Chapter 1

Challenges and Complexity of Aerodynamic Wing Design

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This paper focuses on aerodynamic design methodology. It discusses challenges and complexity of aerodynamic wing design for a transonic aircraft, which arise from the complex nature of flow around the wing. It introduces the concept of automatic design based on computational fluid dynamics (CFD) and the concept of adjoint method. At the conceptual level, the adjoint method largely simplifies the complexity of the design, yet makes the optimization process possible at the practical level. A redesign for a shock-free wing is presented, showing the effectiveness of the automatic design. An extension to include a large scale wing design such as planform optimization is also presented. However, a new cost function needs to be properly chosen and the problem actually leads to multi-objective optimization. Successful planform design results also confirm the robustness of this automatic design strategy.

1.1 Introduction

Designing a good airplane is not trivial. The problem arises both from the complexity of the flow over the airplane and from the need to treat complex multi-disciplinary interactions such as the trade-off between aerodynamic performance and structural weight. Flow past the airplane is governed by a system of highly non-linear equations, and for various problems such as viscous separated flows, their solution is still beyond our reach.

In a relatively simple case, such as a wing in inviscid flow, flow prediction can be performed fast enough that allows interactive calculations to improve the design. However, it requires tremendous experience, even talent, from the designer to achieve desired aerodynamic properties. To provide the necessary guidance on how to change the shape if it is unsatisfactory, it is necessary to integrate the predictive capability into an automatic design method, based on computer optimization.

Another potential of the automatic design method is that it can lead to a truly optimum design by increasing the number of design variations. Traditionally the process of selecting design variations has been carried out by trial and error, relying on the intuition and experience of the designer. With currently available equipment the turn around for numerical simulations is becoming so rapid that it is feasible to examine an extremely large number of variations. It is not at all likely that repeated trials in an interactive design and analysis procedure can lead to a truly optimum design. In order to take full advantage of the possibility of examining a large design space the numerical simulations need to be combined with automatic search and optimization procedures. This can lead to automatic design methods which will fully realize the potential improvements in aerodynamic efficiency.

This paper presents a design methodology for aerodynamic shape optimization based on automatic design method. Section 1.2 discuses a traditional approach for optimization which becomes prohibitive when the number of design variables increase. Section 1.3 states the optimization problem, which is governed by complex system of non-linear partial differential equations. Then it introduces the control theory approach, leading to the simplicity in the conceptual level, yet efficiently reduces the computational cost. Results and extension to multi-disciplinary optimization are demonstrated in section 1.4, validating the concept of automatic design method.

1.2 Optimization and design

The simplest approach to optimization is to define the geometry through a set of design parameters, which may, for example, be the weights α_i applied to a set of shape functions $b_i(x)$ so that the shape is represented as

$$f(x) = \sum \alpha_i b_i(x).$$

Then a cost function I is selected which might, for example, be the drag coefficient at a final lift coefficient, and I is regarded as a function of the parameters α_i. The sensitivities $\frac{\partial I}{\partial \alpha_i}$ may now be estimated by making a small variation $\delta\alpha_i$ in each design parameter in turn and recalculating the flow to obtain the change in I. Then

$$\frac{\partial I}{\partial \alpha_i} \approx \frac{I(\alpha_i + \delta\alpha_i) - I(\alpha_i)}{\delta\alpha_i}.$$

The gradient vector $\frac{\partial I}{\partial \alpha}$ may now be used to determine a direction of improvement. The simplest procedure is to make a step in the negative gradient direction by setting

$$\alpha^{n+1} = \alpha^n - \lambda \frac{\partial I}{\partial \alpha},$$

so that to first order

$$I + \delta I = I - \frac{\partial I^T}{\partial \alpha} \delta \alpha = I - \lambda \frac{\partial I^T}{\partial \alpha} \frac{\partial I}{\partial \alpha}.$$

More sophisticated search procedures may be used such as quasi-Newton methods, which attempt to estimate the second derivative $\frac{\partial^2 I}{\partial \alpha_i \partial \alpha_j}$ of the cost function from changes in the gradient $\frac{\partial I}{\partial \alpha}$ in successive optimization steps. These methods also generally introduce line searches to find the minimum in the search direction which is defined at each step. The main disadvantage of this approach is the need for a number of flow calculations proportional to the number of design variables to estimate the gradient. The computational costs can thus become prohibitive as the number of design variables is increased. In drag minimization problem, the design variables are location of points on the wing surface, in which the number of points is regularly in an order of two thousand and the computational time for one solution is about two to five minutes. Therefore, the traditional way to calculate the gradient is not practical. In the next section, we present the idea of Adjoint method, which dramatically reduces the computational cost.

1.3 Application of control theory

A common problem in aerodynamic optimization is to minimize drag of the airplane

$$I \;=\; C_D \;=\; \frac{1}{q_\infty S_{ref}} \int_{\mathcal{B}} p d\mathcal{S}_{\mathcal{B}}, \tag{1.1}$$

subject to the flow equations

$$\frac{\partial w}{\partial t} + \frac{\partial f_i}{\partial x_i} = 0 \quad \text{in } \mathcal{D}, \tag{1.2}$$

where the state vector w and inviscid flux vector f are described as

$$w = \left\{ \begin{array}{c} \rho \\ \rho u_1 \\ \rho u_2 \\ \rho u_3 \\ \rho E \end{array} \right\} \quad \text{and} \quad f_i = \left\{ \begin{array}{c} \rho u_i \\ \rho u_i u_1 + p\delta_{i1} \\ \rho u_i u_2 + p\delta_{i2} \\ \rho u_i u_3 + p\delta_{i3} \\ \rho u_i H \end{array} \right\}, \tag{1.3}$$

together with proper boundary conditions on $\mathcal{S}_{\mathcal{B}}$. In these definitions, ρ is the density, u_1, u_2, u_3 are the Cartesian velocity components along the Cartesian coordinates (x_1, x_2, x_3), E is the total energy, and δ_{ij} is the Kronecker delta function. The pressure is determined by the equation of state

$$p = (\gamma - 1)\,\rho \left\{ E - \frac{1}{2}\,(u_i u_i) \right\},$$

and the stagnation enthalpy is given by

$$H = E + \frac{p}{\rho},$$

where γ is the ratio of the specific heats.

Symbolically, equations (1.1) and (1.2) can be represented as a problem of minimizing

$$I = I\,(w, \mathcal{S})\,, \tag{1.4}$$

subject to

$$R\,(w, \mathcal{S}) = 0. \tag{1.5}$$

A change in $\mathcal{S}$ results in a change

$$\delta I = \left[\frac{\partial I^T}{\partial w}\right] \delta w + \left[\frac{\partial I^T}{\partial \mathcal{S}}\right] \delta \mathcal{S}, \tag{1.6}$$

and δw is determined from the equation

$$\delta R = \left[\frac{\partial R}{\partial w}\right] \delta w + \left[\frac{\partial R}{\partial \mathcal{S}}\right] \delta \mathcal{S} = 0. \tag{1.7}$$

Generally to completely represent a wing surface, the size of the design parameter $\mathcal{S}$ is in the order of two thousand. If we were to follow the traditional way of calculating gradient described in section 1.2, it would require us to solve equation (1.7) about two thousand times. Unfortunately, equation (1.7) is expensive to solve.

In oder to reduce the computational costs, it turns out that there are advantages in formulating the problems within the framework of the mathematical theory for the control of systems governed by partial differential equations [10]. A wing, for example, is a device to produce lift by controlling the flow, and its design can be regarded as a problem in the optimal control of the flow equations by variation of the shape of the boundary.

Using techniques of control theory, the gradient can be determined indirectly by solving an adjoint equation which has coefficients defined by the solution of the flow equations. The cost of solving the adjoint equation is comparable to that of solving the flow equations. Thus the gradient can be determined with roughly the computational costs of two flow solutions, independently of the number of design variables, which may be infinite if the boundary is regarded as a free surface.

The underlying concepts are clarified by the following abstract description of the adjoint method. Recall equations (1.6) and (1.7). Since the variation δR is zero, it can be multiplied by a Lagrange Multiplier ψ and subtracted from the variation δI without changing the result. Thus equation (1.6) can be replaced by

$$\begin{aligned} \delta I &= \frac{\partial I^T}{\partial w}\delta w + \frac{\partial I^T}{\partial \mathcal{S}}\delta \mathcal{S} - \psi^T \left(\left[\frac{\partial R}{\partial w} \right] \delta w + \left[\frac{\partial R}{\partial \mathcal{S}} \right] \delta \mathcal{S} \right) \\ &= \left\{ \frac{\partial I^T}{\partial w} - \psi^T \left[\frac{\partial R}{\partial w} \right] \right\} \delta w + \left\{ \frac{\partial I^T}{\partial \mathcal{S}} - \psi^T \left[\frac{\partial R}{\partial \mathcal{S}} \right] \right\} \delta \mathcal{S}. \end{aligned} \tag{1.8}$$

Choosing ψ to satisfy the adjoint equation,

$$\left[\frac{\partial R}{\partial w} \right]^T \psi = \frac{\partial I}{\partial w}, \tag{1.9}$$

the first term is eliminated, and we find that

$$\delta I = \mathcal{G} \delta \mathcal{S}, \tag{1.10}$$

where

$$\mathcal{G} = \frac{\partial I^T}{\partial \mathcal{S}} - \psi^T \left[\frac{\partial R}{\partial \mathcal{S}} \right].$$

The advantage is that equation (1.10) is independent of δw, with the result that the gradient of I with respect to an arbitrary number of design variables can be determined without the need for additional flow-field evaluations.

Note that equation (1.5) is a partial differential equation. Thus the adjoint equation (1.9) is also a partial differential equation and determination of the appropriate boundary conditions requires careful mathematical treatment. The detailed derivation of the adjoint equations for compressible Euler and Navier-Stokes equations can be found in [1, 2, 5, 4]

Once equation (1.10) is established, an improvement can be made with a shape change

$$\delta \mathcal{S} = -\lambda \mathcal{G}$$

where λ is positive, and small enough that the first variation is an accurate estimate of δI. The variation in the cost function then becomes

$$\delta I = -\lambda \mathcal{G}^T \mathcal{G} < 0.$$

After making such a modification, the gradient can be recalculated and the process repeated to follow a path of steepest descent until a minimum is reached. In order to avoid violating constraints, such as a minimum acceptable wing thickness, the gradient may be projected into an allowable subspace within which the constraints are satisfied. In this way, procedures can be devised which must necessarily converge at least to a local minimum. It has been established by our research that the descent process can be greatly accelerated by implicitly smoothing the gradient so that it corresponds to the use of a Sobolev inner product [3, 6].

1.4 Case studies

1.4.1 Redesign for shock-free wing

We present a result to show that adjoint method can be used to efficiently redesign the wing using low computational cost. The case chosen is the Boeing 747 wing-fuselage combination, which is considered a good design among existing airplane. Flight condition is at Mach 0.87 and a lift coefficient $C_L = 0.42$. At this transonic speed, flow is very sensitive to small perturbation. If the wing section is not designed properly, shock wave is likely to form, creating drag.

Redesigning to get a shock-free wing is very challenging due to nature of highly sensitive flow. Even with an experienced designer, it is not likely that a shock-free wing can be achieved by a trial-and-error process.

Here, we apply an idea of automatic design and use adjoint method to provide necessary optimization information. Figure 1.1 compares the baseline and

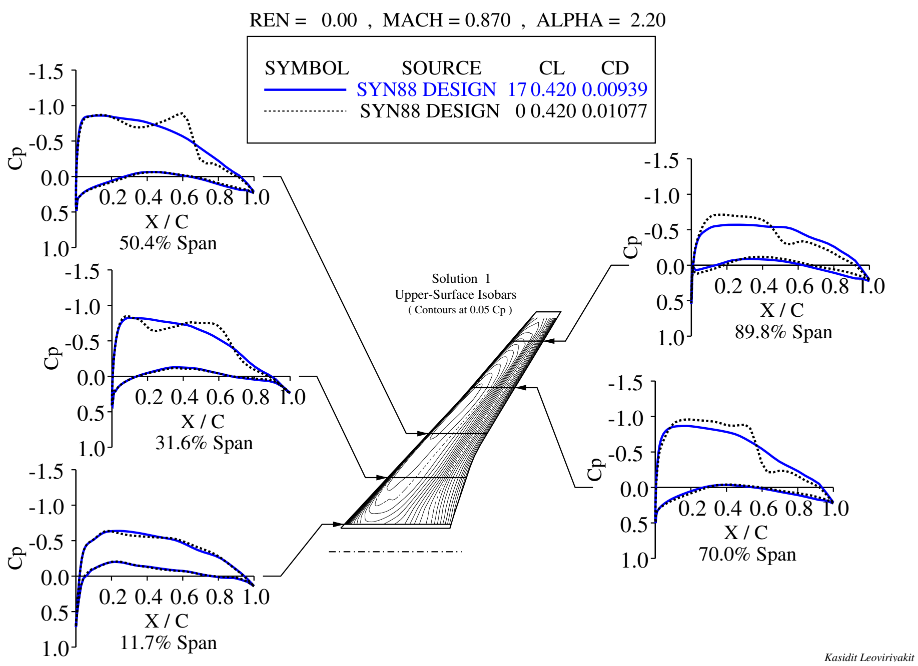

Figure 1.1: Redesign of Boeing 747

the redesign calculations. Pressure distributions at five different span location are shown; dash line for the baseline and solid line for the redesigned pressure distributions. The kinks of the baseline C_p distribution indicate the presence of shock waves. We allow section modification based on adjoint gradient to redesign the sections. Within 17 design cycles, the re-designed sections have smooth C_p distribution, indicating that all shock waves are eliminated. This also leads to a reduction in drag from 108 counts to 94 counts.

For the computational cost, the overall process takes about 20 minutes for 17 design cycles with 2016 mesh points as the design variables. This is only possible with the application of adjoint method, together with fast flow and adjoint solvers. Moreover, with the available of sufficient memory and fast processor speed, the optimization can actually be performed on a laptop.

1.4.2 Planform and aero-structural optimization

The results from section 1.4.1 verify that the adjoint method has been perfected for transonic wing design. The process produces a shock free wing very rapidly.

But for the purpose of drag minimization, shock drag is not the only component of drag. Table 1.1 shows a break-down of the drag for a typical long-range transport aircraft.

Item	C_D	Cumulative C_D
Wing pressure	120 counts	120 counts
	(15 shock,105 vortex)	
Wing friction	45	165
Fuselage	50	215
Tail	20	235
Nacelles	20	255
Other	15	270
	—	
Total	270	

Table 1.1: Break down for drag in counts (1 count = 0.0001)

Clearly, the major portion of drag is vortex drag (roughly 45 % of total drag). It is known that changes in the wing planform (sweepback, span, chord, section thickness, and taper) have the potential to affect the vortex drag. However these also directly affect the structural weight. Therefore, a meaningful result can only be obtained by considering a cost function that accounts for both the aerodynamic characteristics and the structural weight.

In references [7, 8, 9] the cost function is defined as

$$I = \alpha_1 C_D + \alpha_2 \frac{1}{2} \int_{\mathcal{B}} (p - p_d)^2 dS + \alpha_3 C_W,$$

where $C_W \equiv \frac{W}{q_\infty S_{ref}}$ is a dimensionless measure of the wing weight, which can be estimated either from statistical formulas, or from a simple analysis of a representative structure, allowing for failure modes such as panel buckling. The coefficient α_2 is introduced to provide the designer some control over the pressure distribution, while the relative importance of drag and weight are represented by the coefficients α_1 and α_3. They can also be chosen such that the range of the aircraft is maximized by considering the sensitivity of the Breguet range equation

$$R = \frac{V}{sfc}\frac{L}{D}log\frac{W_0 + W_f}{W_0}. \tag{1.11}$$

to variations in $\frac{L}{D}$ and W_0. Here V is the speed, $\frac{L}{D}$ is the lift to drag ratio, sfc is the specific fuel consumption of the engines, W_0 is the landing weight, and W_f is the weight of the fuel burnt. By varying α_1 and α_3 it is possible to calculate the Pareto front of designs which have the least weight for a given drag coefficient, or the least drag coefficient for a given weight.

Figure 1.2 shows the Pareto front obtained from a study of the Boeing 747 wing [8], in which the flow was modeled by the Euler equations. The wing planform and section were varied simultaneously, with the planform defined by six parameters; sweepback, span, the chord at three span stations, and wing thickness. It also shows the point on the Pareto front when $\frac{\alpha_3}{\alpha_1}$ is chosen such that the range of the aircraft is maximized. The optimum wing, as illustrated in figure 1.3, has a larger span, a lower sweep angle, and a thicker wing section in the inboard part of the wing. The increase in span leads to a reduction in the induced drag, while the section shape changes keep the shock drag low. At the same time the lower sweep angle and thicker wing section reduce the structural weight. Overall, the optimum wing improves both aerodynamic performance and structural weight. The drag coefficient is reduced from 108 counts to 87 counts (19%), while the weight factor C_W is reduced from 455 counts to 450 counts (1%).

1.5 Conclusion

The accumulated experience of the last decade suggests that most existing aircraft which cruise at transonic speeds are amenable to a drag reduction of the order of 3 to 5 percent, or an increase in the drag rise Mach number of at least .02. These improvements can be achieved by very small shape modifications, which are too subtle to allow their determination by trial and error methods. The potential economic benefits are substantial, considering the fuel costs of the entire airline fleet. Moreover, if one were to take full advantage of the increase in the lift to drag ratio during the design process, a smaller aircraft could be designed to perform the same task, with consequent further cost reductions. It seems inevitable that some method of this type will provide a basis for aerodynamic designs of the future.

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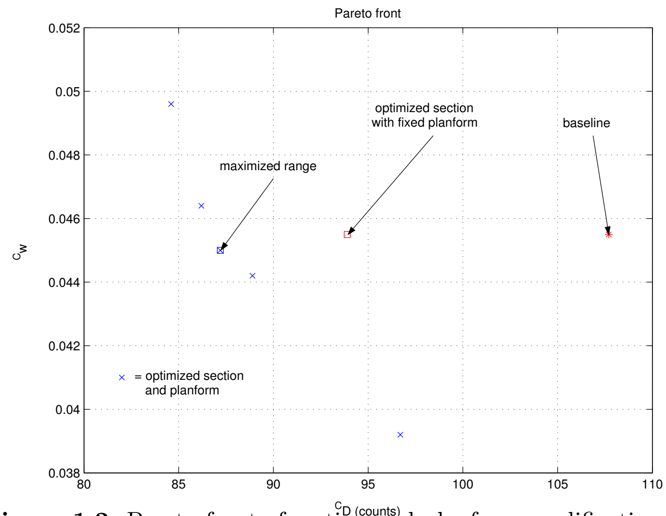

Figure 1.2: Pareto front of section and planform modifications

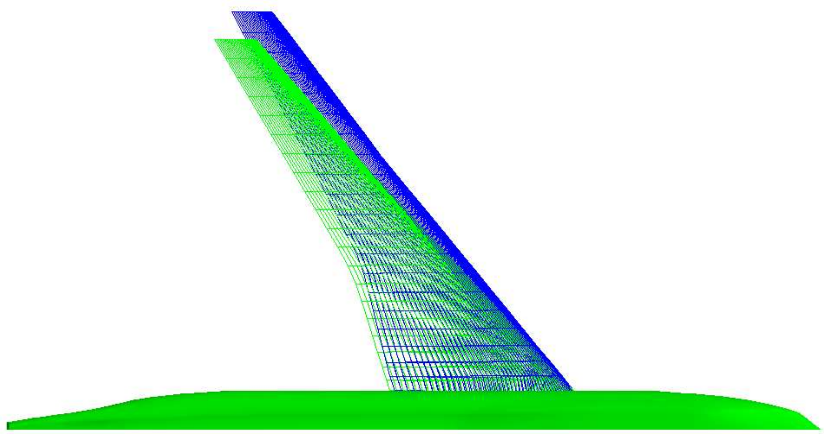

Figure 1.3: Superposition of the baseline (green) and the optimized section-and-planform (blue) geometries of Boeing 747. The redesigned geometry has a longer span, a lower sweep angle, and thicker wing sections, improving both aerodynamic and structural performances. The optimization is performed at Mach .87 and fixed C_L .42, where $\frac{\alpha_3}{\alpha_1}$ is chosen to maximize the range of the aircraft.