is now:

$$\vec{s} = \begin{bmatrix} 0 \\ \rho(\hat{n} \times \vec{\omega}) \cdot \mathbf{i} \\ \rho(\hat{n} \times \vec{\omega}) \cdot \mathbf{j} \\ \rho(\hat{n} \times \vec{\omega}) \cdot \mathbf{k} \\ \rho(\hat{n} \times \vec{\omega}) \cdot \mathbf{u} \end{bmatrix} \quad \text{and} \quad \hat{n} = \frac{\nabla|\vec{\omega}|}{|\nabla|\vec{\omega}||}.$$

Making ϵ Dimensionless and Dynamic

- Fedkiw *et al.* (2001) for incompressible Euler equations on structured meshes

$$\epsilon_h \propto \epsilon h$$

- Löhner & Yang (2002); Löhner *et al.* (2002) for incompressible RANS calculations on unstructured meshes

$$\epsilon_v \propto \begin{cases} \epsilon|\mathbf{u}| \\ \epsilon h|\vec{\omega}| \\ \epsilon h^2|\nabla|\vec{\omega}|| \end{cases}$$

Robinson (2004)

- Chose to scale ϵ is with $|\mathbf{u}|$
- Factor out $|\vec{\omega}|$ from $\mathbf{s}$ $\left(= \frac{\nabla|\vec{\omega}|}{|\nabla|\vec{\omega}||} \times \vec{\omega}\right)$
- $\Rightarrow |\mathbf{u}| \cdot |\vec{\omega}|$
- $|\mathbf{u} \cdot \vec{\omega}| \equiv$ helicity

$$\mathbf{s} = \rho\,|\mathbf{u} \cdot \vec{\omega}| \left\{ \frac{\nabla\,|\vec{\omega}|}{|\nabla\,|\vec{\omega}||} \times \frac{\vec{\omega}}{|\vec{\omega}|} \right\}$$

New Formulation

Combine

1. Helicity form
2. Body force per unit mass term in energy equation
3. Scaling based on cell size

$$\mathbf{s} = |\mathbf{u}\cdot\vec{\omega}| \left[1 + \log_{10}\left(1 + \frac{\mathcal{V}}{\mathcal{V}_{\text{averaged}}}\right)^{1/3}\right] \begin{bmatrix} 0 \\ \rho\left[\hat{n}\times\frac{\vec{\omega}}{|\vec{\omega}|}\right]\cdot\mathbf{i} \\ \rho\left[\hat{n}\times\frac{\vec{\omega}}{|\vec{\omega}|}\right]\cdot\mathbf{j} \\ \rho\left[\hat{n}\times\frac{\vec{\omega}}{|\vec{\omega}|}\right]\cdot\mathbf{k} \\ \rho\left[\hat{n}\times\frac{\vec{\omega}}{|\vec{\omega}|}\right]\cdot\mathbf{u} \end{bmatrix}$$

where

$$\hat{n} = \frac{\nabla|\vec{\omega}|}{|\nabla|\vec{\omega}||}.$$

NACA 0012 Wing

Test Case:

- Euler calculation
- Untwisted, untapered wing with NACA 0012 cross section
- Aspect ratio of 3

$$\begin{aligned} \alpha &= 5^\circ \\ \mathrm{M}_\infty &= 0.8 \end{aligned}$$

Vorticity Magnitude

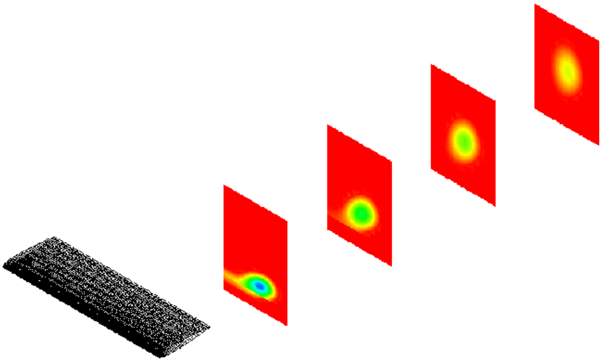


Figure: $\epsilon = 0$

Vorticity Magnitude

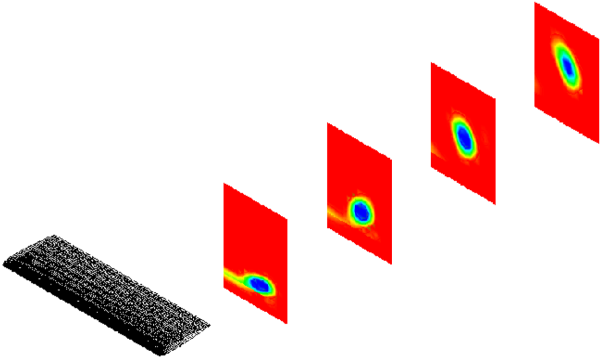


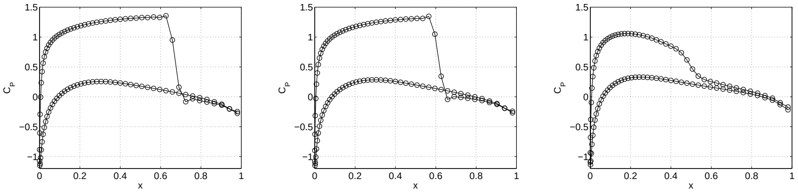
Figure: $\epsilon = 0.075$

c_d and c_l at Three Different Spans

	$z = 0.891$		$z = 1.828$		$z = 2.766$	
ϵ	c_l	c_d	c_l	c_d	c_l	c_d
0	0.7098	0.0792	0.6123	0.0651	0.3869	0.0394
0.025	0.7091	0.0791	0.6114	0.0650	0.3851	0.0393
0.050	0.7083	0.0790	0.6103	0.0649	0.3833	0.0391
0.075	0.7074	0.0788	0.6093	0.0647	0.3817	0.0389

- 0.3% difference in c_l and 0.5% difference in c_d at $z = 0.891$
- 1.3% difference in both c_l and c_d at $z = 2.766$

C_p Plots

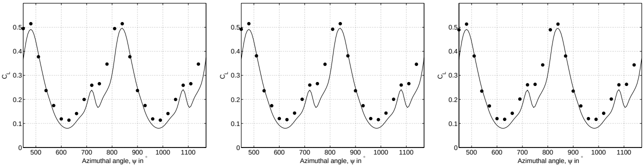


(a) $z = 0.891$ (b) $z = 1.828$ (c) $z = 2.766$

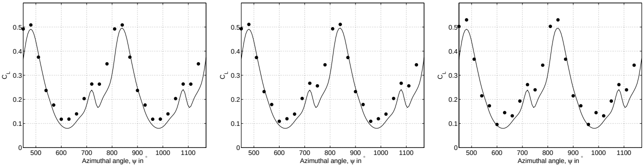
— ○ — $\epsilon = 0$, $\cdots$ $\epsilon = 0.025$, $- \cdot -$ $\epsilon = 0.05$, $- -$ $\epsilon = 0.075$

Application to Lifting Rotor in Forward Flight

C_L Comparison: JST Scheme



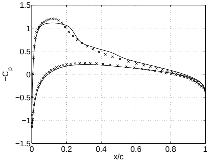
(a) $\epsilon = 0$ (b) $\epsilon = 0.05$ (c) $\epsilon = 0.1$



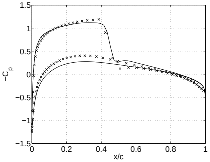
(d) $\epsilon = 0.15$ (e) $\epsilon = 0.2$ (f) $\epsilon = 0.25$

— Allen, • computed result

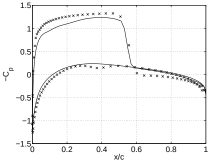
$160 \times 48 \times 48$: JST scheme (Advancing Side), $\epsilon = 0.2$



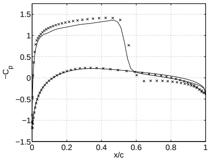
(a) $\psi = 30^\circ$



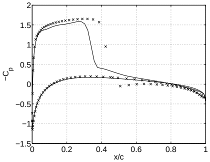
(b) $\psi = 60^\circ$



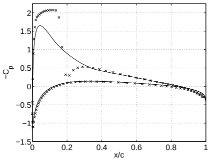
(c) $\psi = 90^\circ$



(d) $\psi = 120^\circ$



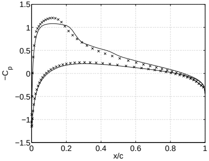
(e) $\psi = 150^\circ$



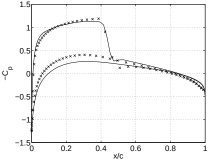
(f) $\psi = 180^\circ$

$\times$ Allen, — computed result

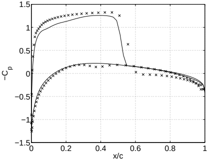
160 × 48 × 48: JST scheme (Advancing Side)



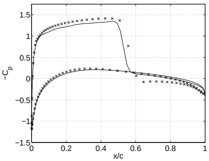
(a) $\psi = 30^\circ$



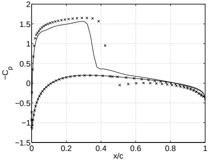
(b) $\psi = 60^\circ$



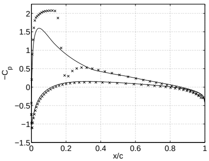
(c) $\psi = 90^\circ$



(d) $\psi = 120^\circ$



(e) $\psi = 150^\circ$

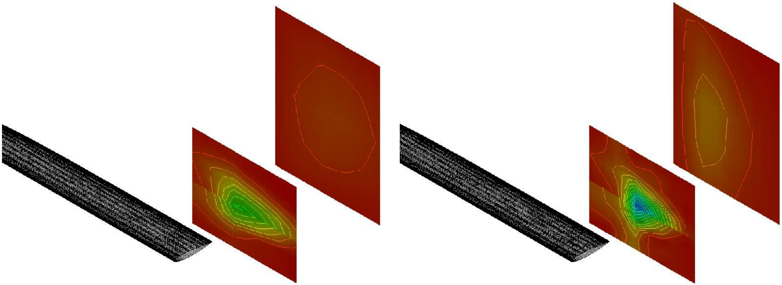


(f) $\psi = 180^\circ$

× Allen, — computed result

Vorticity Magnitude

- $x = 2$ and $x = 5$
- 1st time instance, i.e. $\psi = 90^{\circ}$



(a) $\epsilon = 0$ (b) $\epsilon = 0.2$

Future Work & Summary

- Time Spectral method has proved to be an efficient method for periodic problems, providing that the number of time instances are enough to capture the smallest frequency.
- Vorticity confinement works well for fixed-wing calculations.
- The vortical structure in lifting rotor in forward flight could be controlled such that the effect of blade–vortex interaction became more apparent as ϵ increased.
- ... but further studies are needed for rotorcraft application, at least with the current mesh geometry.
- Perhaps H-mesh would be better suited, or one can resort to overset or unstructured meshes.

Conclusion

- Hover calculation takes much longer than forward flight calculation (surprisingly).
- Time Spectral method is approximately 10 times faster than the traditional backward difference formula (depending on the number of time instances required).
- RANS calculations for nonlifting rotor in forward flight took only 5 hours on four dual-core processors with 500 multigrid cycles.
- Using the time-lagged boundary condition, computational expense can be reduced by N_b times.
- New formulation for vorticity confinement has no effect on the distribution of C_p for fixed-wing transonic flow calculations.
- The maximum error for c_l and c_d for was only 1.3%.

Acknowledgments

- The authors would like to thank Professor Chris Allen for his data for our comparison purpose.