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# On Criteria for Closed-Loop Sensitivity Reduction

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Let $\delta x(t)$ be a first-order variation of a system's trajectory $x(t)$ due to a first-order variation $\delta\mu(t)$ of a system's parameter $\mu$. A sufficient condition is derived for a nonlinear closed-loop system to be less sensitive than the nominally equivalent open-loop system, according to a particular integral-square measure of $\delta x(t)$. It is shown that for this sufficient condition to be satisfied the weighting matrix must be of a specific form; this form is also shown to be necessary when one considers the closed-loop sensitivity reduction of arbitrary members of broad classes of linear and nonlinear systems.

## 1. INTRODUCTION AND SUMMARY

The potential benefits of sensitivity reduction by use of feedback are well known and the sensitivity comparison of open-loop and closed-loop systems is a fundamental problem of feedback theory. We consider here the concept of differential *trajectory sensitivity*, the first-order variation $\delta x(t)$ of the system's trajectory (or motion) $x(t)$ due to first-order continuous variations $\delta\mu(t)$ of an internal parameter or external disturbance $\mu(t)$. The closed-loop system (denoted by subscript $c$) will be considered less sensitive than the nominally equivalent open-loop system (denoted by subscript $o$) if

$$\int_{t_0}^{t'} \delta x_0^{\mathrm{T}}(t)\, Z(t)\, \delta x_0(t)\, dt \geqslant \int_{t_0}^{t'} \delta x_c^{\mathrm{T}}(t)\, Z(t)\, \delta x_c(t)\, dt, \qquad \text{all} \qquad t', t' > t_0, \tag{1.1}$$

where $Z(t)$ is a continuously differentiable, nonnegative definite symmetric matrix, and the superscript T denotes matrix transposition.

Trajectory sensitivity is a well-known topic in the theory of differential equations (dependence of solutions on parameters) and is widely used in the technical literature [1]. The criterion (1.1) for *closed-loop sensitivity reduction* was first investigated by Cruz and Perkins [2–5], and their results have been extended and generalized in several directions [6–10]. It was shown in these papers that (1.1) is closely related to the Bode sensitivity function originally formulated in the frequency domain for linear time-invariant feedback systems.

In this paper, Theorem 3.1 gives a sufficient condtion, *the sensitivity inequality* (3.7), for closed-loop sensitivity reduction in the sense of (1.1); this result is similar to those in [2–10]. The main results are Theorem 4.1, which shows that the sensitivity inequality can be satisfied only if $Z$ has a particular form, and Theorem 4.2, which shows that this form, slightly strengthened, is mandatory when considering broad classes of linear and nonlinear systems, such as the class of optimal systems considered in [11–14].

## 2. Preliminaries

Consider a plant described by the differential equation

$$\dot{x} = f(t, x, u, \mu), \qquad x(t_0) = x_0 , \tag{2.1}$$

where the scalar $t$ is the time, $x$ is the state, an $n$-vector, $u$ is the control, an $r$-vector, $r \leqslant n$, and $\mu$ is a vector of continuously time-varying internal parameters and external disturbances. The control function $u$ is derived from a state-feedback control law

$$u(t) = -\, k(t, x). \tag{2.2}$$

The functions $f$ and $k$ are assumed to be continuously differentiable in $t$, $x$, and $\mu$. It is further assumed that the set (2.1, 2.2) has a unique solution $x(t)$, $t \geqslant t_0$, for all initial points $(t_0 , x_0)$ in some $(n + 1)$-dimensional connected region of $(t, x)$-space and for all $\mu(t)$ in a neighborhood of a nominal parameter denoted by $\mu_*(t)$.

Next, consider a small, continuously time-varying variation

$$\Delta\mu(t) = \mu(t) - \mu_*(t)$$

and let

$$\Delta\mu(t) = \epsilon\delta\mu(t),$$

where $\epsilon$ is a nonzero scalar and $\delta\mu$ is a vector of arbitrary fixed finite length. The trajectory $x(t)$ is, in general, a function of $\mu$ and we define the trajectory sensitivity $\delta x$ as the (weak and, in this case, equal to the strong) differential

$$\delta x(\delta\mu) = \lim_{\epsilon\to 0} \frac{1}{\epsilon} [x(\mu_* + \epsilon\delta\mu) - x(\mu_*)]. \tag{2.3}$$

Suppose that in (2.1) $\mu = \mu_*$ and $u(t)$ is a particular function $u_*(t)$ yielding a solution $x_*(t)$ that starts at some nominal initial point $(t_0 , x_0)$. The trajectory variation $\delta x(t)$ due to a parameter variation $\delta\mu(t)$ is given by the linear time-varying differential equation (the variational equation)

$$\delta\dot{x} = f_x \delta x + f_u \delta u + f_\mu \delta_\mu , \qquad \delta x(t_0) = \xi, \tag{2.4}$$

where the matrices of partial derivatives $f_x$, $f_u$, and $f_\mu$ are understood to be evaluated along $\{x_*(t), u_*(t), \mu_*(t)\}$. If $u$ is given by (2.2), then we readily find that

$$\delta u = - k_x \delta x. \tag{2.5}$$

## 3. Conditions for Closed-Loop Sensitivity Reduction

Consider the closed-loop system (2.1, 2.2) with parameters at nominal value, $\mu = \mu_*$, generating the nominal trajectory $x_*(t)$ and control $u_*(t) = - k(t, x_*(t))$. An open-loop system (2.1) with $\mu = \mu_*$ and forced by $u_*(t)$ is clearly ***nominally equivalent*** to the closed-loop system (2.1, 2.2) under nominal deterministic conditions. Our objective is to compare the trajectory sensitivity, for an arbitrary nominal trajectory and with respect to the criterion (1.1), of a nominally equivalent pair of open-loop and closed-loop systems.

Let the subscripts $o$ (to be distinguished from the subscript zero) and $c$ represent quantities of the nominally equivalent open-loop and closed-loop systems, respectively. Then, from (2.4) and (2.5),

$$\delta\dot{x}_o = f_x \delta x_o + f_\mu \delta_\mu , \qquad \delta x_o(t_0) = \xi, \tag{3.1}$$

and

$$\delta\dot{x}_c = (f_x - f_u k_x)\, \delta x_c + f_\mu \delta_\mu , \qquad \delta x_c(t_0) = \xi, \tag{3.2}$$

where $\delta u \equiv 0$ in the open-loop system. The vector $\xi$ represents an error in the initial value $x(t_0) = x_0$. By nominal equivalence, the matrices $f_x$ and $f_\mu$ in (3.1) are identical to those in (3.2).

From (3.1) and (3.2) we have

$$\delta x_o(t) = \delta x_c(t) + v(t), \tag{3.3}$$

where

$$\dot{v} = f_x v + f_u k_x \delta x_c , \qquad v(t_0) = 0; \tag{3.4}$$

in terms of the transition matrix $\Phi(t, \tau)$, i.e.,

$$\frac{d}{dt} \Phi(t, t_0) = f_x \Phi(t, t_0), \qquad \Phi(t_0 , t_0) = I,$$

$v$ is given by

$$v(t) = \int_{t_0}^{t'} \Phi(t, \tau) f_u k_x \, \delta x_c(\tau)\, dt. \tag{3.5}$$

Further, squaring each side of (3.3) with respect to a weighting matrix $Z(t)$ we find that (1.1) holds if and only if

$$\int_{t_0}^{t'} (2\delta x_c^{\mathrm{T}}(t)\, Z(t)\, v(t) + v^{\mathrm{T}}(t)\, Z(t)\, v(t))\, dt \geqslant 0. \tag{3.6}$$

If one wishes (1.1) to hold for all continuous $\delta\mu(t)$ and for all initial errors $\xi$, then (3.6) is not a useful condition since it must be tested for the $\delta x_c(t)$ due to all $\delta\mu(t)$ and $\xi$. In theoretical work it is convenient to replace $\delta x_c(t)$, which is continuously differentiable, by an arbitrary member $z(t)$ of the space $D_1$ of continuously differentiable functions. We then have

THEOREM 3.1. *A sufficient condition for a closed-loop system* (2.1, 2.2) *to be less sensitive than the nominally equivalent open-loop system, in the sense of* (1.1), *with respect to all continuous parameter variations* $\delta\mu(t)$ *and initial errors* $\xi$, *is that for every continuously differentiable* $z(t)$ *the sensitivity inequality*

$$\int_{t_0}^{t'} (2z^{\mathrm{T}} Z\nu + \nu^{\mathrm{T}} Z\nu)\, dt \geqslant 0, \qquad \text{all} \qquad t', t' > t_0, \tag{3.7}$$

*holds, where*

$$\dot{\nu} = f_x \nu + f_u k_x z, \qquad \nu(t_0) = 0. \tag{3.8}$$

We mention in passing that by defining the linear operator

$$Lz = \int_{t_0}^{t} \Phi(t, \tau)\, f_u k_x z(\tau)\, d\tau \tag{3.9}$$

on $D_1$, the sensitivity inequality (3.7) can be written in the more compact form (see [14, Appendix])

$$[I + L]^a\, Z[I + L] \geqslant Z, \tag{3.10}$$

where superscript $a$ denotes the adjoint operator. For linear time-invariant asymptotically stable systems

$$\dot{x} = A(\mu)\, x + B(\mu)\, u + C\mu, \qquad x(0) = x_0, \tag{3.11}$$

$$u = -Kx, \tag{3.12}$$

where $Z$ is taken to be constant, (3.7) reduces to the condition first given by Cruz and Perkins [2, 3]

$$[I + \hat{\Phi}(-j\omega)\, BK]^{\mathrm{T}}\, Z[I + \hat{\Phi}(j\omega)\, BK] \geqslant Z \qquad \text{for all } \omega, \tag{3.13}$$

where $\hat{\Phi}(s)$ is the Laplace transform of $\Phi(t)$. The matrix $[I + \hat{\Phi}(j\omega) BK]$ is a generalized return difference and hence the connection of (1.1) with the Bode sensitivity function.

*Remark 3.1.* The satisfaction of the sensitivity inequalities (2.7), (3.10), and (3.13) is in general not necessary for closed-loop sensitivity reduction in the sense of (1.1) because $\delta x_c(t)$, being the solution of (3.2), is in general not an arbitrary member of $D_1$. On the other hand, whenever a system is capable of generating every $\delta x_c(t)$ in $D_1$, the above sufficient conditions are also necessary. Considering Eq. (3.2) for $\delta x_c(t)$, it is evident that if

$$f_\mu[t, -k(t, x_*(t)), x_*(t), \mu_*(t)]$$

has rank $n$ and (1.1) is to hold for all continuous $\delta\mu(t)$, then for every $\delta x_c(t)$ in $D_1$ there is a continuous $\delta\mu(t)$ given by

$$\delta\mu(t) = f_\mu^{-1}[\delta\dot{x}_c - (f_x - f_u k_x)\,\delta x_c], \tag{3.14}$$

such that (3.2) is satisfied. Similarly, if the system (2.1) is forced by white noise, every continuous $x(t)$ can be realized, and if in addition $f_\mu$ can be solved for $x$, then every continuously differentiable $\delta x_c(t)$ can be generated.

## 4. Results on the Form of $Z$

Henceforth we make the mild and unrestrictive assumption that the pair of matrices $[f_x, f_u\}$ is completely controllable on every interval $[t_0, t]$ where $t_0$ is an arbitrary fixed initial time and $t$ is variable, and the rows of $k_x(t)$ are linearly independent on every such interval. Together these assumptions imply [15]:

$$\left.\begin{array}{c}\text{Along the nominal solution } \{x_*(t), u_*(t), \mu_*(t)\},\\ \Gamma(t) = \displaystyle\int_{t_0}^{t} \Phi(t,\tau) f_u(\tau)\, k_x(\tau)\, k_x^{\mathrm{T}}(\tau) f_u^{\mathrm{T}}(\tau)\, \Phi^{\mathrm{T}}(t,\tau)\, d\tau \\ \text{is nonsingular for all } t > t_0 .\end{array}\right\} \tag{4.1}$$

THEOREM 4.1. *For systems* (2.1, 2.2) *satisfying assumption* (4.1), *the weighting matrix* $Z(t)$ *in the sensitivity inequality* (3.7) *of Theorem 3.1 must be of the form*

$$Z(t) = k_x^{\mathrm{T}}(t)\, M(t)\, k_x(t), \tag{4.2}$$

*where* $M(t)$ *is some continuously differentiable nonnegative definite and symmetric* *matrix. For linear time-invariant systems* (3.11, 3.12) *where $Z$ is constant, $Z$ in the sensitivity inequality* (3.13) *must be of the form*

$$Z = K^{\mathrm{T}} M K \tag{4.3}$$

*where $M$ is some constant nonnegative definite and symmetric matrix.*

*Proof.* We show that unless $Z$ has the form prescribed by the theorem, there exists a continuously differentiable $z(t)$ which violates the sensitivity inequality (3.7). Let $z(t)$ in (3.7) and (3.8) be chosen as

$$z = \zeta + \rho\eta,$$

where $\rho$ is a scalar and $\eta$ is any finite nonzero vector such that

$$k_x(t)\,\eta(t) = 0. \tag{4.4}$$

In (4.4) we assumed that $k_x$ is singular; when $k_x(t)$ is nonsingular, the form (4.2) poses no restriction on $Z(t)$ because we can always set $M(t) = (k_x^T)^{-1} Z(t)\, k_x^{-1}$. Let

$$\alpha(t) = \zeta^{\mathrm{T}}(t)\, Z(t) \int_{t_0}^{t} \Phi(t, \tau)\, f_u k_x \zeta(\tau)\, d\tau,$$

$$\beta(t) = \eta^{\mathrm{T}}(t)\, Z(t) \int_{t_0}^{t} \Phi(t, \tau)\, f_u k_x \zeta(\tau)\, d\tau, \tag{4.5}$$

and

$$\gamma(t) = \left[\int_{t_0}^{t} \Phi(t, \tau)\, f_u k_x \zeta(\tau)\, d\tau\right]^{\mathrm{T}} Z(t) \int_{t_0}^{t} \Phi(t, \tau)\, f_u k_x \zeta(\tau)\, d\tau.$$

Then, in view of (4.4), (3.7) becomes

$$\int_{t_0}^{t'} \alpha(t)\, dt + 2 \int_{t_0}^{t'} \rho(t)\, \beta(t)\, dt + \int_{t_0}^{t'} \gamma(t)\, dt \geqslant 0. \tag{4.6}$$

Since $\beta(t)$ is continuously differentiable we can set in (4.6)

$$\rho(t) = -\,\epsilon\beta(t), \qquad \epsilon > 0,$$

and if $\beta(t)$ is not identically zero on $[t_0, t']$ then the left side of (4.6) can be made negative by a sufficiently large $\epsilon$. Thus $\beta(t) \equiv 0$ on $[t_0, t']$ is a necessary condition for (4.6) to hold. We now set in (4.5)

$$\zeta(\tau) = k_x^{\mathrm{T}}(\tau)\, f_u(\tau)\, \Phi^{\mathrm{T}}(t, \tau)\, \lambda, \qquad \lambda \neq 0,$$

and by using (4.1), (4.5) becomes

$$\beta(t) = \eta^{\mathrm{T}}(t)\, Z(t)\, \Gamma(t)\, \lambda.$$

Since $\Gamma(t)$ is nonsingular and $\lambda$ is an arbitrary nonzero vector, for every nonzero vector $Z(t)\,\eta(t)$ there exists a nonorthogonal nonzero vector $\Gamma(t)\,\lambda$ such that $\beta(t) \neq 0$. Thus for $\beta(t) = 0$ it is necessary that

$$Z(t)\,\eta(t) = 0.$$

This implies, in view of (4.4), that the rows of $Z(t)$ must be in the subspace spanned by the rows of $k_x(t)$. Hence

$$Z(t) = L(t)\,k_x(t) \tag{4.7}$$

for some continuously differentiable matrix $L(t)$. Because of the symmetry of $Z(t)$

$$Z(t) = k_x^{\mathrm{T}} L^{\mathrm{T}}(t),$$

and it must be possible therefore to factor $Z$ in the form of (4.2). When $Z$ and $k_x$ are constant, then from (4.7) $0 = \dot{L}(t)\,k_x$, and since the rows of $k_x$ are linearly independent $\dot{L}(t) = 0$. Hence $L$ and $M$ must be constant and $Z$ must have the form (4.3). Q.E.D.

The theorem shows that the weighting matrix $Z$ in the sensitivity inequalities (3.7), (3.10), and (3.13) cannot be arbitrarily chosen, as is often implied in the literature; these inequalities are in fact meaningless unless $Z$ is of the proper form given by (4.2) and (4.3), i.e.,

$$\int_{t_0}^{t'} (2z^{\mathrm{T}} k_x^{\mathrm{T}} M k_x \nu + \nu^{\mathrm{T}} k_x^{\mathrm{T}} M k_x \nu)\, dt \geqslant 0 \tag{4.8}$$

must replace our main sensitivity inequality (3.7). We remark that since the sensitivity inequality (3.7) is only a sufficient condition for the criterion (1.1), $Z$ in (1.1) may be of a different form then that prescribed by Theorem 4.1. For systems of the type noted in Remark 3.1, however, for which the sensitivity inequality (3.7) is necessary, the theorem applies to $Z$ also in (1.1).

Another application of Theorem 4.1 is in general investigations where one wishes to examine the properties of arbitrary members of an entire class of systems. Clearly, whenever such a class contains one member for which $Z$ is necessarily of the form (4.2), this form is also necessary for the class as a whole. Moreover, we can then strengthen $M$ to be necessarily positive definite. We have

THEOREM 4.2. *Consider the class of closed-loop linear systems* (3.11, 3.12) *or any class generalized from it up to and including the class of nonlinear systems* (2.1, 2.2), *all satisfying assumption* (4.1). *For every member of such a class to* *exhibit closed-loop sensitivity reduction in the sense of* (1.1) *with a* $Z$ *given by the same formula for all members, for all continuous parameter variations* $\delta\mu(t)$ *and all initial errors* $\xi$, *it is necessary that the weighting matrix* $Z(t)$ *have the form* (4.2) *with* $M(t)$ *a positive definite symmetric matrix.*

*Proof.* Consider Remark 3.1. In the linear case $f_\mu$ is

$$[A(\mu) - B(\mu)\, Kx_*(t)]_\mu + C$$

and it is easy to construct an example where this matrix has rank $n$. The class of systems (2.1, 2.2) includes this example as a special case and hence the form (4.2) for $Z$ is mandatory as stated. We prove that $M$ has to be positive-definite by contradiction of (1.1) in at least one example. Let $M$ be non-negative-definite. We construct an example so that $K\delta x_o(t)$ is in the null-space of $M$ and $K\delta x_c(t)$ is not in this space for some $t$. Then the left side of (1.1) is zero and the right side is positive—a contradiction. Consider a second order linear system (3.11, 3.12) with two inputs (i.e., $B$ and $K$ are square), where $C = 0$, satisfying assumption (4.1) and criterion (1.1) with $Z = K^{\mathrm{T}}MK$; for example, if $K$ is such that the performance index

$$I = \tfrac{1}{2} \int_0^\infty (x^{\mathrm{T}}Qx + u^{\mathrm{T}}Ru)\, dt, \qquad Q \geqslant 0, \quad R > 0,$$

is minimized, it is shown in [11] that (1.1) with $Z = K^{\mathrm{T}}RK$ holds in this case. The equations for trajectory sensitivity are

$$\delta\dot{x}_o = A(\mu_*)\, \delta x_o + [A(\mu)\, x_*(t) + B(\mu)\, u_*(t)]_\mu\, \delta\mu, \qquad \delta x_o(0) = \xi, \tag{4.9}$$

$$\delta\dot{x}_c = (A(\mu_*) - B(\mu_*)\, K)\, \delta x_c + [A(\mu)\, x_*(t) + B(\mu)\, u_*(t)]_\mu\, \delta\mu, \qquad \delta x_c(0) = \xi. \tag{4.10}$$

We construct our example so that the matrix $[Ax + Bu]_\mu$ is nonsingular. Assume $M$ is singular and let $\lambda$ be a real vector such that $K\lambda \neq 0$ is in the null space of $M$, and let $\xi$ and $\delta\mu(t)$ be such that $\delta x_o(t) = \lambda$ for all $t \geqslant 0$; then from (4.9), $\delta\mu(t)$ is given by

$$\delta\mu(t) = -\, [Ax + Bu]_\mu^{-1}\, A\lambda. \tag{4.11}$$

Substitution of (4.11) into (4.10) gives

$$\delta\dot{x}_c = A(\delta x_c - \lambda) - BK\delta x_c\,, \qquad \delta x_c(0) = \xi = \lambda,$$

and at $t = 0$

$$\delta\dot{x}_c(0) = -\, BK\lambda.$$

We construct our example so that $B$ and $K$ are of full rank and $BK$ has complex eigenvectors. Then $\lambda$ is not an eigenvector of $BK$ and $BK\lambda \neq 0$. Thus $\delta\dot{x}_c(0)$ is not in the one dimensional null-space of $MK$ and therefore on some interval $MK\delta x_c(t) \neq 0$. Thus for every singular $M$ there exists in this example a $\delta\mu(t)$ and $\xi$ such that the left side of (1.1) is zero while the right side is positive. Q.E.D.

The fact that generally speaking the weighting matrix $Z$ in (1.1) must be of the form (4.2) is not surprising because it is the quantity $k_x\delta x_c = \eta$ that is being fed back. Theorem 4.1 thus states that, in general, the criterion for closed-loop sensitivity reduction should be

$$\int_{t_0}^{t'} \eta_o^{\mathrm{T}} M \eta_o \, dt \geqslant \int_{t_0}^{t'} \eta_c^{\mathrm{T}} M \eta_c \, dt, \qquad M > 0, \qquad \text{all} \quad t' > t_0, \quad (4.12)$$

rather than (1.1).

It is again remarked that particular members of one of the classes of systems covered by the theorem, or entire subclasses, may satisfy (1.1) also with a $Z$ different from that prescribed. For the classes of linear and nonlinear optimal systems considered in [11–14], $Z$ turned out to be of the form (4.2) with $M$ a positive definite matrix related to the specific problem.[1] In view of the proof of Theorem 4.2, it is evident that the theorem applies to these broad classes of optimal systems and therefore the form of $Z$ which emerged in [11–14] in an ad hoc fashion is actually mandatory. Nevertheless, certain subclasses of optimal systems and a numerical example are shown in [11, 13, 16] to satisfy (1.1) with a $Z$ not of the form (4.2). This is, of course, no contradiction: The more restricted the class of systems under consideration, the more general the weighting matrix $Z$ in (1.1) can be.

In conclusion we observe that only nondynamic state-feedback [see (2.2) and (3.12)] has been considered. Does dynamic state-feedback remove the restriction on the form of $Z$? This question and also evidence in the case of the minimum trajectory sensitivity problem [17] suggest that when sensitivity is taken into account the consideration of dynamic state-feedback is legitimate and potentially fruitful.

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