NLF Airfoil and Wing Design by Adjoint Method and Automatic Transition Prediction

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Outline

- Design via Control Theory
- Automatic Transition Prediction and Coupling with RANS solver
- 2D Airfoil and 3D Wing Design
- Conclusion
- Acknowledgment

Design via Control Theory

Automatic Design Based on Control Theory

- The current approach used in this study is a gradient-based numerical optimization technique based on the control theory.
- Regard the wing as a device to generate lift (with minimum drag) by controlling the flow.
- Apply theory of optimal control of systems governed by PDEs (Lions) with boundary control (the wing shape).
- Merge control theory and CFD.
- Find the Frechet derivative (infinite dimensional gradient) of a cost function with respect to the shape by solving the adjoint equation in addition to the flow equation.
- Modify the shape in the sense defined by the smoothed gradient.
- Repeat until the performance value approaches an optimum.

Gradient-based Optimization

Let **x** be the current design point. We want to find a direction **d** and step size $\alpha > 0$ such that $f(\mathbf{x} + \alpha \mathbf{d}) < f(\mathbf{x})$. Expand f about **x** using Taylor's expansion

$$f(\mathrm{x}+\alpha \mathrm{d}) = f(\mathrm{x}) + \alpha \nabla f(\mathrm{x})^T \mathrm{d} + O(\alpha^2)$$

The change in *f* is

$$\begin{aligned} \delta f &= f(\mathrm{x}+\alpha \mathrm{d}) - f(\mathrm{x}) \\ &= \alpha \nabla f(\mathrm{x})^T \mathrm{d} + O(\alpha^2) \end{aligned}$$

For small enough α, we have

$$\delta f \approx \alpha \nabla f(\mathrm{x})^T \mathrm{d}$$

Gradient-based Optimization

For a reduction in f or $\delta f < 0$, we want to choose a *descent direction such that*

$$\nabla f(\mathbf{x})^T \mathbf{d} < 0$$

and then update new design by

$$\mathbf{x}_{k+1} = \mathbf{x}_k + \alpha_k \mathbf{d}_k$$

For a steepest descent method, **d** is chosen to be

$$\mathbf{d} = -\nabla f(\mathbf{x})$$

Gradient Calculation: Adjoint approach

Let ***I*** be the cost (or objective) function

$$I = I\left(w, \mathcal{F}\right)$$

where

w = flow field variables
$\mathcal{F}$ = grid variables

The first variation of the cost function is

$$\delta I = \frac{\partial I}{\partial w}^{T} \delta w + \frac{\partial I}{\partial \mathcal{F}}^{T} \delta \mathcal{F}$$

The flow field equation and its first variation are

$$R\left(w, \mathcal{F}\right) = 0$$

$$\delta R = 0 = \left[\frac{\partial R}{\partial w}\right] \delta w + \left[\frac{\partial R}{\partial \mathcal{F}}\right] \delta \mathcal{F}$$

Gradient Calculation: Adjoint approach

Introducing a Lagrange Multiplier, Ψ, and using the flow field equation as a constraint

$$\delta I = \frac{\partial I}{\partial w}^T \delta w + \frac{\partial I}{\partial \mathcal{F}}^T \delta \mathcal{F} - \psi^T \left\{ \left[\frac{\partial R}{\partial w}\right] \delta w + \left[\frac{\partial R}{\partial \mathcal{F}}\right] \delta \mathcal{F} \right\}$$

$$= \left\{ \frac{\partial I}{\partial w}^T - \psi^T \left[\frac{\partial R}{\partial w}\right] \right\} \delta w + \left\{ \frac{\partial I}{\partial \mathcal{F}}^T - \psi^T \left[\frac{\partial R}{\partial \mathcal{F}}\right] \right\} \delta F$$

By choosing Ψ such that it satisfies the adjoint equation

$$\left[\frac{\partial R}{\partial w}\right]^T \psi = \frac{\partial I}{\partial w},$$

We have

$$\delta I = \left\{ \frac{\partial I}{\partial \mathcal{F}}^T - \psi^T \left[\frac{\partial R}{\partial \mathcal{F}}\right] \right\} \delta \mathcal{F}$$

This reduces the gradient calculation for an arbitrarily large number of design variables to
=> ONE FLOW SOLUTION + ONE ADJOINT SOLUTION

Gradient Smoothing

In general, the gradient G obtained from equation (5) is of a lower smoothness class than the shape *S*. Hence it is important to restore the smoothness. This can be achieved by replacing G by $\bar{\mathcal{G}}$

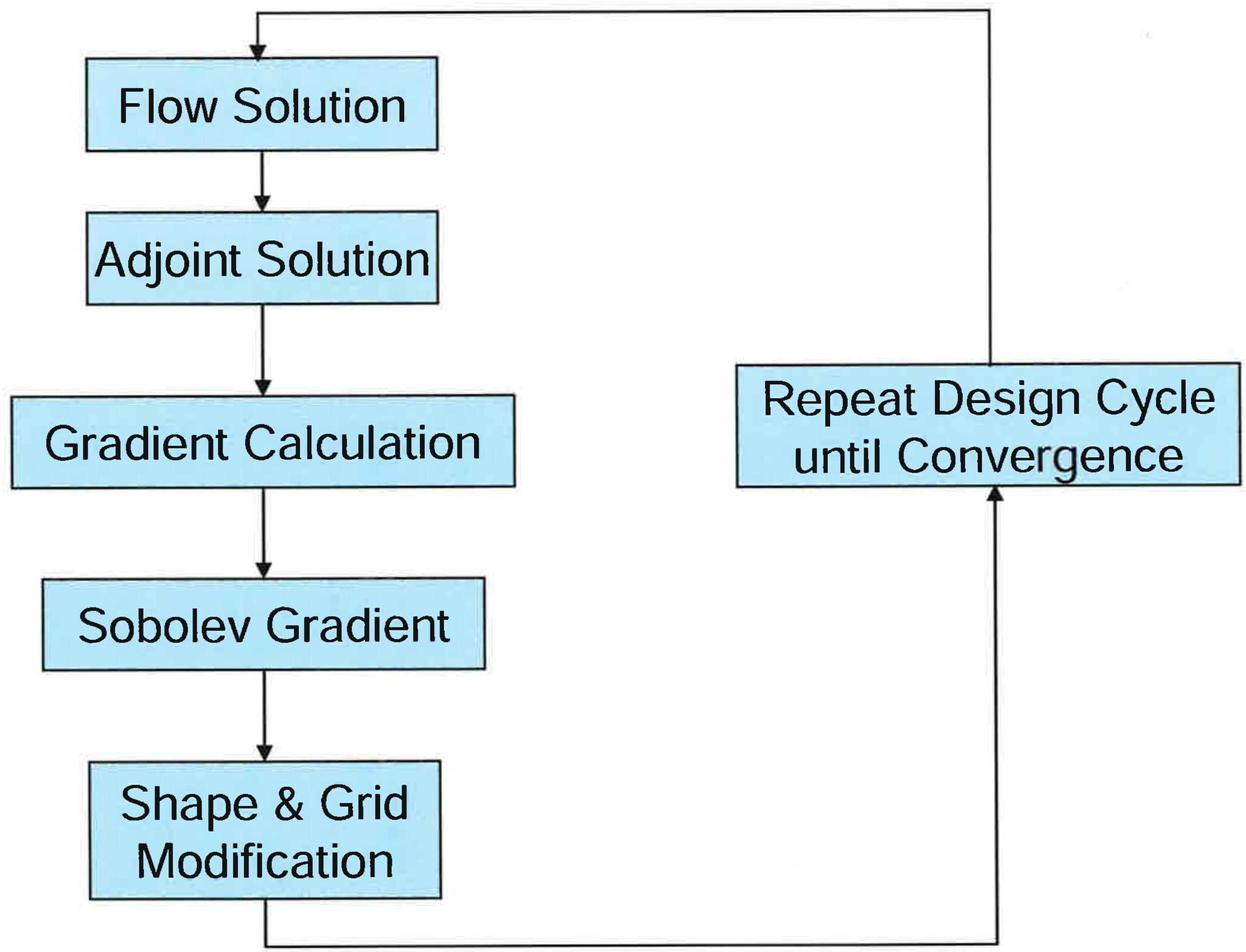
$$\bar{\mathcal{G}} - \frac{\partial}{\partial \xi} \epsilon \frac{\partial \bar{\mathcal{G}}}{\partial \xi} = \mathcal{G} \qquad \bar{\mathcal{G}} = 0, \text{ at end points}$$

Now makes a shape change δ S = -$\bar{\mathcal{G}}$ and for small positive λ

$$\begin{aligned} \delta I &= -\lambda \langle \bar{\mathcal{G}}, \bar{\mathcal{G}} \rangle \\ &\leq 0 \end{aligned}$$

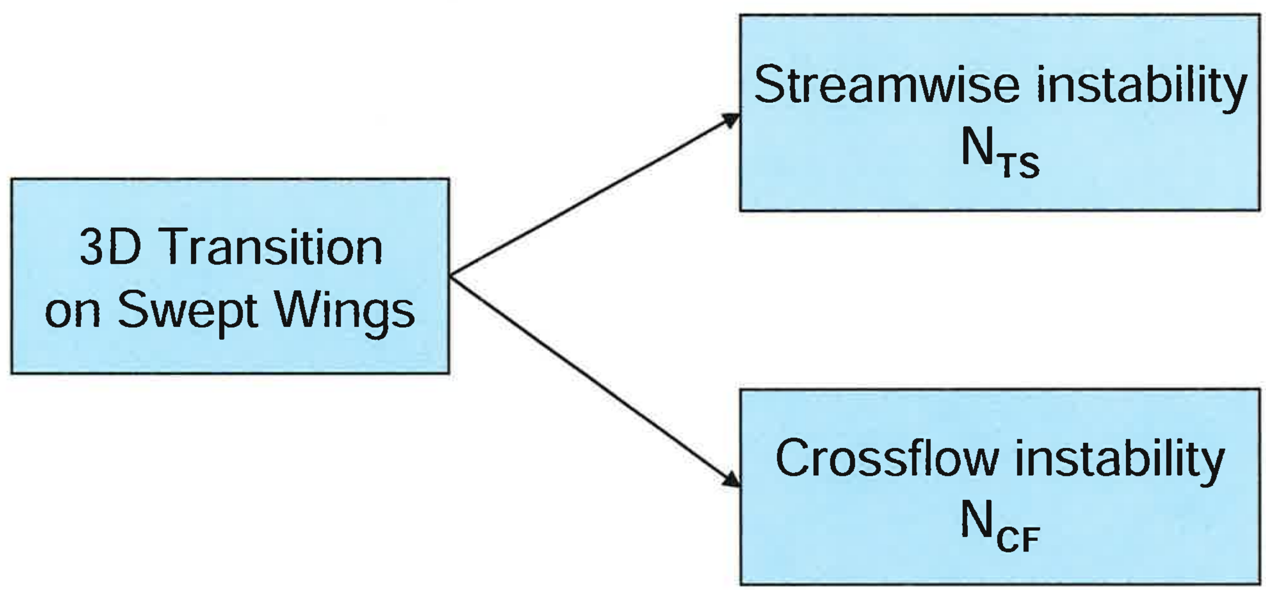
guarantees an improvement.

Outline of the Design Process



Transition Prediction

Present Transition Prediction Methodology



The e^N Criterion

- In the mid 1950s, Smith and Gamberoni, and independently van Ingen, compared linear stability calculations with experimental results for which transition locations were known.

- They found that transition locations in many cases happened at the location where Linear Stability Theory predicts the amplification of Tollmien-Schlichting waves to reach about 8100.

- This value corresponds to e^N, where $N = 9$.

Calculation of Viscous Boundary-Layer Parameters

- The first step in transition prediction using e^N-database method is to calculate viscous laminar boundary-layer parameters, e.g. displacement thickness, δ^*, and momentum thickness, θ,

$$\delta^\star = \int_0^{\delta_e} (1 - \frac{U(y)}{U_e})dy$$
$$\theta = \int_0^{\delta_e} \frac{U(y)}{U_e}(1 - \frac{U(y)}{U_e})dy$$

- Use RANS solver to provide viscous data
  - Straightforward
  - Need large number of mesh points and grid adaptation may also be needed

- Use QICTP* code to provide viscous data
  - A compressible laminar boundary-layer code for swept, tapered wings

* Horton, H.P., Stock, H.W. "Computation of Compressible, Laminar Boundary Layers on Swept, Tapered Wings", J. Aircraft, Vol. 32, pp. 1402-14-5, 1995

Streamwise Amplification Factor Calculation

Given the high quality boundary-layer parameters, the next step toward transition prediction is to calculate amplification factor for Tollmien-Schlichting waves, N_{TS}, by using parametric fitting results from Sturdza*.

$$N_{TS} = \int_{R_{e\theta_0}}^{R_{e\theta}} \frac{dn_{ts}}{dR_{e\theta}} \, dR_{e\theta}$$

where

$R_{e\theta}$ = momentum thickness Reynolds number,

$R_{e\theta 0}$ = critical Reynolds number = f (H_k , T_w/T_e)

$dn_{ts}/dR_{e\theta}$ = g (H_k , Tw/Te)

H_k = kinematic shape factor = δ^*/θ

* P. Sturdza., An Aerodynamic Design Method For Supersonic Natural Laminar Flow Aircraft., PhD thesis, PhD thesis, Stanford, 2004, 3781-2004

Crossflow Amplification Factor Calculation

The crossflow amplification factor, N_{CF}, is calculated by integrating amplification rate, α, of corssflow instability

$$N_{CF} = \int_{x_0}^{x} \alpha \, dx$$

starting from x_0, where x_0 is the location at which crossflow Reynolds number exceed its critical value

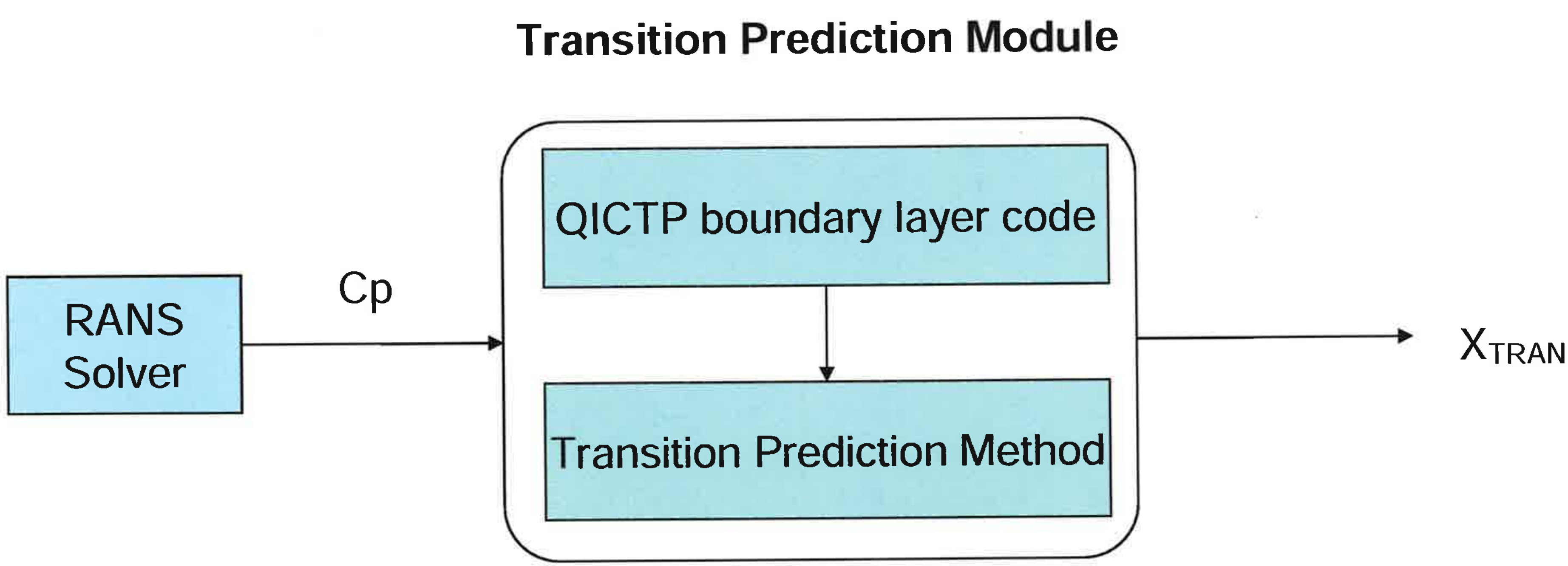
$$R_{cf0} = 46 \frac{T_w}{T_e}$$

and

$$\alpha = \alpha \left(R_{cf}, \frac{w_{max}}{U_e}, H_{cf}, \frac{T_w}{T_e} \right)$$

Transition Prediction Module

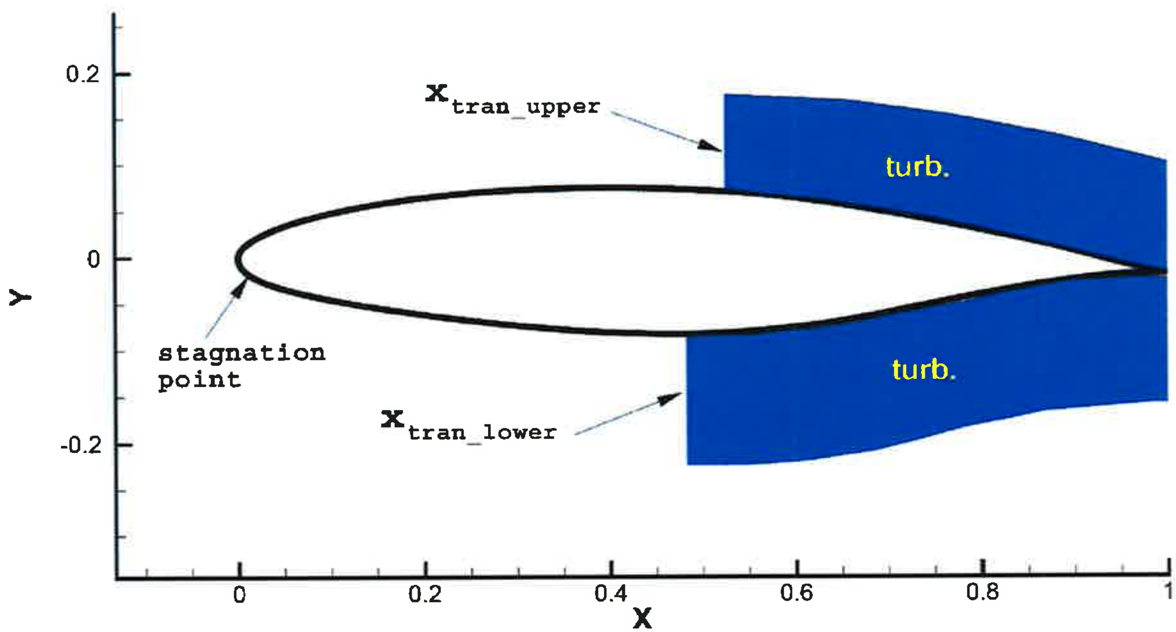
- Use Cp provided by RANS solver as inputs
- Split the airfoil into upper and lower surfaces from stagnation point
- Analyze each surface separately
- Outputs x_{tran_upper} and x_{tran_lower}



Transition Prescription

Transition Prescription

- Transition prescription on surface
  - Transition prediction module => x_{tran_upper} , x_{tran_lower}
  - Split the airfoil surface into laminar and turbulent patches
- Transition prescription in flow domain
  - Project the turbulent patches into the flow field



Transition Prescription

- Flow domain decomposition
  - laminar + turbulent
  - apply turbulence model to turbulent subdomains
- Baldwin-Lomax model

$$\tau_{ij} = (\mu_{lam} + \mu_{turb}) \left\{ \frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} - \frac{2}{3} [\frac{\partial u_k}{\partial x_k}] \right\} \delta_{ij}$$

- Laminar-turbulent switch:

  lt_switch(x) = 0 $x \in$ laminar

  = 1 $x \in$ turbulent

- Set:

$$\mu_{turb} = lt\_switch(x)\ \mu_{turb}$$

Coupling with RANS Solver

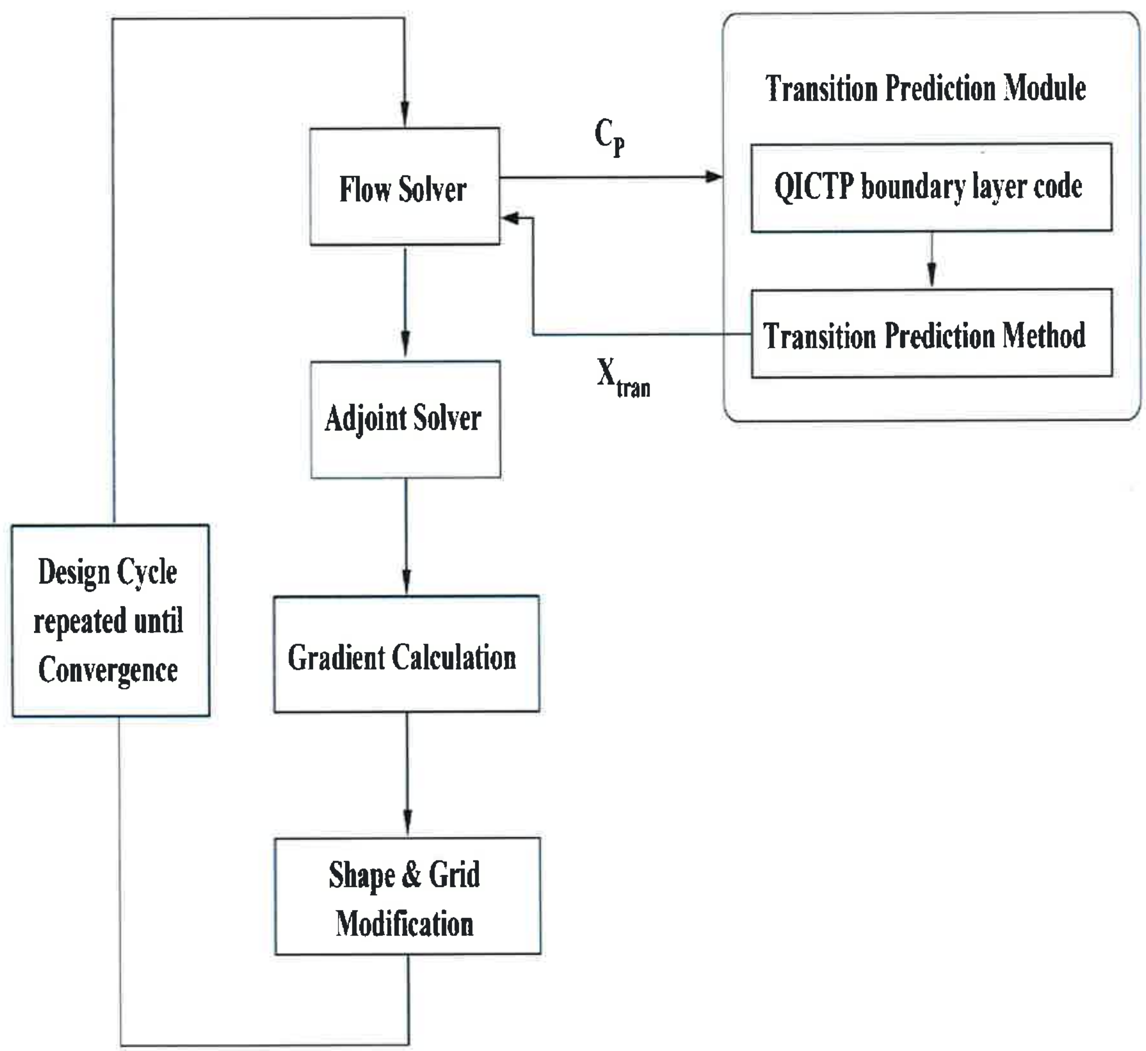
The coupling of RANS solver with transition prediction module can be summarized as follows:

- The RANS solver iterates with prescribed transition locations, e.g. 80% from the leading edge until residual drops below certain level.
- The transition prediction module is called. Surface pressure distribution from RANS solver is used as input for boundary-layer code.
- With the calculated boundary-layer parameters, two e^N-database are used to calculate transition locations.
- The new transition locations are fed into RANS solver and this complete one iteration of transition prediction module.
- This process continues until convergence criteria

$$| x_{tran}(k) - x_{tran}(k-1) | \leq \delta$$

Is reached.

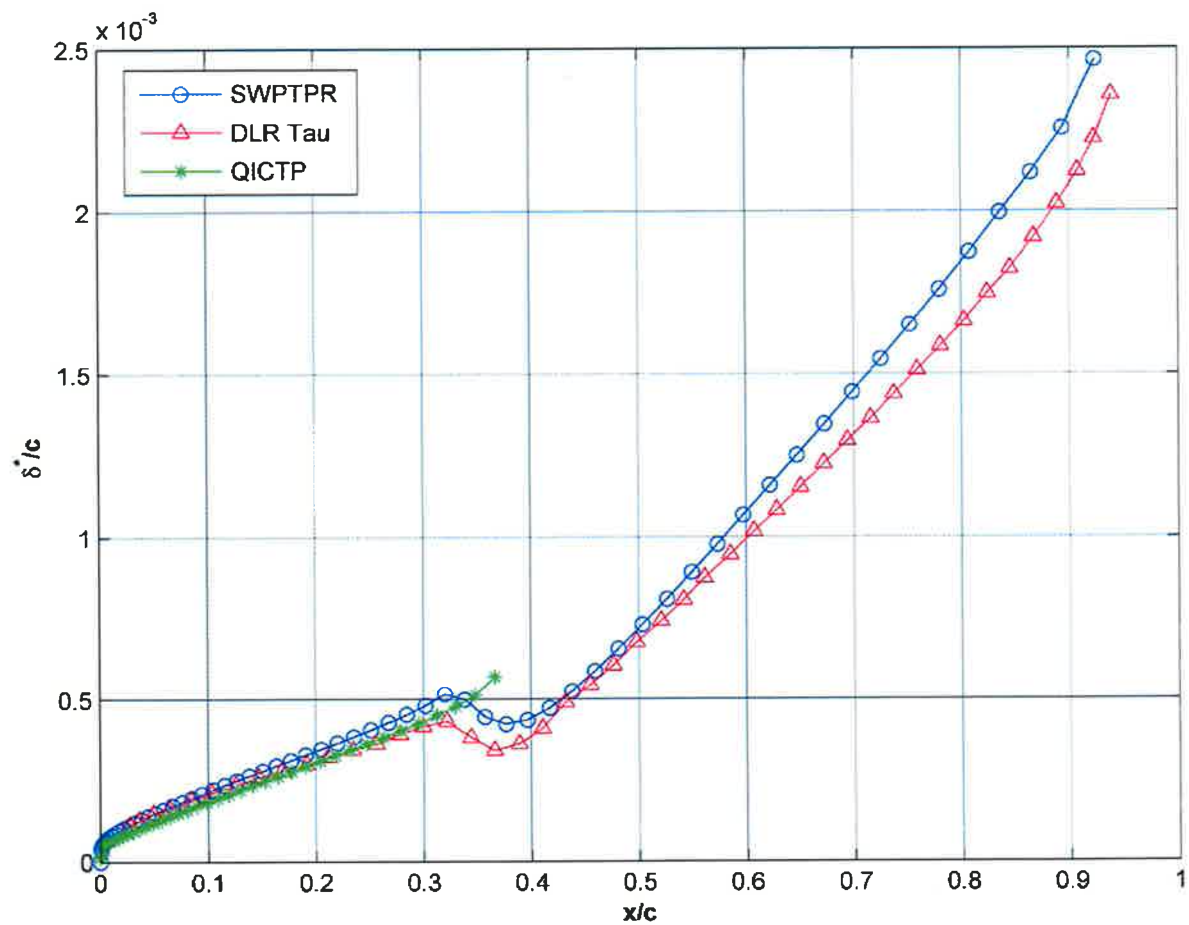
Coupling with RANS Solver



Numerical Results

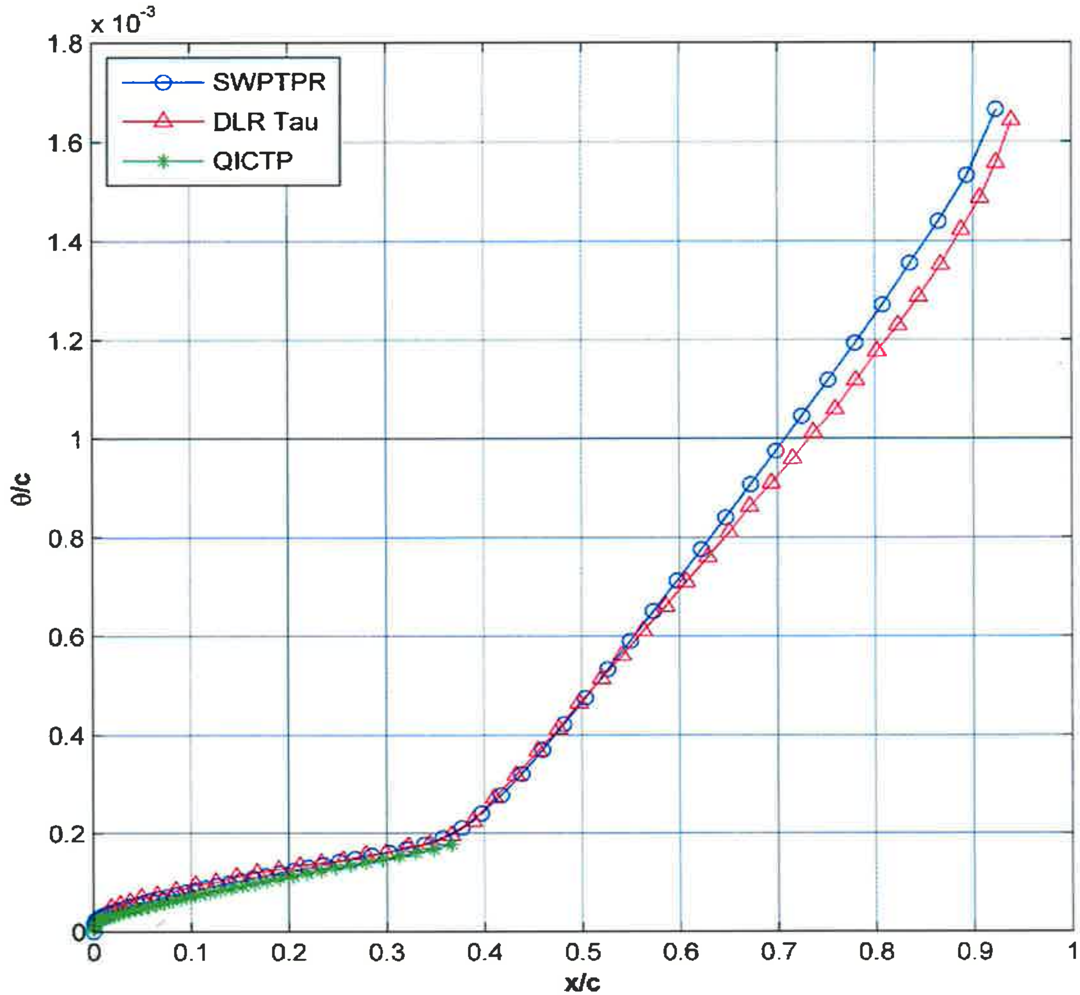
Verification of Boundary-Layer Parameters

- Comparison of incompressible displacement thickness δ^*
- NLF-0416 at $M_\infty = 0.3$, $Re = 4 \times 10^6$, and $\alpha = 2.03°$



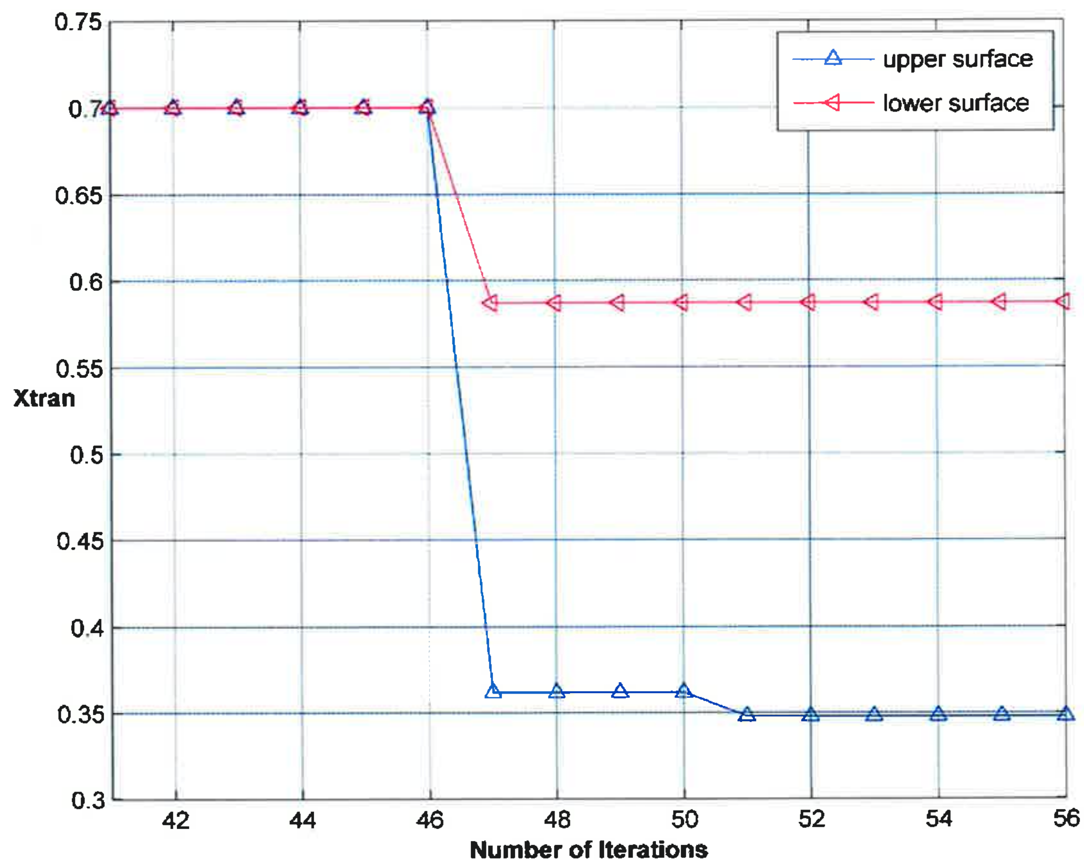
Verification of Boundary-Layer Parameters

- Comparison of momentum thickness θ
- NLF-0416 at $M_\infty = 0.3$, Re = 4x106, and $\alpha = 2.03°$



Verification of NLF-0416 Transition Locations

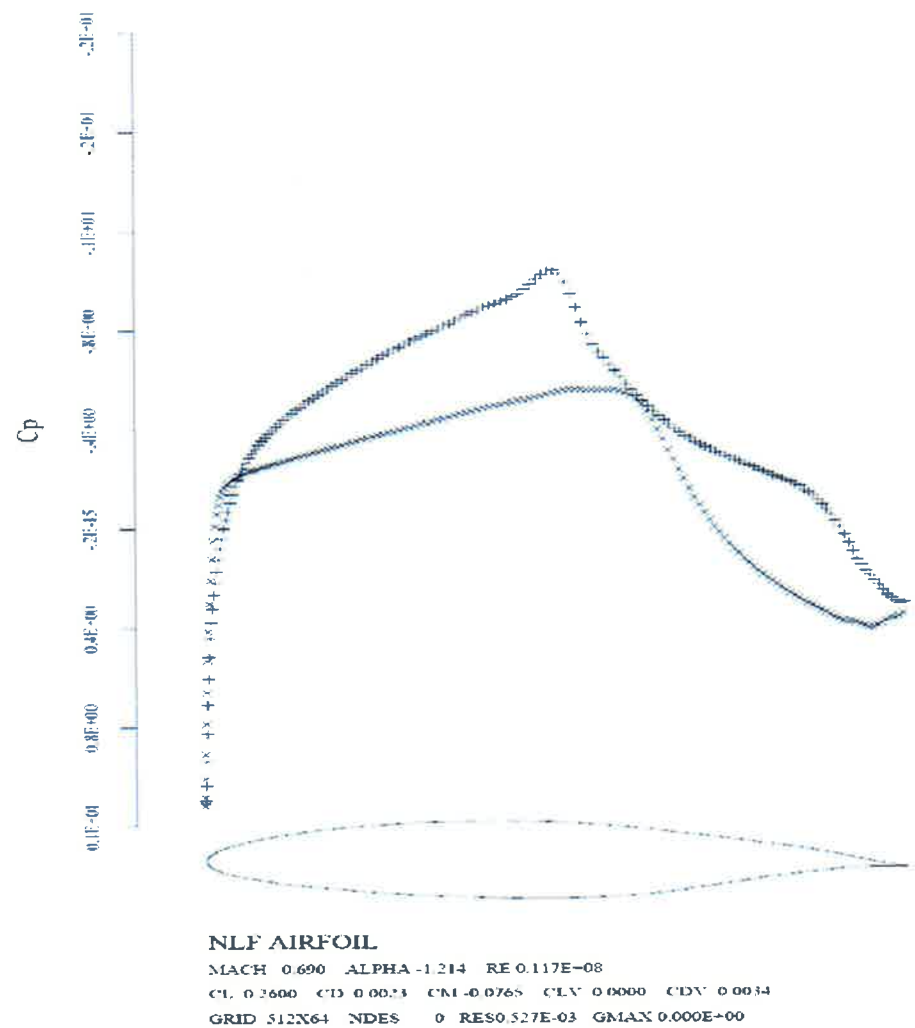
	X_{tran_upper}	X_{tran_lower}
Current Method	0.348	0.587
Experiment	0.35	0.6



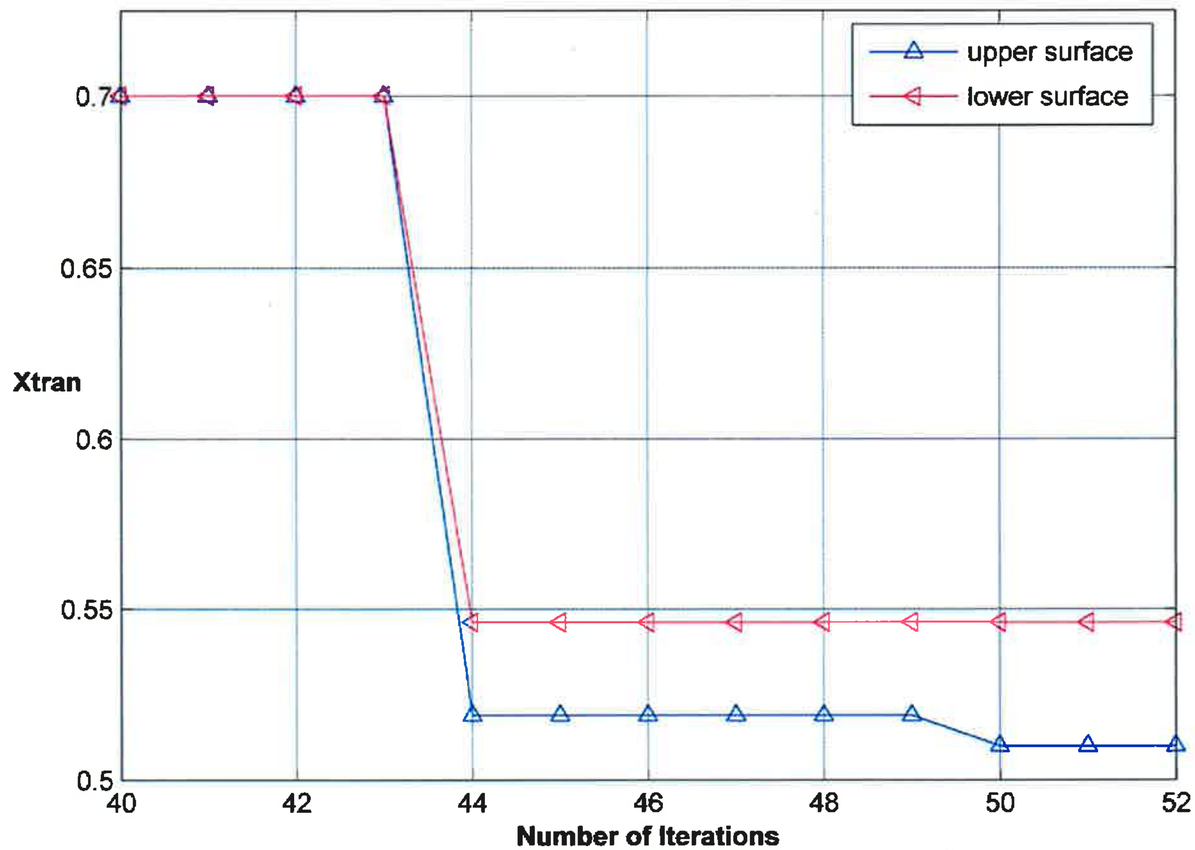
2D NLF Airfoil Design

- Design targets:
  - $M_\infty = 0.69$
  - $Re_\infty = 11.7 \times 10^6$
  - $Cl_{target} = 0.26$
- Cost function:

$$I = \frac{1}{2} \oint_B (p - p_d)^2 \, dS$$



2D NLF Airfoil Design



- X_{tran_upper} = 0.510
- X_{tran_lower} = 0.546

Breguet Range Equation

A good first estimate of performance is provided by the Breguet range equation:

$$R = \frac{VL}{D}\frac{1}{SFC}log\frac{W_1}{W_2}$$

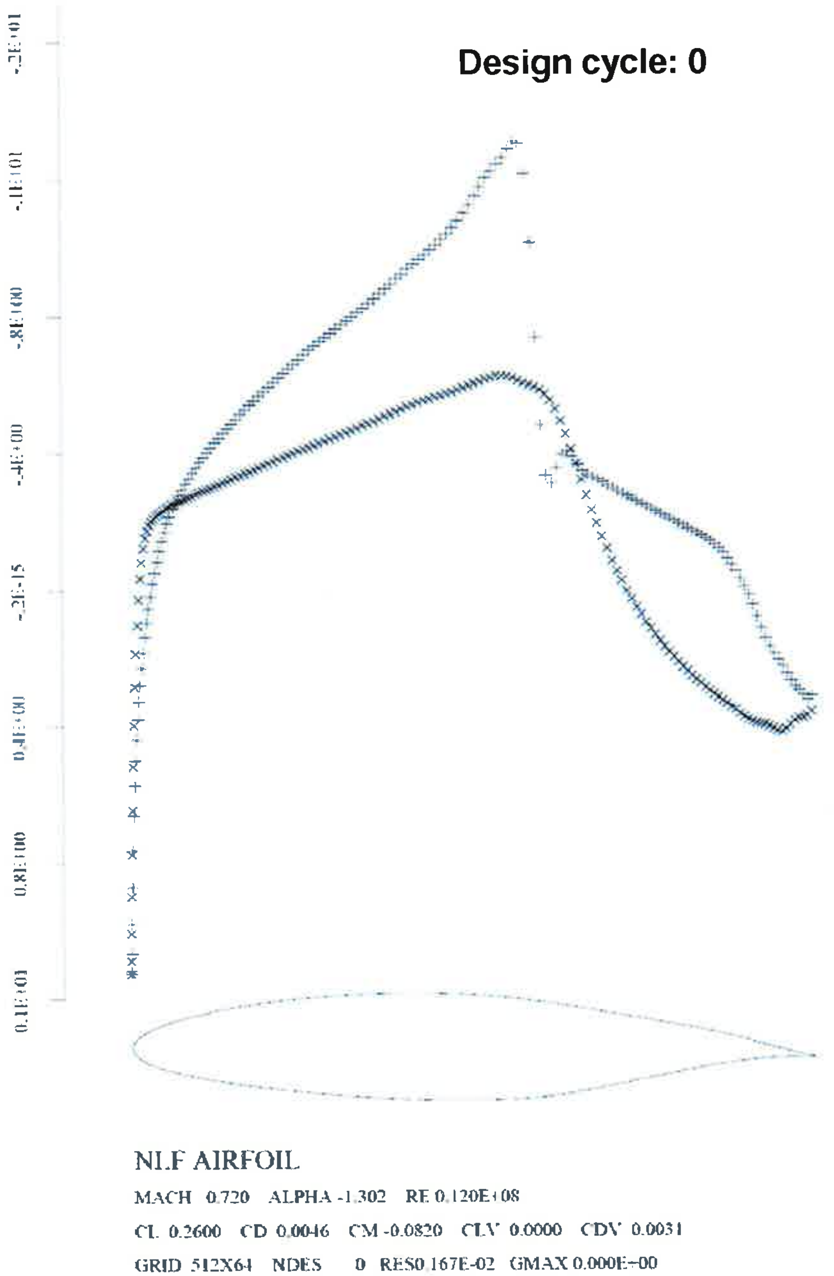
From aerodynamic point of view, the designer should try to maximize VL/D. This means the cruising speed V should be increased until the onset of drag rise.

Redesign of NLF Airfoil

- Improve the range parameter $M_\infty L/D$
  - $M_\infty = 0.69 \implies 0.72$, $Cl_{target} = 0.26$
- Still maintain a reasonable amount of laminar flow
- Use the ajoint method to minimize wave drag

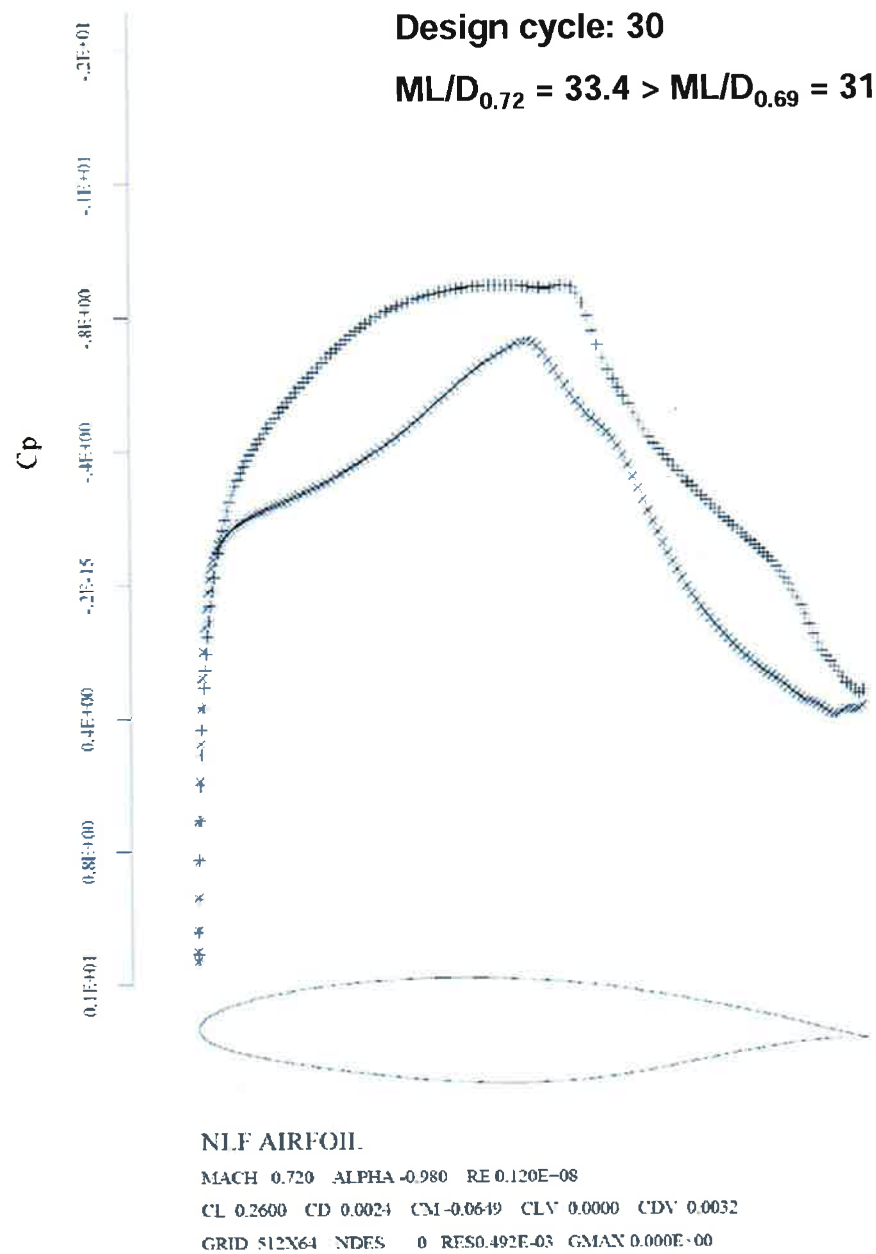
Redesign of NLF Airfoil

Design cycle: 0



Design cycle: 30

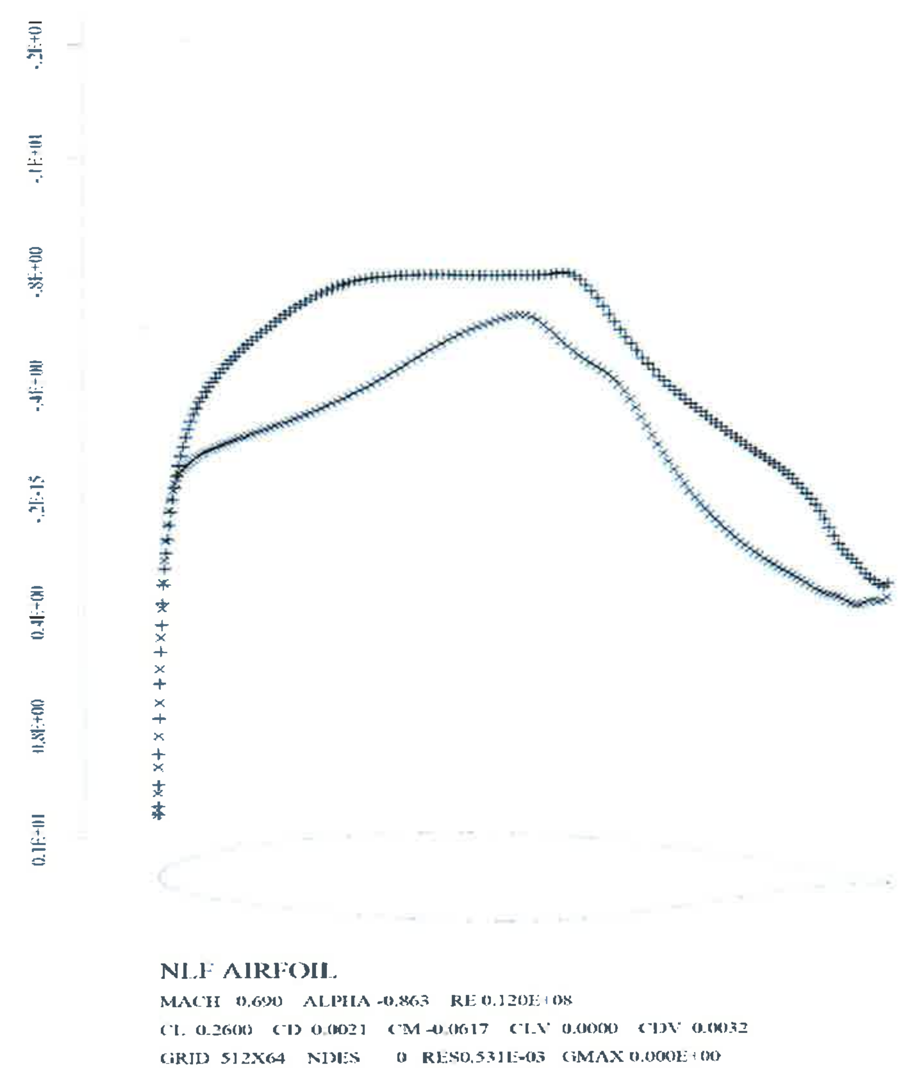
$ML/D_{0.72} = 33.4 > ML/D_{0.69} = 31.4$



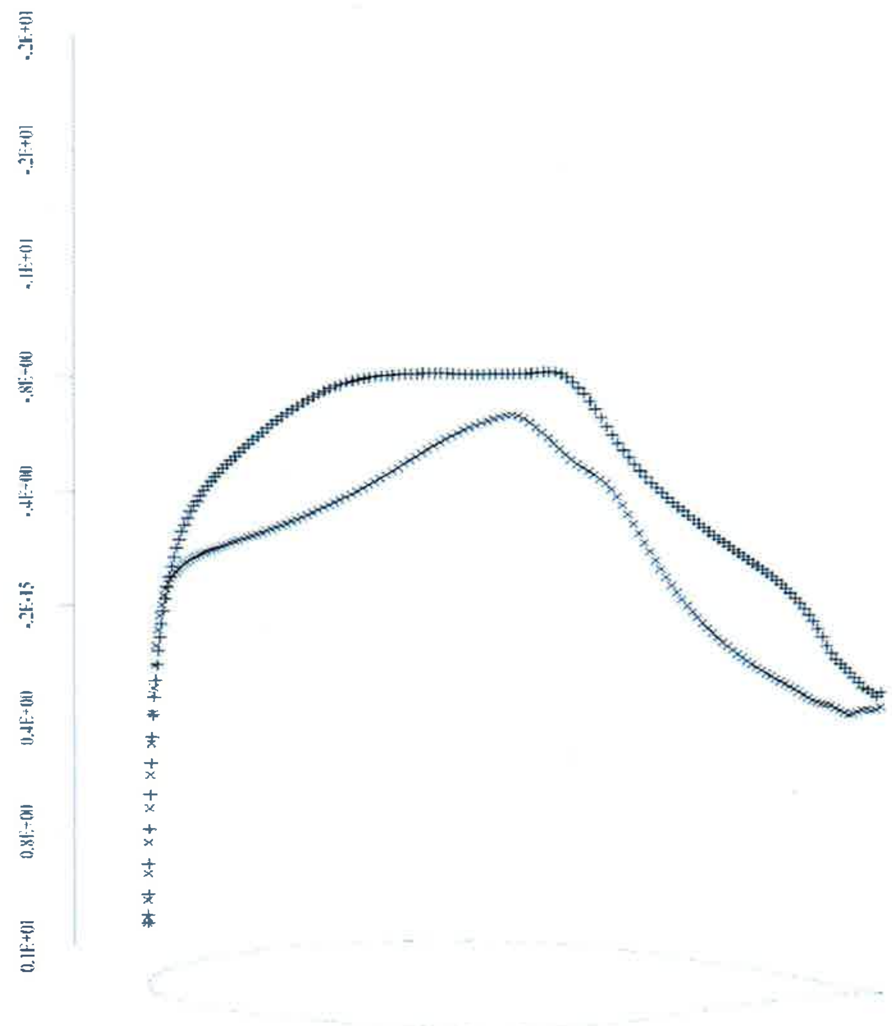
Off-design Condition I: M = 0.69, Cl = 0.26

- Natural-laminar-flow airfoils may have undesirable characteristics, such as formation of shock waves, when flying at off-design conditions.

- Test three off-design flight conditions
  - M = 0.69, Cl_{target} = 0.26
  - M = 0.70, Cl_{target} = 0.26
  - M = 0.71, Cl_{target} = 0.26



Off-design Condition II: M = 0.70, Cl = 0.26



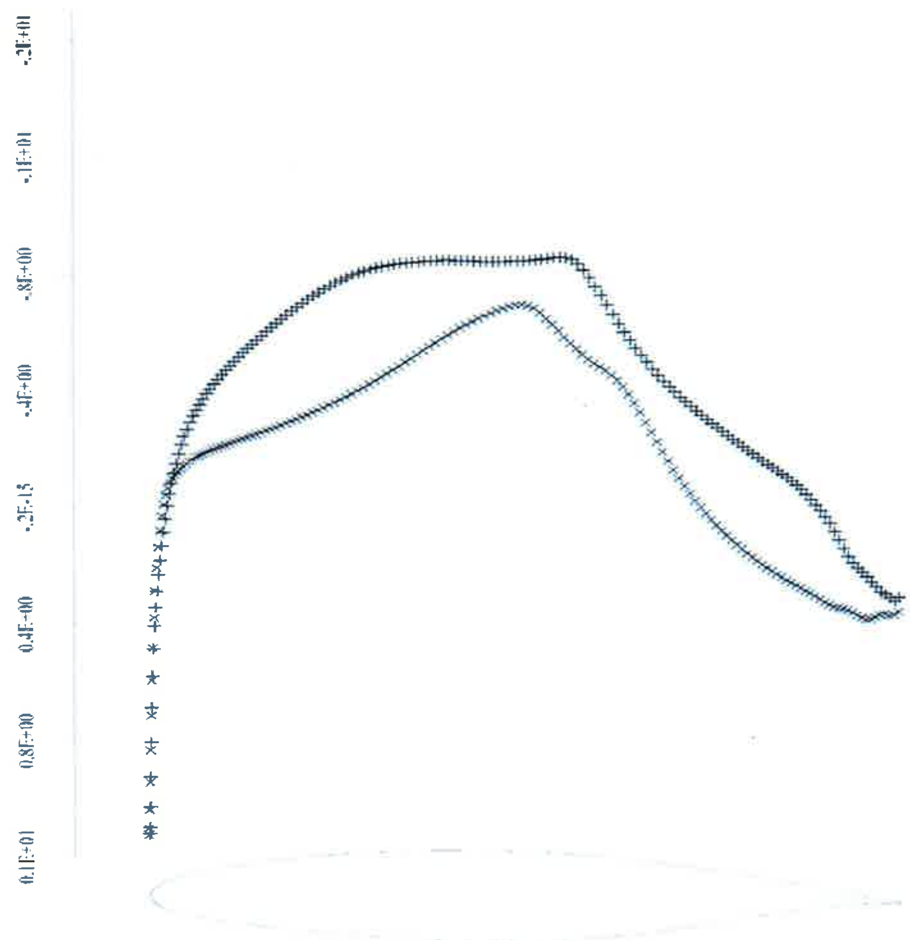
NFL AIRFOIL

MACH 0.700   ALPHA -0.895   RE 0.120E+08

CL 0.2600   CD 0.0022   CM -0.0625   CLV 0.0000   CDV 0.0032

GRID 512X64   NDES   0   RES0.513E-03   GMAX 0.000E+00

Off-design Condition III: M = 0.71, Cl = 0.26



NLF AIRFOIL

MACH 0.710 ALPHA -0.935 RE 0.120E+08

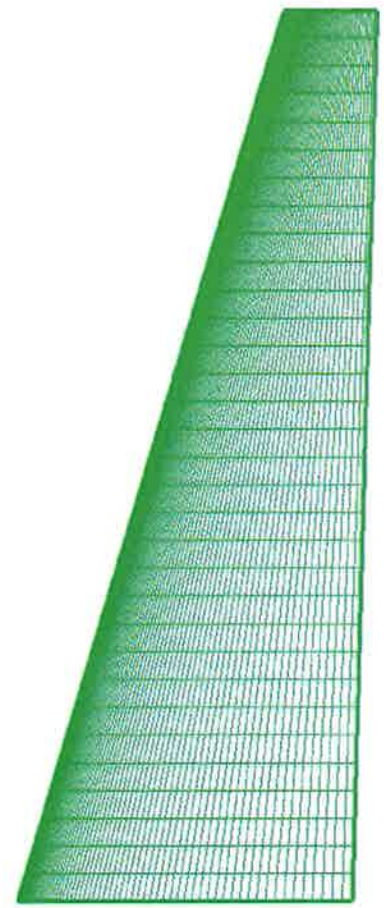
CL 0.2600 CD 0.0023 CM -0.0636 CLV 0.0000 CDV 0.0032

GRID 512X64 NDES 0 RES0.497E-03 GMAX 0.000E+00

3D NLF Wing Calculation

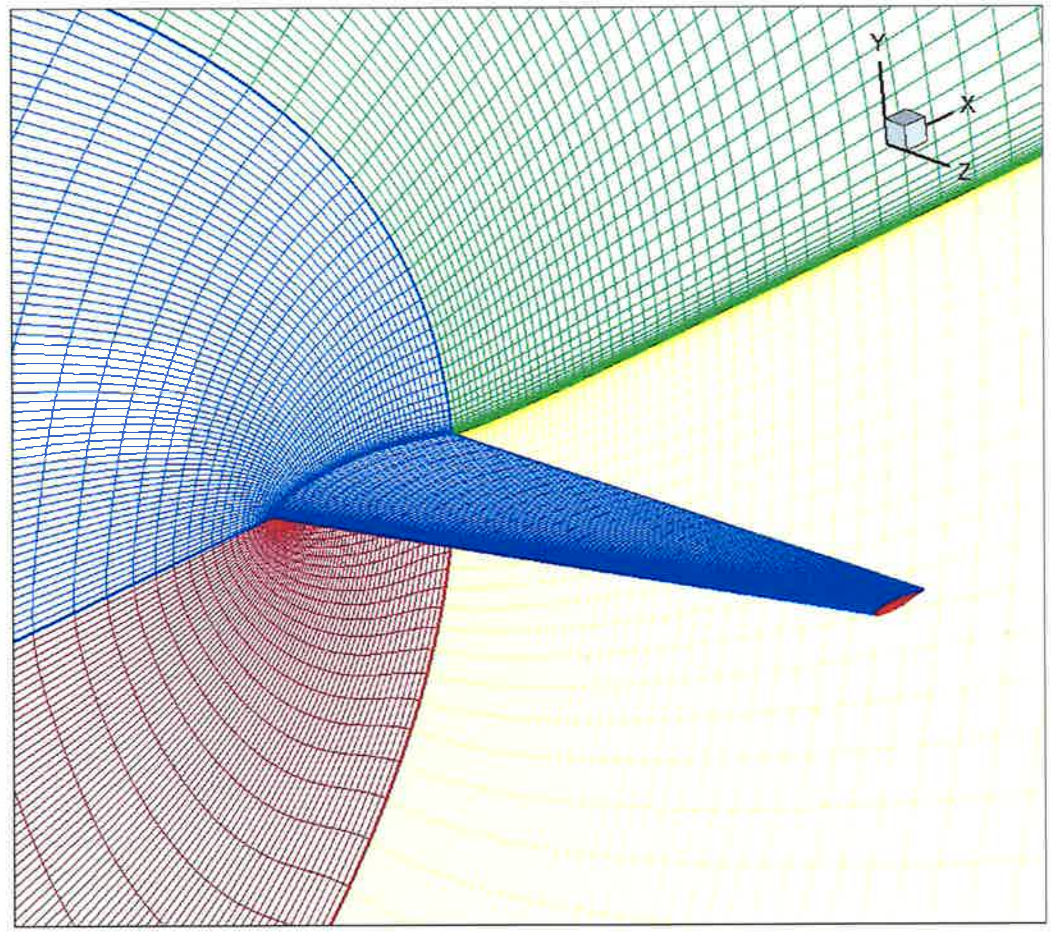
3D NLF Wing Definition

- Semi-span
- Tapered, λ = 0.278
- Swept, Λ_{LE} = 16.69°, Λ_{TE} = 1.67°
- Cross sections: NLF airfoil designed at M_∞ = 0.69

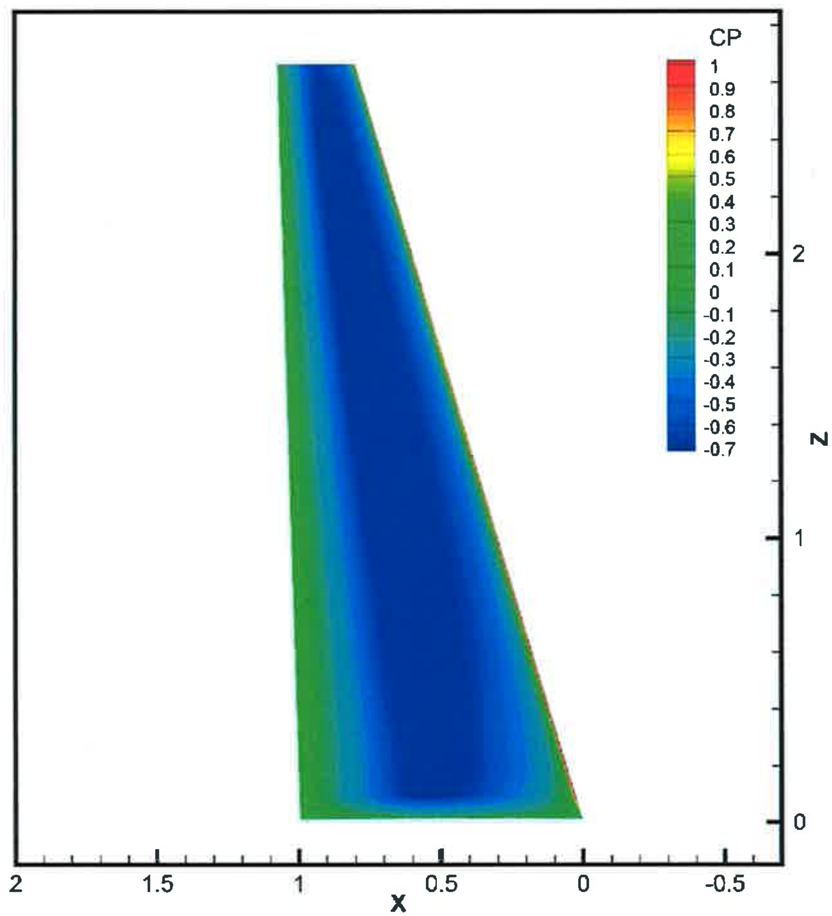


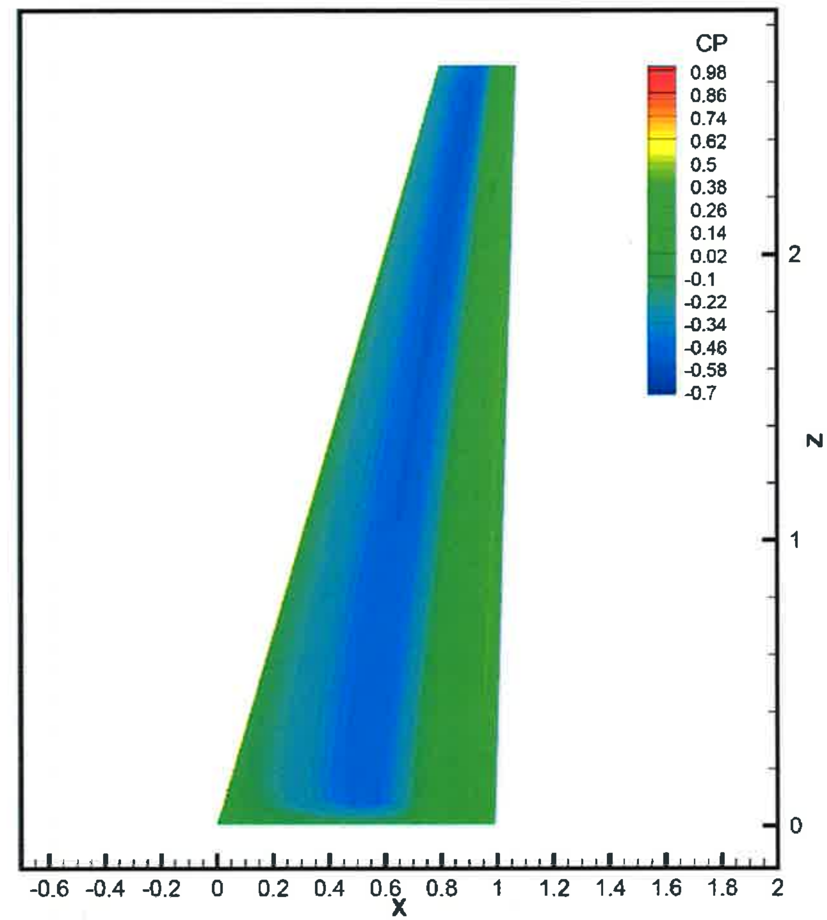
3D NLF Wing Definition

- C-type structured mesh, total = 256 x 64 x 48 mesh cells
- Wing definition: 128 x 33
- Parallel computation with MPI

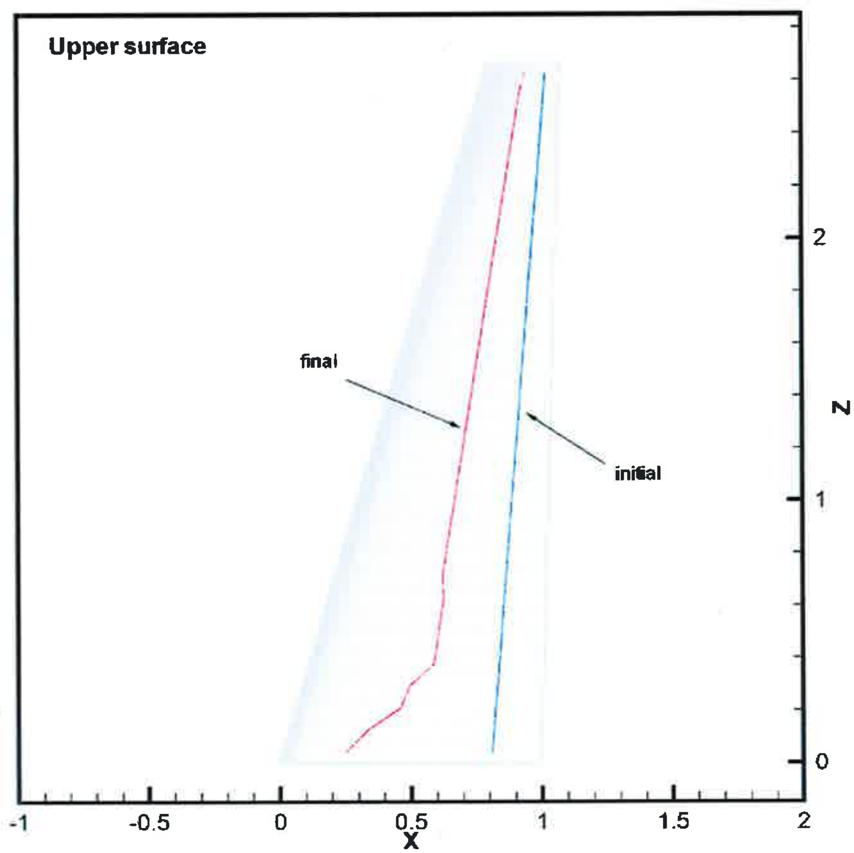


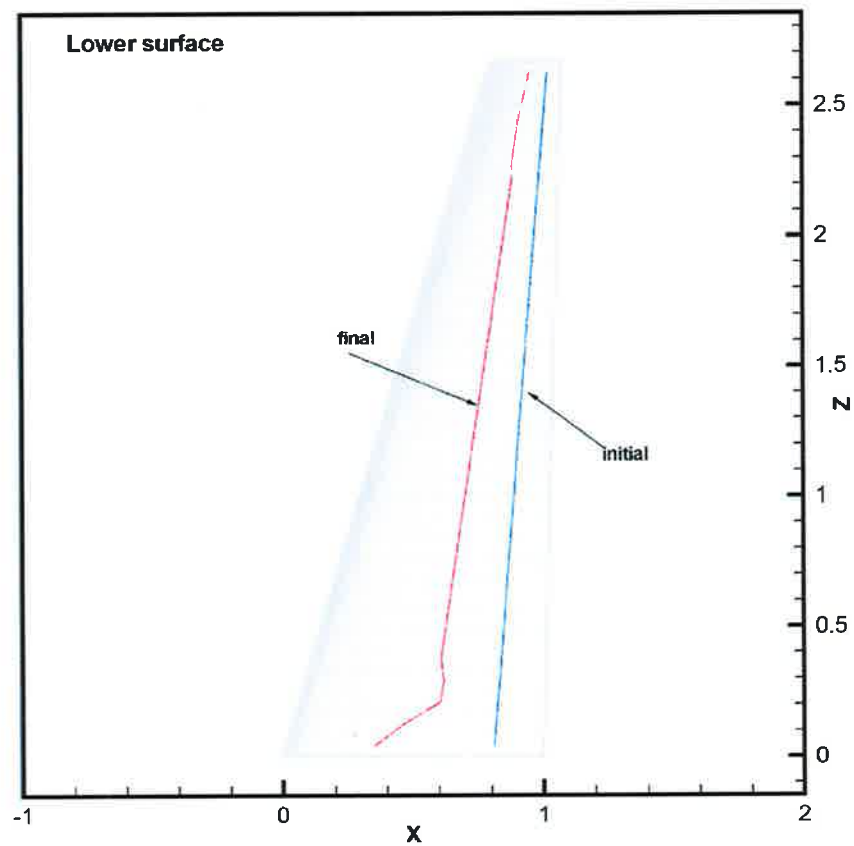
Pressure Distribution: M = 0.69, C_L = 0.26



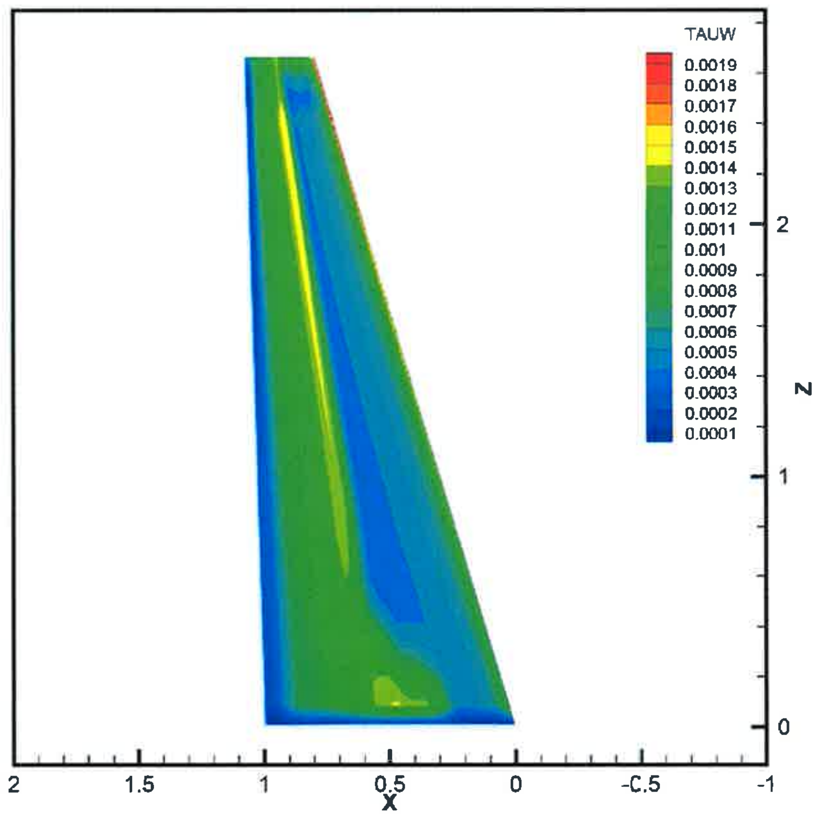


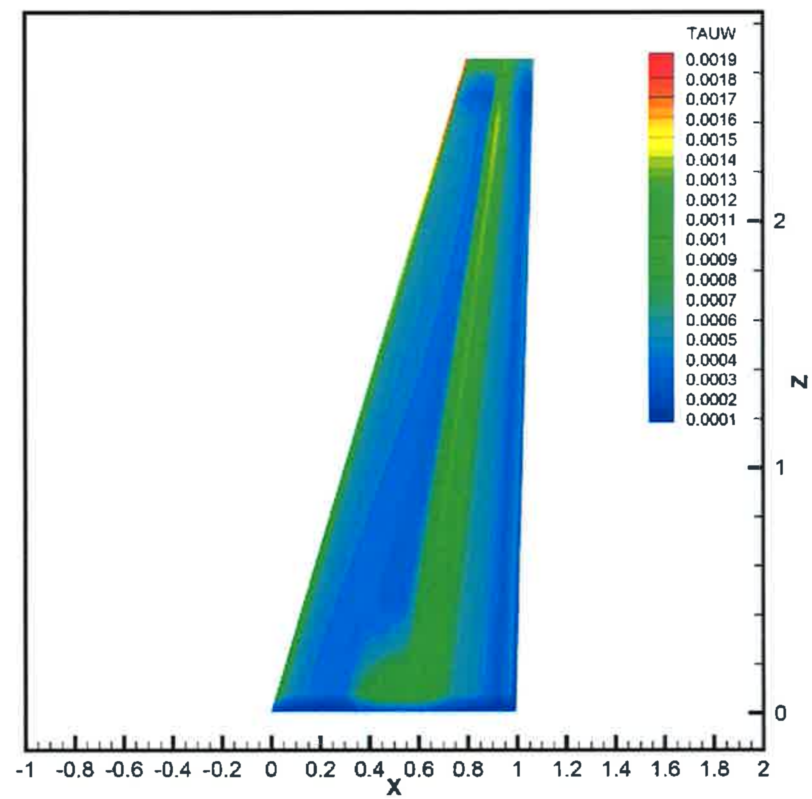
Initial and Final Transition Locations: M = 0.69, C_L = 0.26



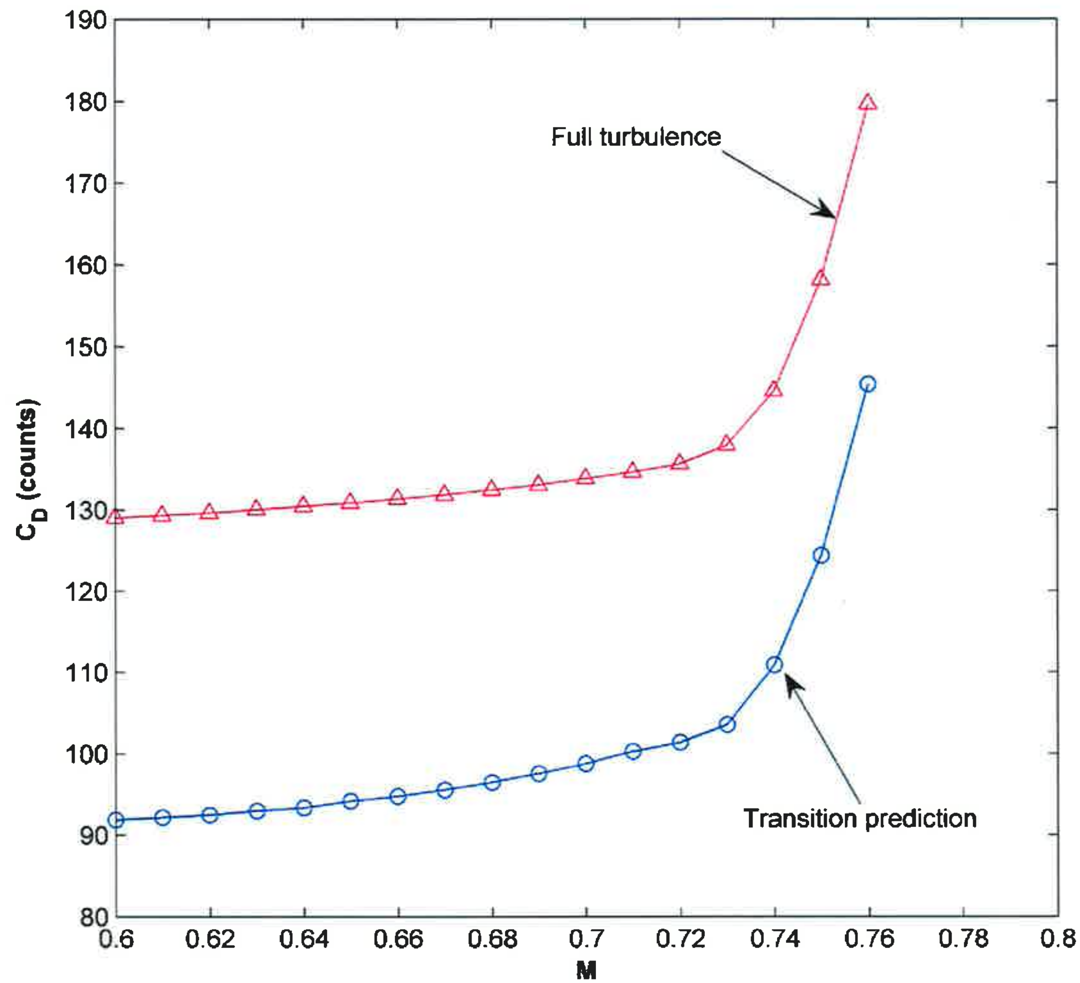


Shear Stress Distribution: $M = 0.69$, $C_L = 0.26$





Comparison of Aerodynamic Coefficients



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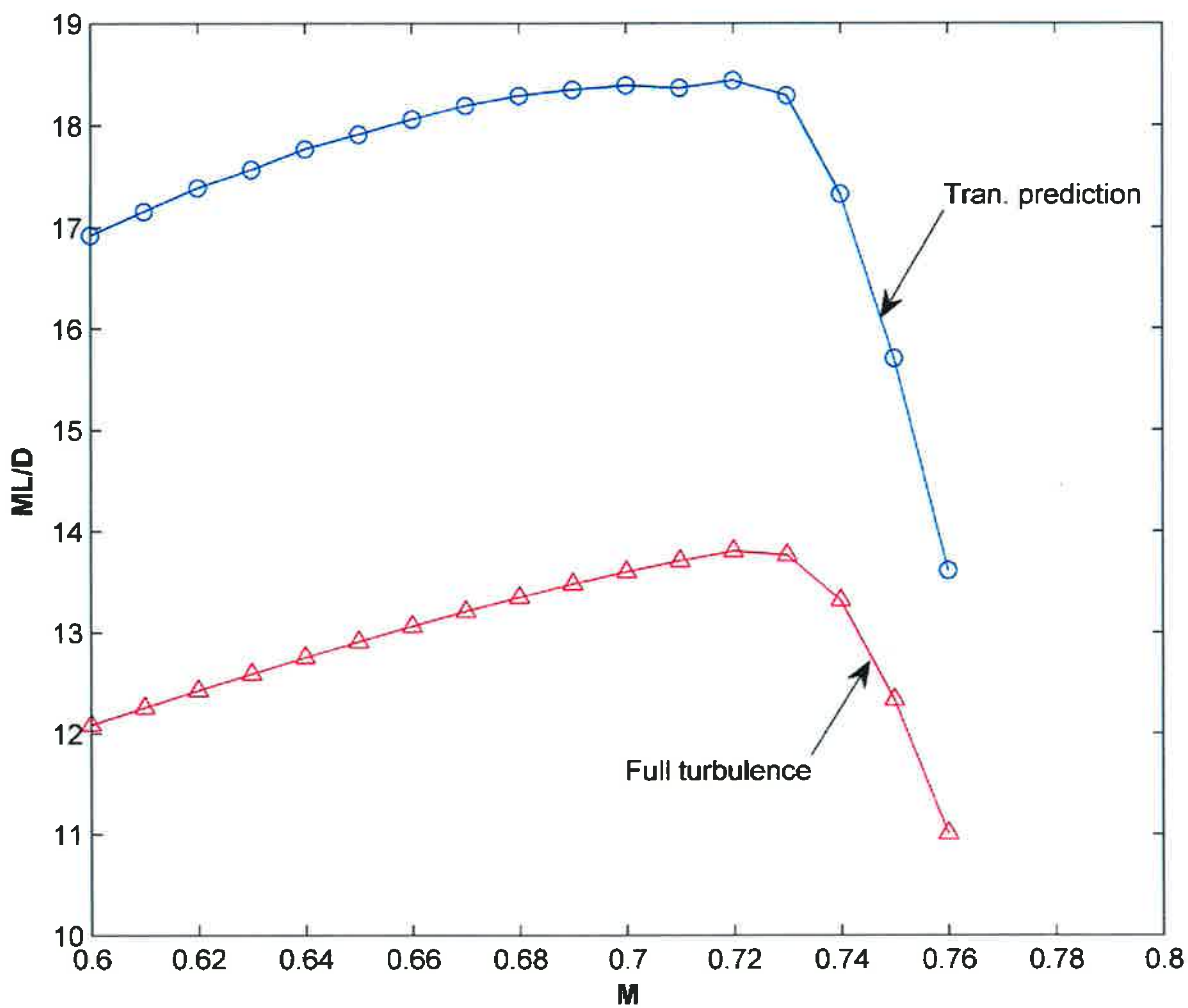


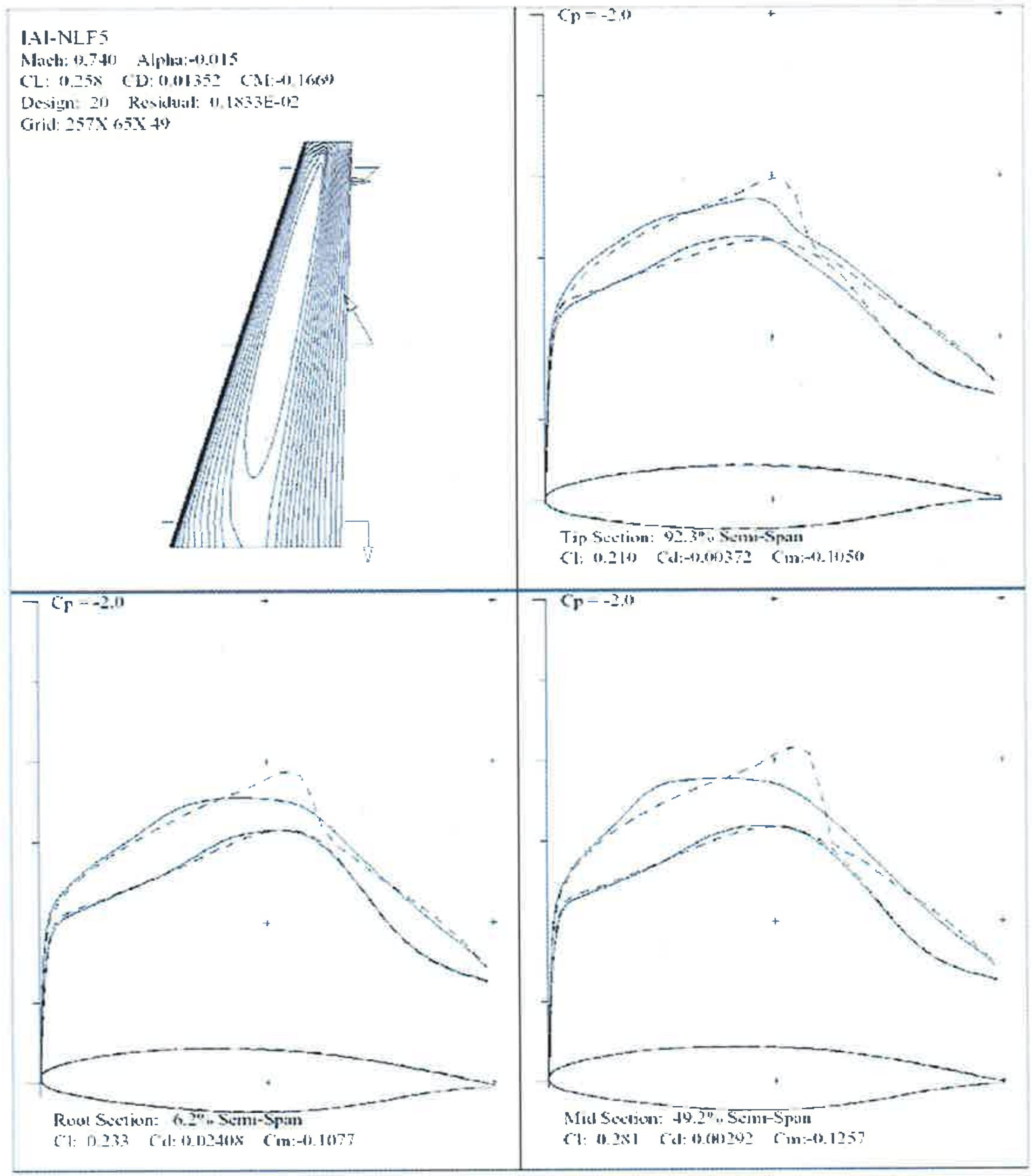
NLF Wing Design

Natural-Laminar-Flow Wing Design

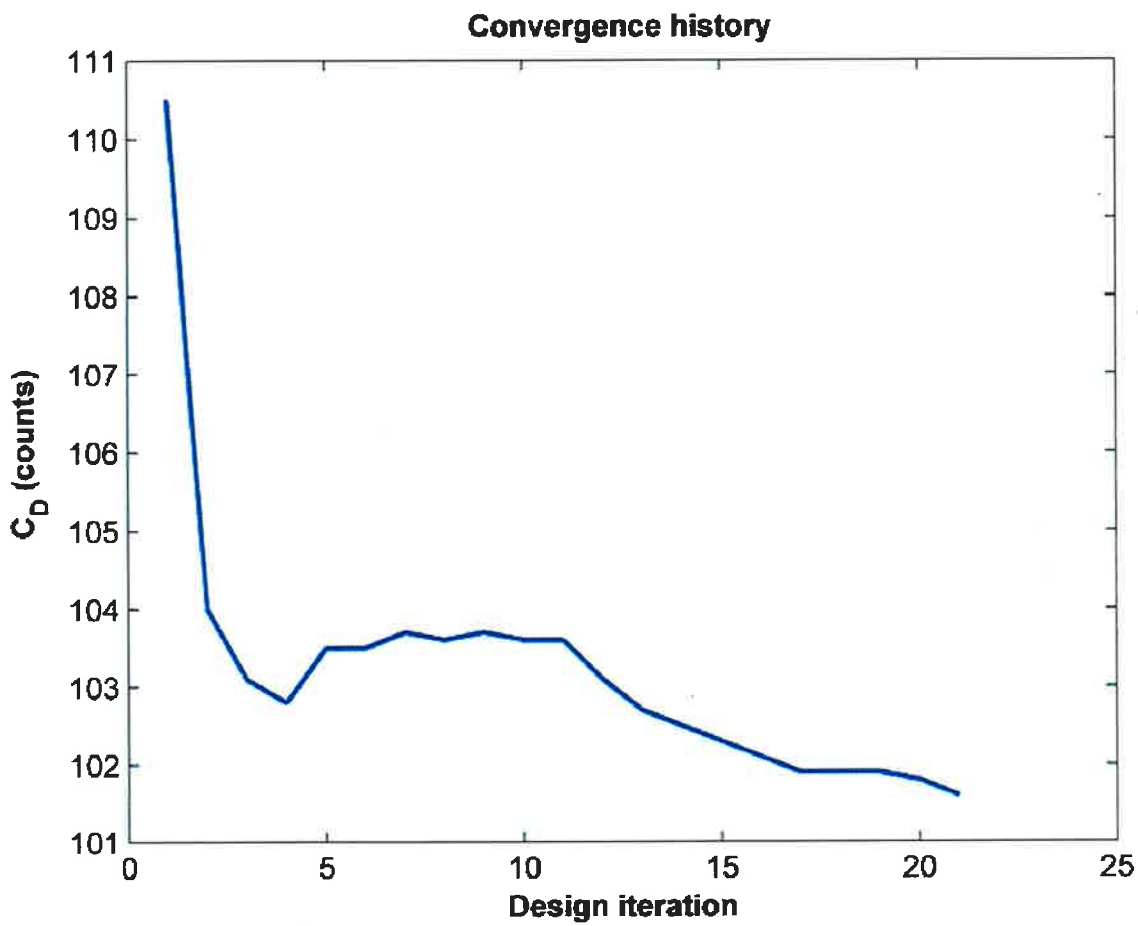
- - - Baseline

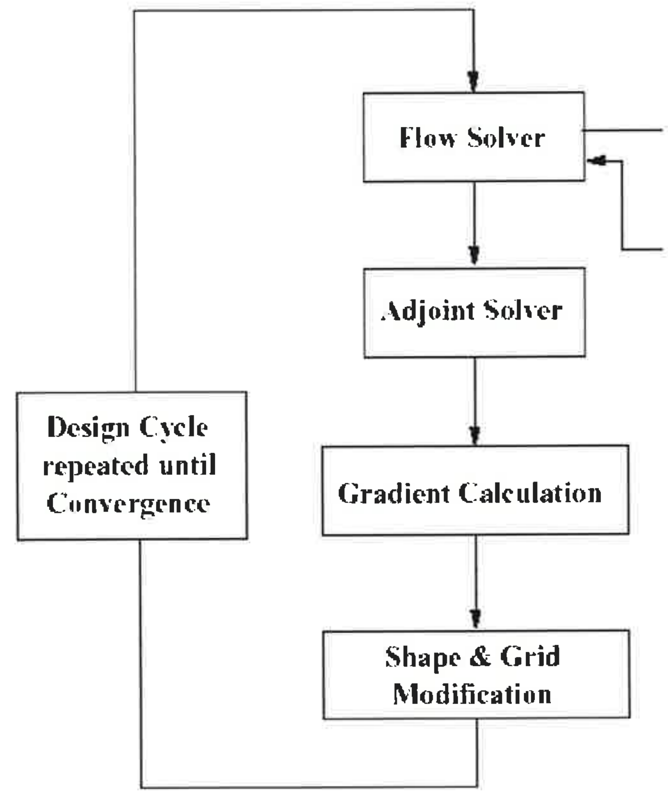
—— Re-designed

Mach = 0.74

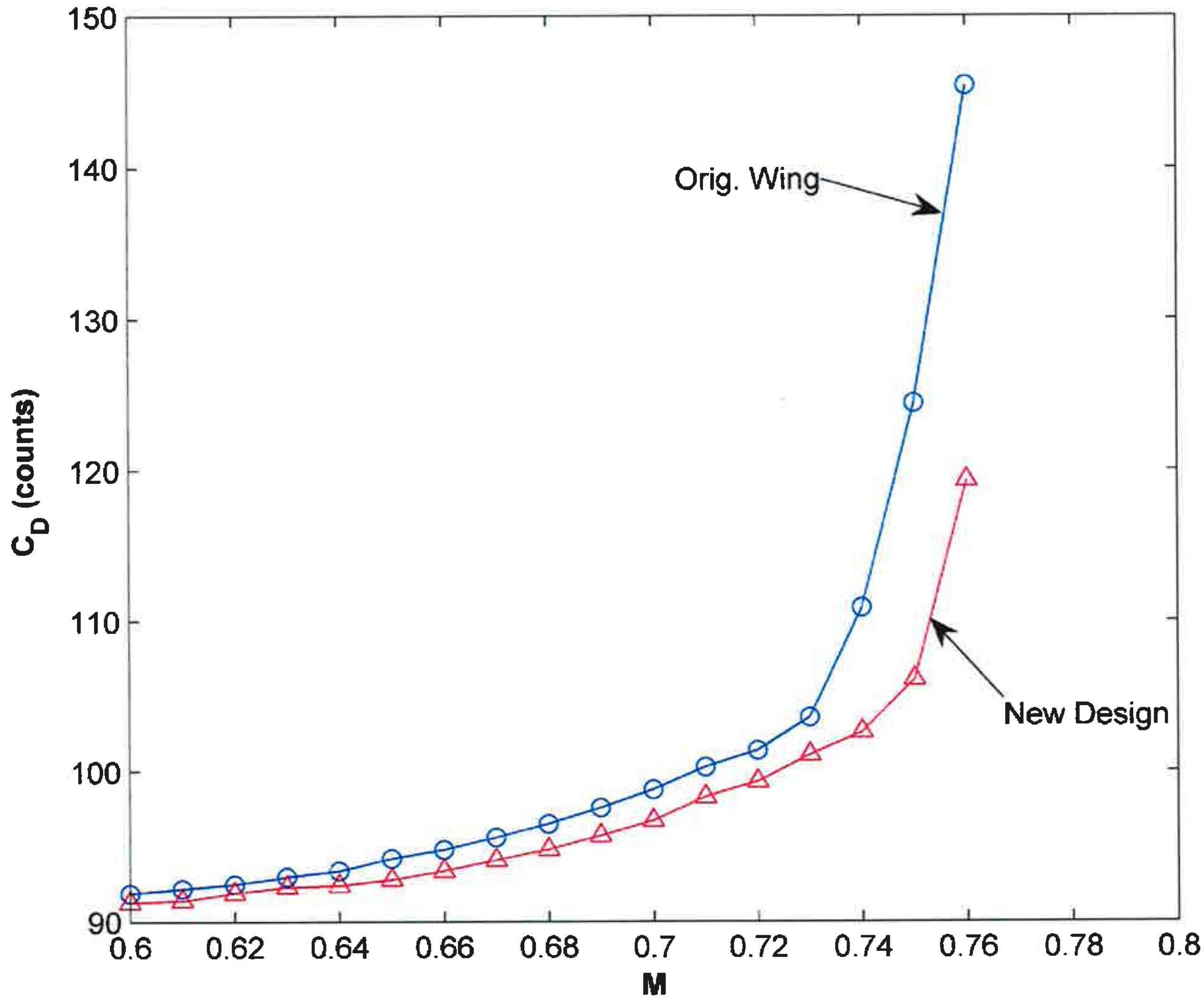


Natural-Laminar-Flow Wing Design

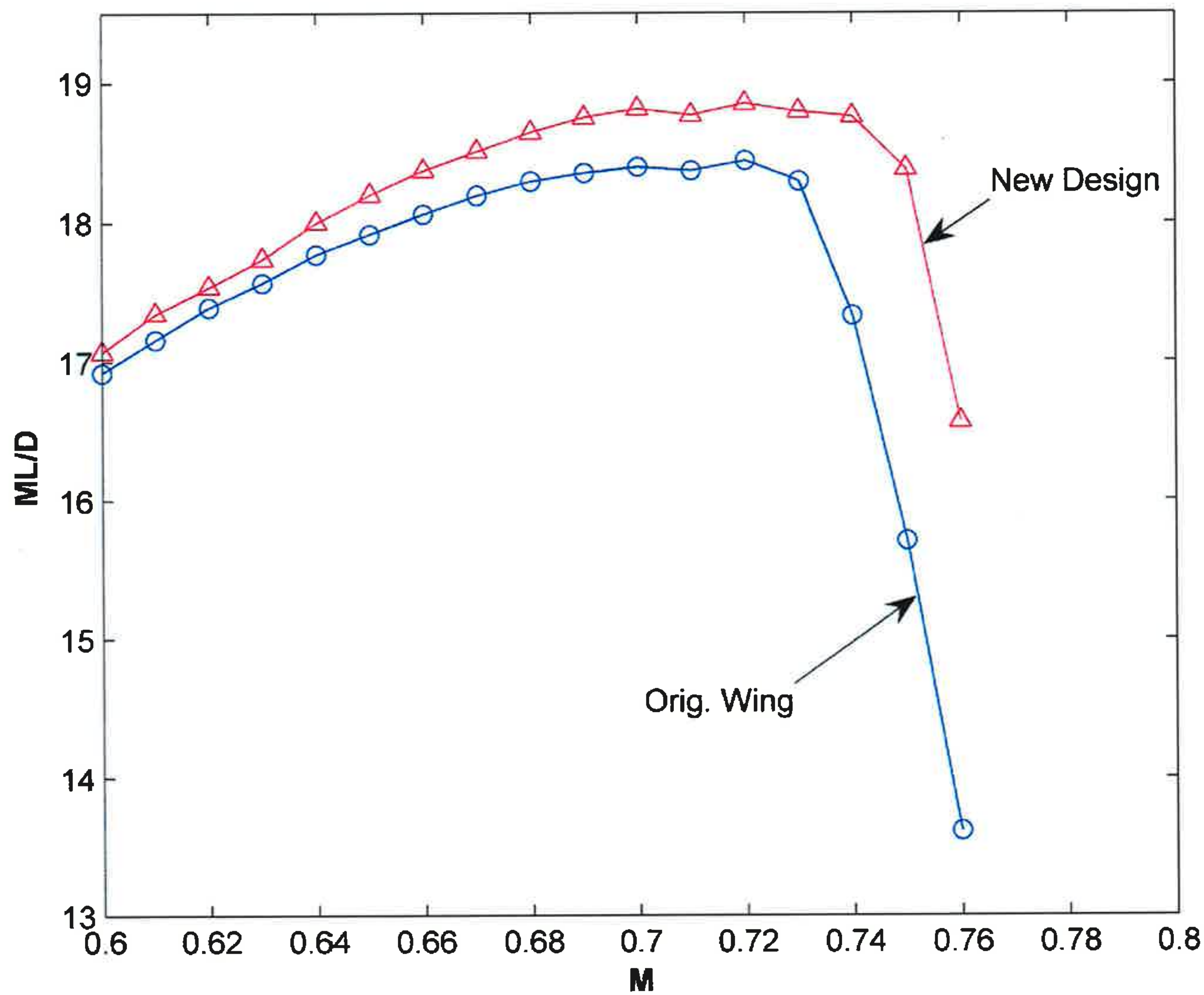




Natural-Laminar-Flow Wing Design



Natural-Laminar-Flow Wing Design

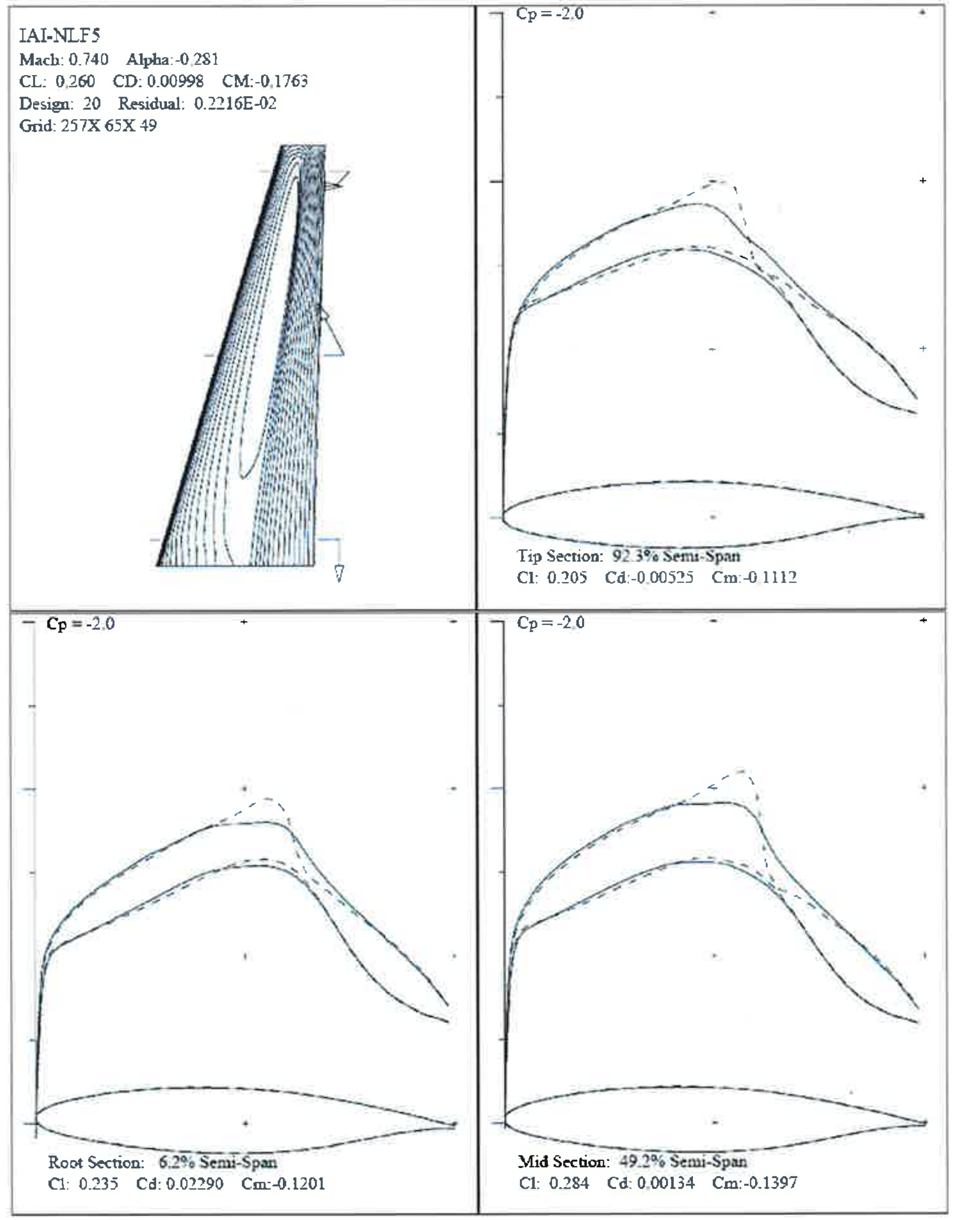


Redesign of Natural-Laminar-Flow Wing Design

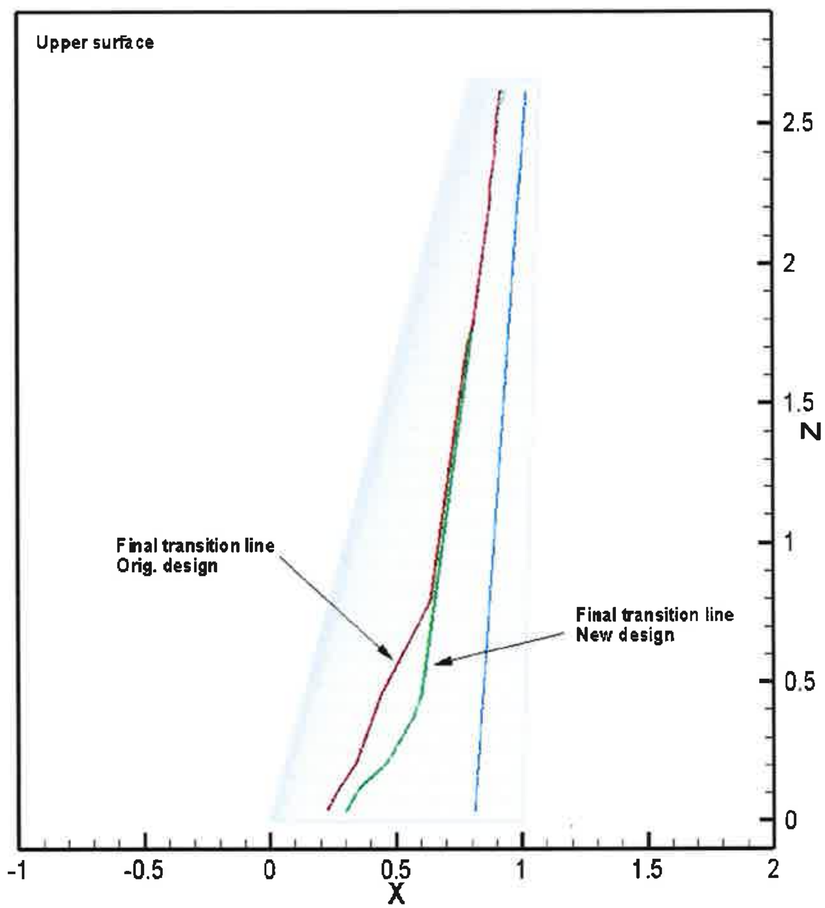
- - - Baseline

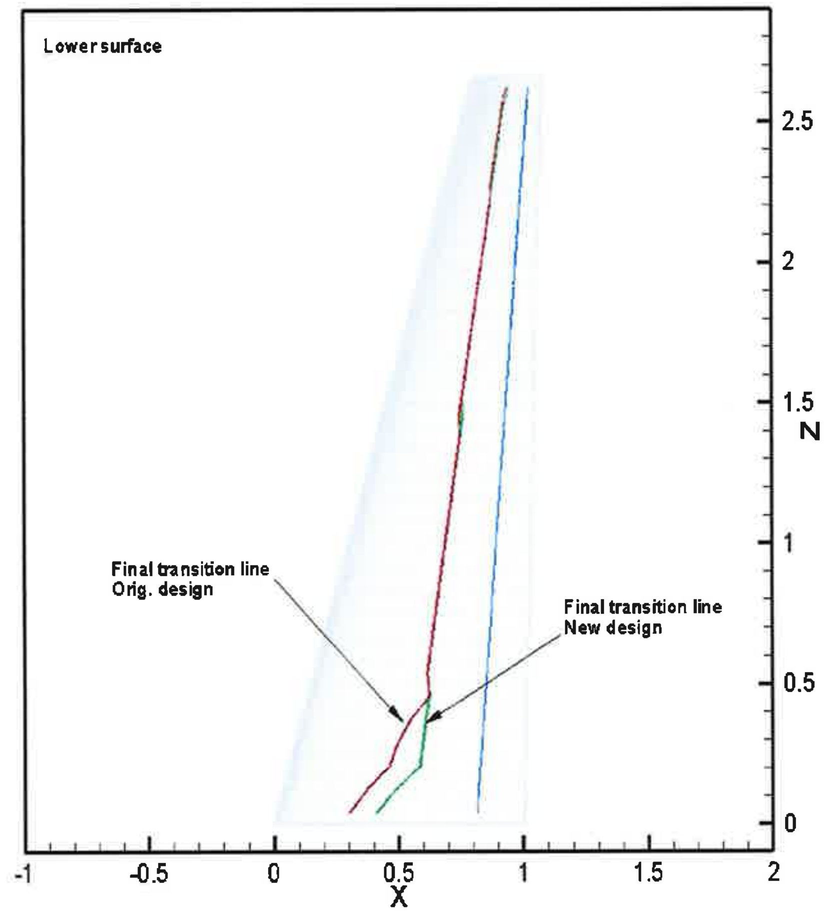
——— Re-designed

Mach = 0.74



Redesign of Natural-Laminar-Flow Wing Design





Summary and Conclusion

Summary and Conclusion

- This paper focuses on the application of adjoint method for natural-laminar-flow airfoil and wing design.
- A transition prediction module consisting of a boundary-layer code and two e^N-database methods for streamwise and crossflow instabilities are coupled with the flow solver to predict and prescribe transition locations automatically.
- The coupling of a 2D RANS solver with a transition prediction module provides reasonable accurate transition locations.
- For 3D wing configurations, the results are still reasonable and can be used for the estimation of aerodynamic performance at the initial design stage.
- The differences in aerodynamic coefficients are evident between 100% turbulence and laminar-turbulent transition model, and this indicates the necessary of incorporating transition prediction mechanism with flow solver for more realistic results.
- The new designed wing has an improvement over wide range of Mach numbers and the airfoil has thickness-to-chord ratio about 15% which gives the airplane sufficient fuel volume for the required range.

Acknowledgement

This work has benefited greatly from the support of

- Krumbein, A. and Sturdza, P., who share their expertise on transition prediction.
- Prof. Horton, H. P., who kindly enough provide us the laminar boundary-layer code to make this research possible.