An Investigation of the Attainable Efficiency of Flight at Mach One or Just Beyond

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I. Summary

The Breguet range equation indicates that to the first order of approximation range will be maximized by maximizing ML/D , where M is the Mach number and L/D is the lift/drag ratio. Since the drag coefficient increases very rapidly as the Mach number approaches one, this leads to the design of a long range aircraft which fly at high subsonic speeds, typically in the range Mach 0.8 to 0.85, just before the onset of drag rise. The results of the present investigation indicate that a wing swept back at an angle of 50 to 55 degrees and a carefully shaped fuselage it may be possible to delay the onset of drag rise, to Mach one or even to a Mach number slightly greater than one. This would enable the design of a long range aircraft which cruises at a speed just above Mach one, but low enough that there would be no supersonic boom.

II. Introduction

Throughout the history of aviation there has been a continuing quest to fly both faster and further. These goals are not entirely incompatible since a good first estimate of the range (not allowing for the overload of climb and descent) is provided by the Breguet range equation.

$$R = \frac{V}{sfc} \frac{L}{D} \log \frac{W_1}{W_2}$$

Here R is the range, V is the speed, sfc is the specific fuel consumption of the engines, L/D is the lift to drag ratio, and W_1 and W_2 are the initial and final weights allowing for fuel burn. Since the specific fuel consumption increases rather slowly with speed, to a first approximation range is maximized by the aerodynamic design which maximizes VL/D , or equivalently ML/D where M is the Mach number.

It is well known that, at least for conventional configurations, the drag coefficient increases rapidly as the Mach number approaches one due to the formation of shock waves and the consequent wave drag. The onset of drag rise can be delayed by the use of swept back wings, and this has led to the dominant design of the last five decades, a swept wing configuration with strut mounted engines carefully located below the wing or on the rear fuselage to minimize interference, or even in some cases to promote a favorable interaction. In the current state of the art lift/drag ratios of about 20 are attainable in the range Mach 0.8 to 0.85 with a leading edge sweep back angle of around 35 degrees.

On the other hand, because of the wave drag due to both lift and volume, the attainable lift/drag ratio at Mach 2 even for a very slender configuration is in the range of 7.3 (the Concorde) to 9.5 (second generation supersonic transport designs). The Lockheed SR71 achieved a lift to drag ratio of slightly above 6 at Mach 3. These numbers are not sufficient to achieve a range efficiency equivalent to that of subsonic aircraft, with the consequence that it has so far proved impossible to build an economically competitive supersonic transport aircraft, aside from the environmental issues of sonic boom and contamination of the upper atmosphere.

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In the numerous studies of transonic wing designs which the author has conducted during the last decade, it has become apparent that by increasing the sweep back angle to 40 degrees or more (at the wing leading edge), it is certainly possible to delay drag rise beyond Mach 0.9, while maintaining a moderate thickness to chord ratio of around 8 percent, sufficient to prevent excessive wing structure weight, and to provide fuel volume for long range. This has motivated the present investigation of whether it might be possible to delay drag rise to Mach one, or even beyond, by increasing the sweep back angle and using sophisticated shape optimization methods to refine the aerodynamic design. There is a widespread perception that efficient flight at Mach one is impossible. This is partly because it is well known that according to one dimensional analysis, the area of an isentropic stream tube reaches a minimum when the local Mach number is unity (corresponding to a convergent-divergent nozzle). In fact for steady isentropic flow of a perfect gas the condition that the total enthalpy is constant can be written as

$$\frac{c^2}{\gamma - 1} + \frac{q^2}{2} = \frac{c_\infty^2}{\gamma - 1} + \frac{q_\infty^2}{2}$$

where q is the speed, c is the speed of sound, γ is the ration of specific heats, and the subscript ∞ denotes values for the far-field. Moreover,

$$c^2 = \frac{\gamma p}{\rho}$$

where p is the pressure and ρ is the density, and p varies as ρ^γ . Accordingly we can write

$$\frac{\rho}{\rho_\infty} = \left(\frac{c^2}{c_\infty^2} \right)^{\frac{1}{\gamma-1}} = \left(1 + \frac{\gamma-1}{2} M_\infty^2 \left(1 - \frac{q^2}{q_\infty^2} \right) \right)^{\frac{1}{\gamma-1}}$$

Differentiating with respect to q

$$\frac{\partial \rho}{\partial q} = -\frac{\rho q}{c^2}$$

and

$$\frac{\partial}{\partial q} (\rho q) = \rho \left(1 - \frac{q^2}{c^2} \right)$$

This is zero when the local Mach number $M = 1$, corresponding to a maximum of the mass flow ρq which is zero when $q = 0$, and at the limiting speed when $\rho = 0$. Denoting the local values when $M = 1$ by the superscript *,

$$q^* = c^* = \frac{1 + \frac{\gamma-1}{2} M_\infty^2}{1 + \frac{\gamma-1}{2}}$$

and

$$\frac{\rho^* q^*}{\rho_\infty q_\infty} = \frac{1}{M_\infty} \left(\frac{1 + 0.2 M_\infty^2}{1.2} \right)^3 \quad \text{if } \gamma = 1.4$$

Recognizing that in the flow over an object at Mach one the stream tubes cannot be compressed, with the consequence that the disturbance of the flow must have an infinite lateral extension. Whitcomb proposed the celebrated area rule, that the drag coefficient at Mach one will be minimized by enforcing a smooth variation of the total cross-sectional area. This was dramatically confirmed with the flight testing of the Convair F102. The transonic area rule in its basic form is too simple (as Whitcomb himself was well aware), because it does not take account of the interaction of effects due to volume and lift. Suppose, moreover, that equations (2-6) are applied to a two dimensional flow through a throat, but that a constant transverse velocity w_∞ is added, as in the theory of an infinite swept wing. Corresponding to 45 degrees of sweep, for example, the volume in the far-field could be

$$u_\infty = \frac{c_\infty}{\sqrt{2}}; \quad v_\infty = 0; \quad w_\infty = \frac{c_\infty}{\sqrt{2}}; \quad M_\infty = 1$$

Then the total Mach number at the throat will be in excess of one.

If one considers two dimensional supersonic flow over an airfoil with a blunt leading edge, there will be a detached bow shock followed by a subsonic pocket. The flow rapidly expands around the leading edge,

becoming supersonic, and then it typically continues to accelerate all the way to the trailing edge as it follows the curved surface in a Prandtl-Meyer expansion, finally recompressing through a shock wave at the trailing edge. This is accompanied by a steadily decreasing pressure over the rear part of the profile with the consequence of a very large drag coefficient.

The approach taken in this work is to shape the fuselage very carefully to generate expansion waves that counter compression waves over the wing, and in particular cause the shock waves to move forward from the trailing edge over the inboard part of the wing. Then advanced shape optimization techniques, as outlined in the subsequent sections, are used to generate a typical transonic pressure distribution over the wing, with shock waves near the 60 percent chord location. The Mach numbers upstream of the shock waves are quite large, but because the shock waves are oblique at roughly the sweep back angle, they are quite weak.

Experimental data has been obtained in the past that suggests that drag rise can be delayed until Mach one with a swept wing mounted on a "coke-bottle" fuselage. One such result, for a non-lifting wing with a bi-circular section was given by Hoerner.¹ The results of the present investigation are presented in Section 3. Two different wing fuselage combinations have been designed and analyzed using the Reynolds averaged Navier Stokes (RANS) equations. Because the optimization should lead to attached flows, the Baldwin-Lomax turbulence model is considered sufficient.

The first configuration, with a leading edge sweepback angle of 50 degrees, has been designed for a minimum drag at Mach 1.0, while the second configuration, with the leading edge sweep back angle of 54 degrees, has been designed for minimum drag in the range Mach 1.05 to 1.10. The results confirm that drag rise for the wing can apparently be delayed beyond Mach one at a useful lift coefficient in excess of .30

III. Optimization Methodology

The shape optimization methodology used in this work is based on the theory of optimal control of systems governed by partial differential equations, where the control is by varying the shape of the boundary,^{2,4} the gradient of a cost function, in this case the drag coefficient with respect to the shape is obtained by solving the adjoint equation. An outline of the procedure is as follows. Suppose that the cost function are functions of the state variables, w , and the control variables, which may be represented by the function, $\mathcal{F}$, say. Then

$$I = I(w, \mathcal{F}),$$

and a change in $\mathcal{F}$ results in a change

$$\delta I = \frac{\partial I^T}{\partial w} \delta w + \frac{\partial I^T}{\partial \mathcal{F}} \delta \mathcal{F}, \quad (1)$$

in the cost function. Using control theory, the governing equations for the state variables are introduced as a constraint in such a way that the final expression for the gradient does not require re-evaluation of the state. In order to achieve this, δw must be eliminated from equation 1. Suppose that the governing equation R which expresses the dependence of w and $\mathcal{F}$ within the domain D can be written as

$$R(w, \mathcal{F}) = 0 \quad (2)$$

Then δw is determined from the equation

$$\delta R = \left[\frac{\partial R}{\partial w} \right] \delta w + \left[\frac{\partial R}{\partial \mathcal{F}} \right] \delta \mathcal{F} = 0 \quad (3)$$

Next, introducing a Lagrange Multiplier ψ , we have

$$\begin{aligned} \delta I &= \frac{\partial I^T}{\partial w} \delta w + \frac{\partial I^T}{\partial \mathcal{F}} \delta \mathcal{F} - \psi^T \left(\left[\frac{\partial R}{\partial w} \right] \delta w + \left[\frac{\partial R}{\partial \mathcal{F}} \right] \delta \mathcal{F} \right) \\ \delta I &= \left(\frac{\partial I^T}{\partial w} - \psi^T \left[\frac{\partial R}{\partial w} \right] \right) \delta w + \left(\frac{\partial I^T}{\partial \mathcal{F}} - \psi^T \left[\frac{\partial R}{\partial \mathcal{F}} \right] \right) \delta \mathcal{F} \end{aligned}$$

Choosing ψ to satisfy the adjoint equation

$$\left[\frac{\partial R}{\partial w} \right]^T \psi = \frac{\partial I}{\partial w} \quad (4)$$

the first term is eliminated and we find that

$$\delta I = \mathcal{G} \delta \mathcal{F} \quad (5)$$

where

$$\mathcal{G} = \frac{\partial I^T}{\partial \mathcal{F}} - \psi^T \left[\frac{\partial R}{\partial \mathcal{F}} \right] \quad (6)$$

This process allows for elimination of the terms that depend on the flow solution with the result that the gradient with respect with an arbitrary number of design variables can be determined without the need for additional flow field evaluations.

After taking a step in the negative gradient direction, the gradient is recalculated and the process repeated to follow the path of steepest descent until a minimum is reached. In order to avoid violating constraints, such as the minimum acceptable wing thickness, the gradient can be projected into an allowable subspace within which the constraints are satisfied. In this way one can devise procedures which must necessarily converge at least to a local minimum and which can be accelerated by the use of more sophisticated descent methods such as conjugate gradient or quasi-Newton algorithms. There is a possibility of more than one local minimum, but in any case this method will lead to an improvement over the original design.

The derivation of the adjoint equation and boundary conditions for optimization using the Reynolds averaged Navier-Stokes equations is presented in detail in Jameson.⁴

A key issue for successful implementation of the continuous adjoint method is the choice of an appropriate inner product for the definition of the gradient. It turns out that there is an enormous benefit from the use of a modified Sobolev gradient, which enables the generation of a sequence of smooth shapes. This can be illustrated by considering the simplest case of a problem in calculus of variations.

Choose $y(x)$ to minimize

$$I = \int_a^b F(y, y') dx$$

with fixed end points $y(a)$ and $y(b)$. Under a variation $\delta y(x)$,

$$\begin{aligned} \delta I &= \int_a^b \left(\frac{\partial F}{\partial y} \delta y + \frac{\partial F}{\partial y'} \delta y' \right) dx \\ &= \int_a^b \left(\frac{\partial F}{\partial y} - \frac{d}{dx} \frac{\partial F}{\partial y'} \right) \delta y dx \end{aligned}$$

Thus defining the gradient as

$$g = \frac{\partial F}{\partial y} - \frac{d}{dx} \frac{\partial F}{\partial y'}$$

and the inner product as

$$(u, v) = \int_a^b u v dx$$

we find that

$$\delta I = (g, \delta y)$$

Then if we set

$$\delta y = -\lambda g, \quad \lambda > 0$$

we obtain a improvement

$$\delta I = -\lambda(g, g) \leq 0$$

unless $g = 0$, the necessary condition for a minimum. Note that g is a function of y, y', y'' ,

$$g = g(y, y', y'')$$

In the case of the Brachistrone problem, for example

$$g = -\frac{1 + y'^2 + 2yy''}{2(y(1 + y'^2))^{3/2}}$$

Now each step

$$y^{n+1} = y^n - \lambda^n g^n$$

reduces the smoothness of y by two classes. Thus the computed trajectory becomes less and less smooth, leading to instability.

In order to prevent this we can introduce a modified Sobolev inner product

$$\langle u, v \rangle = \int (uv + \epsilon u' v') dx$$

where ϵ is a parameter that controls the weight of the derivatives. If we define a gradient $\bar{g}$ such that

$$\delta I = \langle \bar{g}, \delta y \rangle$$

Then we have

$$\begin{aligned} \delta I &= \int (\bar{g} \delta y + \epsilon \bar{g}' \delta y') dx \\ &= \int (\bar{g} - \frac{\partial}{\partial x} \epsilon \frac{\partial \bar{g}}{\partial x}) \delta y dx \\ &= (g, \delta y) \end{aligned}$$

where

$$\bar{g} - \frac{\partial}{\partial x} \epsilon \frac{\partial \bar{g}}{\partial x} = g$$

and $\bar{g} = 0$ at the end points. Thus $\bar{g}$ is obtained from g by a smoothing equation.

Now the step

$$y^{n+1} = y^n - \lambda^n \bar{g}^n$$

gives an improvement

$$\delta I = -\lambda^n \langle \bar{g}^n, \bar{g}^n \rangle$$

but y^{n+1} has the same smoothness as y^n , resulting in a stable process.

In applying control theory for aerodynamic shape optimization, the use of a Sobolev gradient is equally important for the preservation of the smoothness class of the redesigned surface, and we have employed it to obtain all the results in this study. With this approach there is no need to parametrize the geometry. Instead it is treated as a free surface, with shape modifications controlled by movement of the surface mesh points.

A. Outline of the Design Process

The design procedure can finally be summarized as follows:

1. Solve the flow equations for ρ, u_1, u_2, u_3, p .
2. Solve the adjoint equations for ψ subject to appropriate boundary conditions.
3. Evaluate $\mathcal{G}$ and calculate the corresponding Sobolev gradient $\bar{\mathcal{G}}$.
4. Project $\bar{\mathcal{G}}$ into an allowable subspace that satisfies any geometric constraints.
5. Update the shape based on the direction of steepest descent.
6. Return to 1 until convergence is reached.

IV. Results

The results presented in this section have been obtained with the author's code SYN107, which implements the adjoint based optimization method described in the previous section for the RANS equations with a Baldwin-Lomax turbulence model. The configurations which were studied consisted of wing-fuselage combinations. Fuselage mounted engines were simulated by bumps on the rear fuselage. These were sized to give a good match in the results with calculations including nacelles. All the calculations were performed on C-H meshes with $256 \times 64 \times 48$ cells. These were automatically generated by SYN107, and the height of the first mesh cell above the wing surface was set to correspond to a value of the dimensionless coordinate of y^+ in the range of 1, with a Reynolds number of 30 million based on root chord.

Analysis calculations require 300 multigrid cycles for convergence of the lift and drag coefficients. A typical optimization run uses 240 multigrid cycles for the initial flow solution, 120 cycles for the initial adjoint solution, and then only 20 cycles after each shape modification for both flow and adjoint solutions. The cost function in the optimization calculations was the drag coefficient of the exposed wing. The lift coefficient was constrained to assume a specified value by adjusting the angle of attack after each multigrid cycle of the flow calculations. The thickness to chord ratio was also constrained to be no smaller than a specified value by blocking shape modifications which would reduce the thickness. Shape modifications during the optimization were limited to the wing, but the fuselage was shaped prior to the optimization to produce favorable interaction with the wing.

The first configuration, Xjet-model D, was designed to give minimum drag at Mach 1.0 with a lift coefficient of 0.3 on the exposed wing (approximately 0.34 for the wing-fuselage combination). The leading edge sweep back angle is 50 degrees, and the thickness/chord ration varies between 10 percent at the root and 7 percent at the tip. The drag coefficient of the optimized wing is 0.00889 or 88.9 drag counts (47.6 counts of pressure drag and 41.3 counts of skin friction). The pressure distribution on the wing is typical of a well designed supercritical wing, as can be seen in Figures (1,2,3). There is a weak lambda shock pattern on the upper surface, and no perceptible shock on the lower surface. The section at the 40 percent span station is displayed in Figure 4. Finally Figure 5 shows the variation of the drag coefficient in the range Mach 0.9 to 1.05 at a fixed lift coefficient of 0.30. It can be seen that the drag increases slowly up to Mach one and more rapidly beyond Mach one.

Figure 1. Pressure distribution on the wing

Figure 2. Pressure distribution on the upper surface of the wing

The second configuration, Xjet-model E, is the result of a two point optimization for minimum average drag at Mach numbers of 1.05 and 1.10, with CL fixed at 0.33 on the exposed wing, with some further refinement at Mach 1.10. In order to accommodate the higher Mach numbers the leading edge sweepback angle has been increased to 54 degrees, and the span reduced to prevent the wing becoming excessively slender. In this case the drag coefficient of the optimized wing is 135.3 counts at Mach 1.05 (97.1 counts of pressure drag and 38.2 counts of skin friction) and 142.7 counts at Mach 1.10 (105.7 counts of pressure drag and 37.0 counts of skin friction). The increase in drag comparison with the first configuration is partly due to the increase in CL and reduction in span, with a consequent increase in the induced drag. It can be seen from Figures 6,7,8 that the pressure distribution on the wing remains quite benign. The wing thickness is comparable to the first configuration, as can be seen from Figure 9. Finally Figure 10 shows the variation in drag in the range Mach 0.85-1.15 at a lift coefficient of 0.33. It can be seen that there is a plateau between Mach 1.05 and 1.10 before the drag rise begins to increase more rapidly.

As an indication of the range that might be attained with these configurations the Brequet formula has been applied, assuming a take-off weight of 100000 pounds, a payload of 2000 pounds, and an initial fuel load of 42500 pounds, including an allowance of 3000 pounds of reserve fuel. For the first configuration, Xjet-model D assuming a specific fuel consumption of 0.695, CD wing 105 counts at CL 0.33 (CL aircraft 0.375), CD for the rest of the aircraft of 150 counts, the estimated range is 6095 nautical miles at Mach 1.0. For the second configuration, Xjet-model E, assuming sfc equal to 0.725, CD wing 142 counts at CL of 0.33 (CL aircraft 0.375), and CD for the rest of the aircraft 155 counts, the estimated range is 5518 nautical miles at Mach 1.10.

V. Conclusions

The preliminary results of this investigation suggest that fairly efficient lift to drag ratios, perhaps sufficient for trans-Pacific flight, may be attainable for cruising speeds in the range Mach 1.0 to 1.1. At these speeds there should be no perceptible sonic boom on the ground. The computational results should be interpreted with caution because of the singular nature of flows at Mach One, including the fact that the stand off distance of the bow wave would approach the far-field. For the completed paper it is also

COMPARISON OF LOWER SURFACE CONTOURS

X-JET : MODEL D

REN = 30.00 , MACH = 1.000

(Contours at 0.05 C_p)

Figure 3. Pressure distribution on the lower surface of the wing

planned to carry out simulations for the complete configurations, including engine nacelles and tail, in order to determine the extent to which drag rise can be alleviated for the other components of the aircraft. In order to reduce the level of uncertainty it is planned to use more than one flow solver for the simulations. If the target aerodynamic performance can be achieved, several major issues remain to be addressed in the design of a feasible aircraft, including the location and retraction of the undercarriage, the design of a high lift system for a highly swept wing, and stability and control. A very high sweepback angle has been successfully used in the past, for example 60 degrees on the English Electric Lightning. Also modern fly-by-wire systems have been demonstrated to provide adequate stability and control for extreme configurations such as the Lockheed F117 and Northrop B2.

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Figure 4. Wing section at 40% of span

Figure 5. Drag rise

COMPARISON OF CHORDWISE PRESSURE DISTRIBUTIONS

X-JET : MODEL E

REN = 30.00 , MACH = 1.100

Figure 6. Pressure distribution on the wing

COMPARISON OF UPPER SURFACE CONTOURS

X-JET : MODEL E

REN = 30.00 , MACH = 1.100

(Contours at 0.05 Cp)

Figure 7. Pressure distribution on the upper surface of the wing

COMPARISON OF LOWER SURFACE CONTOURS
X-JET : MODEL E
REN = 30.00 , MACH = 1.100
(Contours at 0.05 Cp)

Figure 8. Pressure distribution on the lower surface of the wing

FOILDAGN
Airfoil Geometry

Figure 9. Wing section at 40% of span

Figure 10. Drag rise