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Active Flutter Control using an Adjoint Method

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Flutter is one of the most frequently encountered problems in aeroelasticity. Flutter control is therefore of great importance. It has been shown that flutter can be adequately modeled using a simple two degree of freedom, spring mass system with forcing from the aerodynamics. In the following paper, control by means of blowing and suction is used to control flutter. This is based on deriving a feedback control law from a linearized model of the aero-structural system. The feedback matrix thus derived is tested on the nonlinear model and is found to control flutter effectively.

I. Introduction

An airplane, by its nature of being, is constructed so that it is as light as possible. The structural design is guided by static and dynamic factors. The more stringent constraints on the structural design are due to dynamic loads, caused by aero-elastic interactions. One of the most commonly encountered problems in aeroelasticity is flutter [1], a term that is used to recognize the transfer of energy from unsteady aerodynamics associated with the surrounding fluid to the wing structure, resulting in rapidly divergent behaviour. If flutter can be controlled at cruise speeds, we can design lighter wings and consequently more efficient airplanes. It is therefore, in the aircraft designer's best interest to design innovative ways in which flutter can be controlled without making the resulting structure too heavy.

There are three important choices to make while designing active control strategies for suppresing flutter. The first is the choice of actuator. In this paper, the actuators we use are jets in the walls through which there is a small mass flow, either by way of blowing or suction. The second is to define a clear control objective.

Finally, we need to design a control law that will make suitable state measurements and drive the actuators so that the desired control objective is achieved. The concept of flow control, as described in this paper, relies heavily on the Adjoint Methods derived by Jameson and his associates over the last few years [2] and [3].

II. Description of Problem

In the present paper we will investigate the aeroelastic behavior and control of a 2-d airfoil whose schematics is shown in Figure II.1. A 2-d airfoil model can be shown to be a fair representation for flutter prediction as shown by Theodorson and Garrik [4] of a straight wing of a large span by giving it the geometric and inertial properties of the cross-section three quarters of the way from the centerline to the wing tip. The equations of motion of this simple system can be shown to be as follows.

$$m\ddot{h} + S_\alpha\ddot{\alpha} + K_h h = -L \tag{II.1}$$
$$S_\alpha\ddot{h} + I_\alpha\ddot{\alpha} + K_\alpha\alpha = M_{ea} \tag{II.2}$$

K_h and K_α are representative of the bending and torsional stiffness of the wing about its elastic axis. The elastic axis is the locus of points about which, if a force is applied, doesn't result in any rotation about

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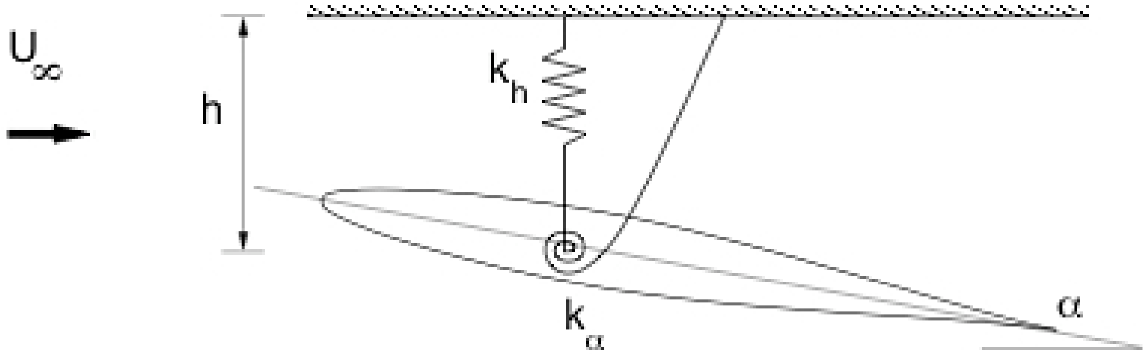

Figure II.1. Typical Section Wing Model Geometry

that point. m and I_α are the mass and moment of inertia of the wing section about the elastic axis. S_α is the coupling term which depends on the relative position of the center of gravity and the elastic axis.

For the present paper we assume the structural properties to be fixed and we have some amount of control of the right hand sides of the equations via blowing and suction. The objective is to find a suitable control law which will modify the aerodynamic terms so as to prevent flutter.

III. Adjoint Based Control Laws

We can define the flutter velocity as that point where we have sustained oscillations of the system. Let us define the state vector $\mathbf{x}$ as follows

$$\mathbf{x} = [\alpha \quad \dot{\alpha} \quad \mathbf{h} \quad \dot{\mathbf{h}}]^{\mathbf{T}} \tag{III.1}$$

The control vector $\mathbf{u}$ is the vector of blowing/suction velocities at the wall. The dynamics of the system is represented by (II.1) and (II.2): this can be rephrased in state space form as follows:

$$\dot{\mathbf{x}} = A\mathbf{x} + \mathbf{B}\mathbf{u} \tag{III.2}$$

Here the matrix B represents the sensitivities of the state vectors with respect to the control variables. This can be obtained by solving the adjoint equations which are formulated in the following sections.

The objective of the problem is to control the system given by (III.1), so that the final value of the state vector is given by

$$\mathbf{x_f} = [\alpha_\mathbf{f} \quad \mathbf{0} \quad \mathbf{h_f} \quad \mathbf{0}]^{\mathbf{T}} \tag{III.3}$$

If this is rephrased as an optimization problem, the objective would be to minimize the following function:

$$f_{obj} = (\mathbf{x} - \mathbf{x_f})^{\mathbf{T}}\mathbf{W}(\mathbf{x} - \mathbf{x_f}) \tag{III.4}$$

where W is a positive definite weighting matrix.

IV. Optimal Feedback Control of Dynamical Systems

One of the most commonly encountered problem in Control Theory is that of finding the optimum path taken by a system to reach a given state. ([5]). Consider a linear system be described by the following dynamical equation

$$\dot{\mathbf{x}} = A\mathbf{x} + B\mathbf{u} \tag{IV.1}$$

where $\mathbf{x}$ is the state of the system at any time and $\mathbf{u}$ is the control input required. An optimal path is one that minimizes a cost function of the form

$$J = \int_0^T \left(\mathbf{x}^{\mathbf{T}}Q\mathbf{x} + \mathbf{u}^{\mathbf{T}}R\mathbf{u}\right) dt + \mathbf{x}^{\mathbf{T}}Q_f\mathbf{x} \tag{IV.2}$$

where Q and Q_f are positive semi-definite and R is positive definite.

The optimal control $\mathbf{u}(\mathbf{t})$ can be found by solving the Riccati equations:

$$-\dot{P}_t = A^T P_t + P_t A - P_t B R^{-1} B^T P_t + Q$$

For all times t not close to the horizon T of the problem of interest the above differential equation reduces to the algebraic Riccati equation:

$$A^T P_{ss} + P_{ss} A - P_{ss} B R^{-1} B^T P_{ss} + Q = 0$$

It is found that the optimal control $\mathbf{u}(\mathbf{t})$ can be represented as a feedback control

$$\mathbf{u}(\mathbf{t}) = K_{ss}\mathbf{x}(\mathbf{t}). \tag{IV.3}$$

The determination of Optimal Feedback Laws for nonlinear systems is generally intractable.

The theory of Optimal Control for systems governed by PDEs may be found in the book by J. L. Lions. Optimal control theory was applied to Aerodynamic Shape Optimization by Jameson and his associates. The complete development is presented in appendices A and B. Equation (B.9) clearly shows that the Adjoint Gradient depends on the flow variables at the boundaries and is hence clearly Feedback Control.

$$\begin{aligned} \delta I &= \int_{\mathcal{B}_\xi} \left(\frac{\partial \mathcal{M}}{\partial \rho q_n} \delta(\rho q_n) \right) d\mathcal{B}_\xi \qquad \text{(IV.4)} \\ &- \int_{\mathcal{B}_\xi} \left(\psi_1 + \psi_2 u + \psi_3 v + \psi_4 w + \psi_5 \left(E + \frac{P}{\rho} \right) \right) \delta \rho q_n d\mathcal{B}_\xi \ . \end{aligned}$$

It should be noted however, that the adjoint equation is solved using the computed state of the flow field and not the measured state. This bypasses the need for global measurements of the flow-field.

V. Computational Framework for the development of a feedback control law for controlling Flutter

A. Flow Solution

The flow is simulated by solving the unsteady Euler equations. The Euler equations are solved using a dual time stepping method, using a third order backward difference formula in time, and a symmetric Gauss Seidel scheme for solving the inner iterations.

B. Aero-Structural Integration

The above mentioned flow simulation code is integrated with a two degree of freedom structural model given by equations (II.1) and (II.2). The coupled aero-structural system is integrated using the Newmark scheme.

C. Choice of state variables for the Flutter Control Problem

For the flutter control problem being studied, the following state vector is used:

$$x = [\alpha, \quad h, \quad \dot{\alpha}, \quad \dot{h}]^T$$

D. Linearization of the Aerodynamic response with respect to the flow variables

In the previous section an optimal feedback control law has been derived for a linear system. We make a claim here, that if flutter is controlled before the onset of instability, all disturbances will be small and hence linearization of the system aerodynamic response with respect to the state and control variables is acceptable.

The matrices A and B in equation (IV.1) depend on a suitable linearization of the system responce. The contribution due to structural component to the state co-efficient matrix is given in equation (II.1) and (II.2). The contribution to the state co-efficient matrix due to the aerodynamic component need to be found using a linearization. This is done in two ways. In the first case the system response is studied for a certain time period and then the derivatives of the aerodynamic terms with respect to the state variables are obtained using a least quares fit to the data thus obtained. In the second case, estimates derived by Theodorsen for thin airfoils in ideal incompressible flow are used. This is clearly presented in the book by Ashley et al.

The B matrix has for two of its columns, the transposes of the gradients of the lift and the moment with respect to the blowing and suction velocities. This is derived using the adjoint equations as detailed in the appendices.

Having derived a linearized system, we can use the theory developed in the preceding section to find the Feedback Control Gain matrix K_{ss}.

E. Backsubstitution of the control law into the nonlinear system

The Feedback Control Gain matrix obtained by solving the optimum linear control problem is then substituted in the full nonlinear coupled aero-elastic system and blowing/suction controls are applied at the walls to see if such control is effective. The results obtained were positive and are explained in the next section.

VI. Results and Conclusions

A. Adjoint Gradients

Like discussed in the last section, the adjoint method is used to find the gradients of lift and moment with respect to the control variables, namely the blowing and suction velocities on the surface. It should be noted that this is done using a steady flow assumption about the nominal rest point of the system. We used a symmetric NACA 0012 section. So for our case, this nominal rest point was at $\alpha = 0$, and $h = 0$. These gradients are shown in figures VI.1 and VI.2.

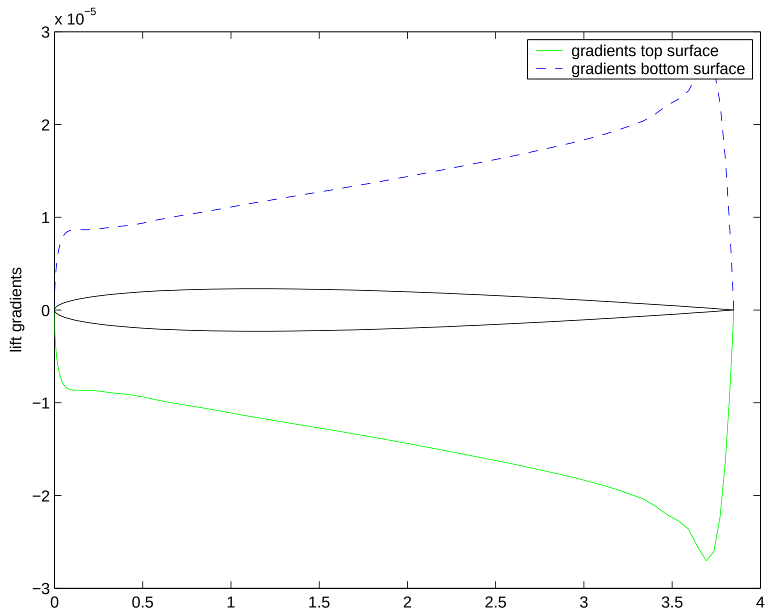

Figure VI.1. Gradient of lift with respect to control mass fluxes

B. Application of Feedback control to the nonlinear flutter problem

The uncontrolled and controlled aero-structural simulations are represented in figures VI.3, VI.4, VI.5, and VI.6. It should be noted that even though, the Feedback law is derived from a linearized model of the system, the control is applied to a complete nonlinear model. Two different methods are used to find the aerodynamic derivatives. It can be seen that the least squares method does a better job than the Thoedorsen method for flutter control. This is obvious because this represents more closely the nonlinear system. The corresponding blowing/suction velocities are shown in figures VI.7 and VI.8.

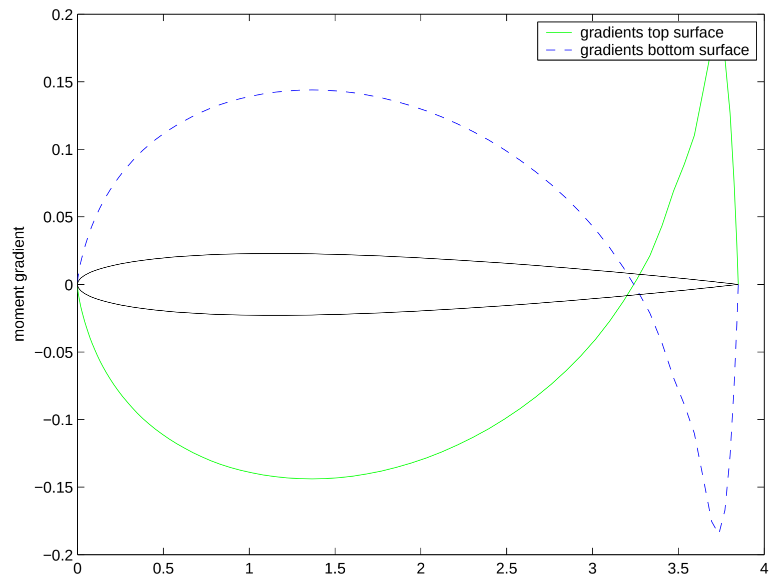

Figure VI.2. Gradient of moment with respect to control mass fluxes

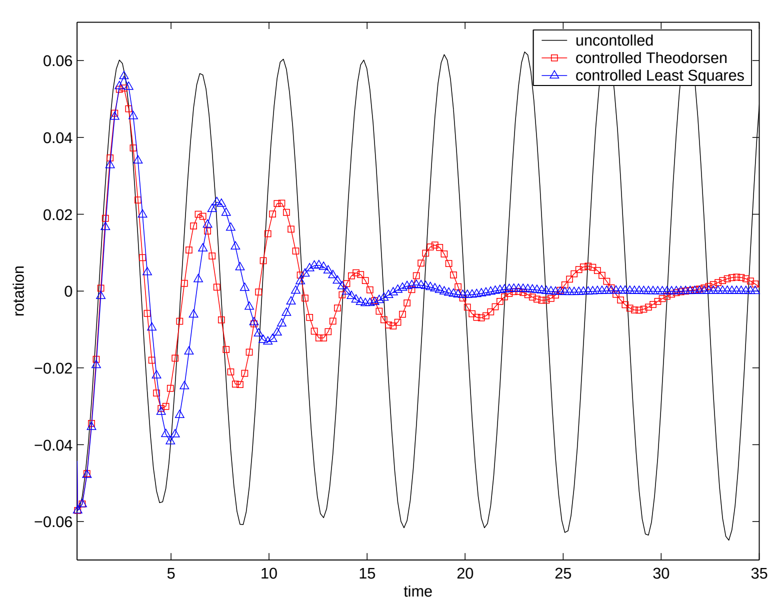

Figure VI.3. Variation of angle of attack with time: controlled and uncontrolled cases

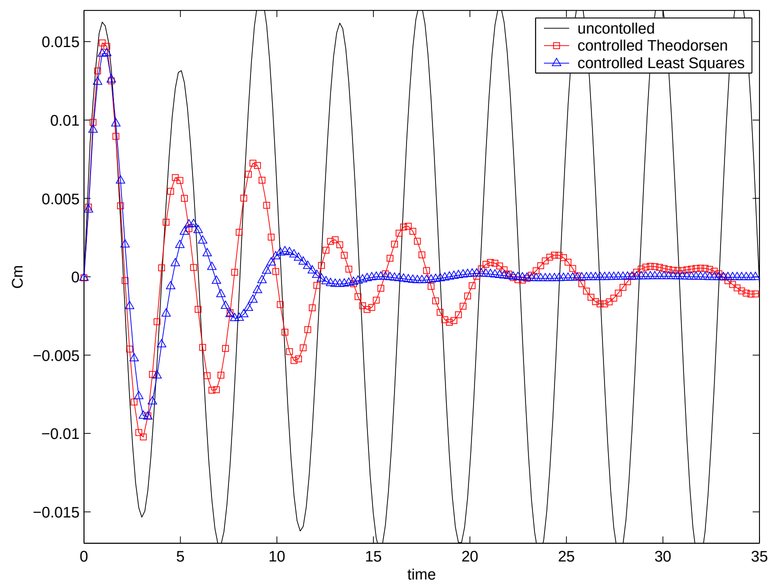

Figure VI.4. Variation of C_m with time: controlled and uncontrolled cases

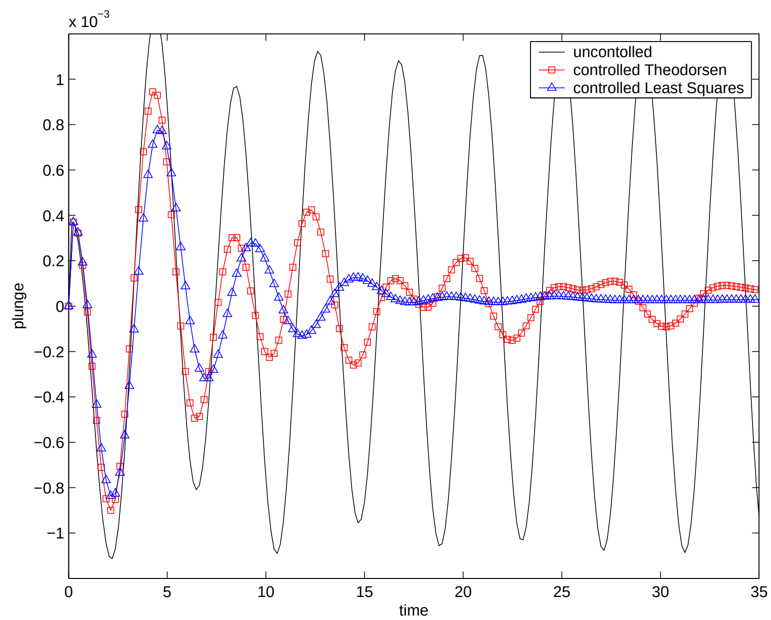

Figure VI.5. Variation of plunge with time: controlled and uncontrolled cases

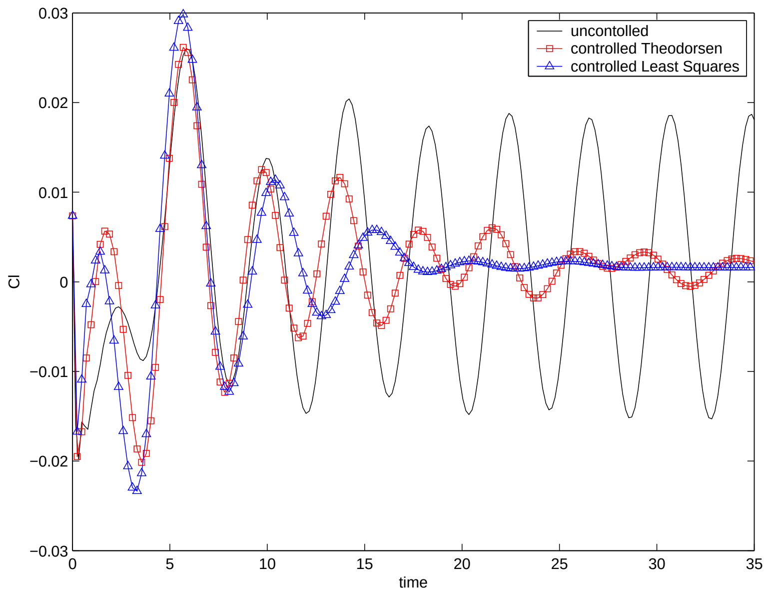

Figure VI.6. Variation of C_l with time: controlled and uncontrolled cases

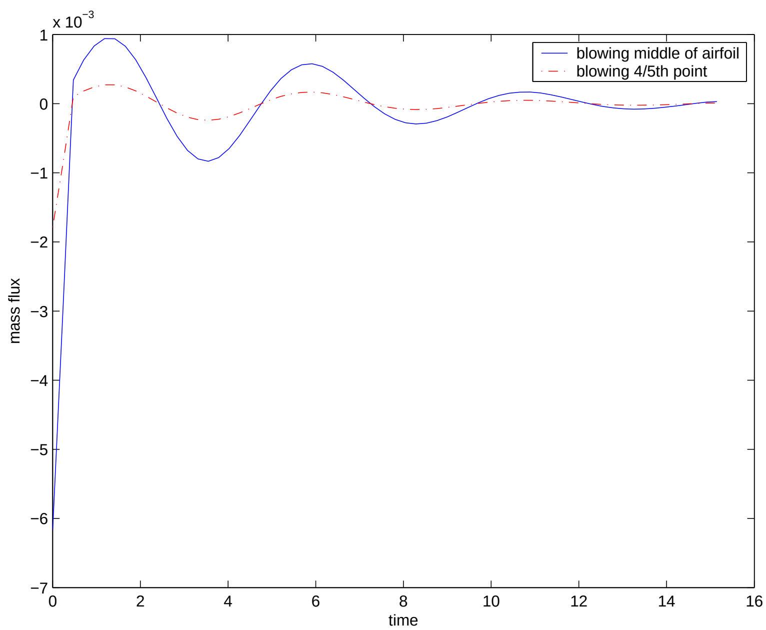

Figure VI.7. Blowing/Suction velocities for the Least Squares model

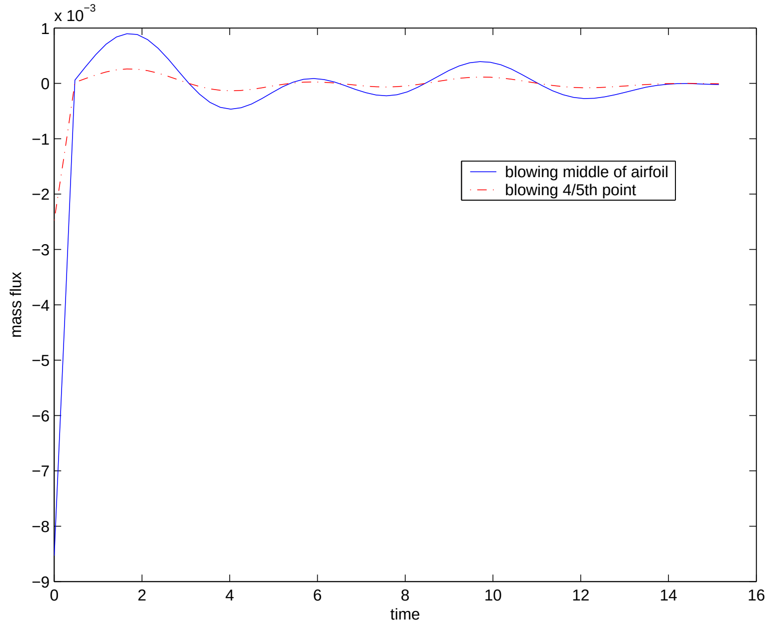

Figure VI.8. Blowing/Suction velocities for the Theodorsen model

C. Time step refinement studies

Just to make sure that the flutter control simulations are correct, the time step for the nonlinear aero-structural solver is made smaller and smaller and the controlled behaviour is observed. It can be seen that the pattern of variation of the angle of attack with time is fairly well predicted by the solver. (See figure).

D. Conclusions

A feedback control law for flutter was derived based on a linearized representation of the aero-structural system and then tested on a completely nonlinear system. This control law has been successfully demonstrated.

References

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[4]T Theodorsen and I. E. Garrick. Mechanism of flutter, a theoretical and experimental investigation of the flutter problem. Technical report, N.A.C.A. Report 685, 1940.

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A. The Euler Equations for Fluid Flow with Blowing at the Walls

In this paper, the fluid flow is modeled using the Euler Equations. The Euler Equations model the behaviour of invicid, compressible fluids. They are

$$\frac{\partial w}{\partial t} + \frac{\partial f_i}{\partial x_i} = 0 \ . \qquad \text{(A.1)}$$

Here x_i represent the cartesian co-ordinate directions, w the state variables, and f_i are the corresponding flux vectors, given by

$$w = (\rho, \rho u, \rho v, \rho w, \rho E) \ , \qquad \text{(A.2)}$$

and,

$$f_i = (\rho u_i, \rho u_i u + \delta_{i1} P, \rho u_i v + \delta_{i2} P, \rho u_i w + \delta_{i3} P, \rho u_i H) \ . \qquad \text{(A.3)}$$

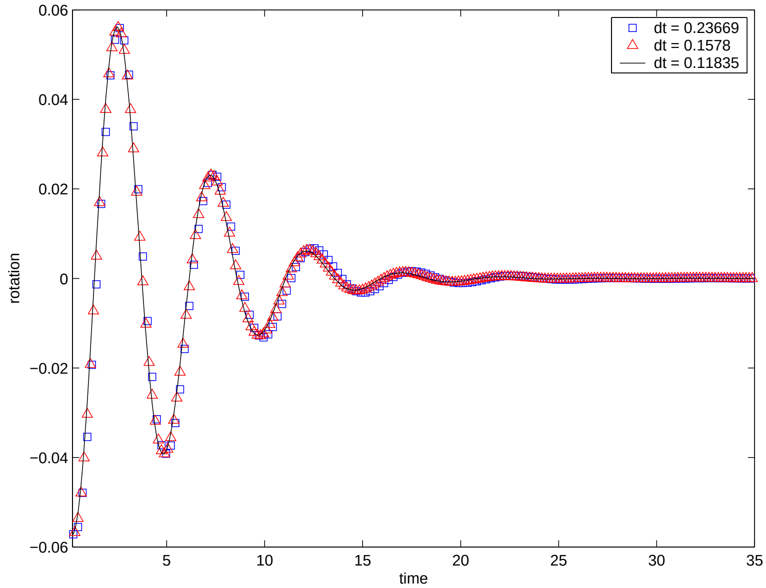

Figure VI.9. Time step refinement studies for the variation of angle of attack with time

The Steady State Euler Equations can be written in weak conservation form as follows

$$\int_{\mathcal{B}} n_i \phi^T f_i(w) d\mathcal{B} = \int_{\mathcal{D}} \frac{\partial \phi^T}{\partial x_i} f_i(w) d\mathcal{D} , \tag{A.4}$$

where ϕ is any test function. If a transformation is made from physical space to computational space, defined by the mapping functions

$$K_{ij} = \left[\frac{\partial x_i}{\partial \varepsilon_j} \right] , \quad J = det(K) , \quad K_{ij}^{-1} = \left[\frac{\partial \varepsilon_i}{\partial x_j} \right] , \tag{A.5}$$

and

$$S = JK^{-1} , \tag{A.6}$$

the Euler Equations (A.4) become

$$\int_{\mathcal{B}_\xi} n_i \phi^T S_{ij} f_j(w) d\mathcal{B}_\xi = \int_{\mathcal{D}_\xi} \frac{\partial \phi^T}{\partial \xi_i} S_{ij} f_j(w) d\mathcal{D}_\xi . \tag{A.7}$$

The boundary conditions for the case where we have blowing or suction at the boundary can then be prescribed in terms of the blowing velocity as follows

$$F_2 = (\rho q_n, \rho q_n u + S_{21} P, \rho q_n v + S_{22} P, \rho q_n w + S_{23} P, \rho q_n H) , \tag{A.8}$$

where ρq_n is the prescribed mass flow at the boundary, initially set to zero in the design problem.

B. The Adjoint Equations for Boundaries with Blowing and Suction: Existence of a Feedback Control Law for the Euler Equations

Let us assume that we are trying to minimize a cost function of the form

$$I = \int_{\mathcal{B}_\xi} \mathcal{M}(w, \rho q_n) d\mathcal{B}_\xi , \tag{B.1}$$

The constraint is given by the Euler Equations (A.7,A.8). Since equation A.7 is true for any test function ϕ, we can choose ϕ to be the adjoint variable ψ. We can then add the constraint given by the Euler Equations to B.1 to form the augmented cost function given by

$$\begin{aligned} I &= \int_{\mathcal{B}_\xi} \mathcal{M}(w, \rho q_n) d\mathcal{B}_\xi \\ &- \int_{\mathcal{B}_\xi} n_i \psi^T S_{ij} f_j(w, \rho q_n) d\mathcal{B}_\xi \\ &+ \int_{\mathcal{D}_\xi} \frac{\partial \psi^T}{\partial \xi_i} S_{ij} f_j(w, \rho q_n) d\mathcal{D}_\xi \ . \end{aligned} \tag{B.2}$$

Taking the first variation of the Cost Function we have

$$\begin{aligned} \delta I &= \int_{\mathcal{B}_\xi} \left(\frac{\partial \mathcal{M}}{\partial w} \delta w + \frac{\partial \mathcal{M}}{\partial \rho q_n} \delta(\rho q_n) \right) d\mathcal{B}_\xi \\ &- \int_{\mathcal{B}_\xi} n_i \psi^T S_{ij} \left(\frac{\partial f_j}{\partial w} \delta w + \frac{\partial f_j}{\partial \rho q_n} \delta(\rho q_n) \right) d\mathcal{B}_\xi \\ &+ \int_{\mathcal{D}_\xi} \frac{\partial \psi^T}{\partial \xi_i} S_{ij} \left(\frac{\partial f_j}{\partial w} \delta w + \frac{\partial f_j}{\partial \rho q_n} \delta(\rho q_n) \right) d\mathcal{D}_\xi \ . \end{aligned} \tag{B.3}$$

Choosing our computational co-ordinate systems so that $\xi_2 = 0$ corresponds to the wing surface, we have

$$F_2 = \begin{bmatrix} \rho q_n \\ \rho q_n u + S_{21} P \\ \rho q_n v + S_{22} P \\ \rho q_n w + S_{23} P \\ \rho q_n E + q_n P \end{bmatrix} . \tag{B.4}$$

Therefore,

$$\begin{aligned} \delta F_2 &= \begin{bmatrix} 1 \\ u \\ v \\ w \\ E + \frac{P}{\rho} \end{bmatrix} \delta(\rho q_n) \\ &+ q_n \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ -u & 1 & 0 & 0 & 0 \\ -v & 0 & 1 & 0 & 0 \\ -w & 0 & 0 & 1 & 0 \\ (\gamma-1)\left(u^2+v^2+w^2\right) - \gamma E & -(\gamma-1)u & -(\gamma-1)v & -(\gamma-1)w & \gamma \end{bmatrix} \delta(w) \\ &+ (\gamma - 1) \begin{bmatrix} 0 \\ S_{21} \\ S_{22} \\ S_{23} \\ 0 \end{bmatrix} \begin{bmatrix} \frac{1}{2}\left(u^2+v^2+w^2\right) & -u & -v & -w & 1 \end{bmatrix} \delta(w) \end{aligned} \tag{B.5}$$

We then have the adjoint equations

$$S_{ij} \frac{\partial f_j}{\partial w}^T \frac{\partial \psi}{\partial w} = 0 \text{ , on } \mathcal{D}_\xi \ , \tag{B.6}$$

and

$$\frac{\partial \mathcal{M}}{\partial w} = \psi^T \frac{\partial F_2}{\partial w} \text{ , on } \mathcal{B}_\xi \ . \tag{B.7}$$

We also observe that

$$\frac{\partial f_j}{\partial \rho q_n} = 0 \text{ , on } \mathcal{D}_\xi \ . \tag{B.8}$$

The expression for the adjoint gradient then becomes

$$\begin{aligned} \delta I &= \int_{\mathcal{B}_\xi} \left(\frac{\partial \mathcal{M}}{\partial \rho q_n} \delta(\rho q_n) \right) d\mathcal{B}_\xi \\ &- \int_{\mathcal{B}_\xi} \left(\psi_1 + \psi_2 u + \psi_3 v + \psi_4 w + \psi_5 \left(E + \frac{P}{\rho} \right) \right) \delta \rho q_n d\mathcal{B}_\xi \ . \end{aligned} \tag{B.9}$$

The gradient is then modified to account for the fact that the nett. mass flow through the boundaries is zero.

From equation (B.7) and (B.9) it is clear that the Adjoint Gradient depends only on the Flow and Adjoint Variables at the Boundaries. Therefore this is clearly a case of Feedback Control, where the feedback is the values of the state variables at the boundary.