Decentralized Control of a String of Vehicles

Antony Jameson and Kui Ou

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Aeronautics and Astronautics Department, Stanford, CA

1 Introduction

The second part of this study examines the use of decentralized control strategies to enable stable control of a platoon of vehicles in close formation. Energy stability was proved for three different strategies using feedbacks from no more than two vehicles ahead or behind any given vehicle. This report presents the results of numerical simulations of the performance of these three strategies.

2 Step response of a string of vehicles

Let's consider the response of a string of vehicles as the lead vehicle assumes an instantaneous displacement of its position. In reality a step vehicle displacement cannot be achieved since it requires an infinite velocity and acceleration. Nevertheless it is useful to investigate how the system of vehicles responses to a step input using various decentralized control techniques. Velocity response of a string of vehicles will be studied in the subsequent section.

The problem has been set up as follows. We consider a string of N vehicles with an initial uniform distribution of unit spacings between neighboring vehicles. At the start of the simulation, at time $t = 0$, the lead vehicle is displaced from $d = 0$ to $d = 1$, where d is the displacement of the vehicle. Three control strategies have been used to control the system of vehicles in response to the lead vehicle displacement. These are the forward 3-point scheme, symmetric 3-point scheme, and symmetric 5-point scheme respectively.

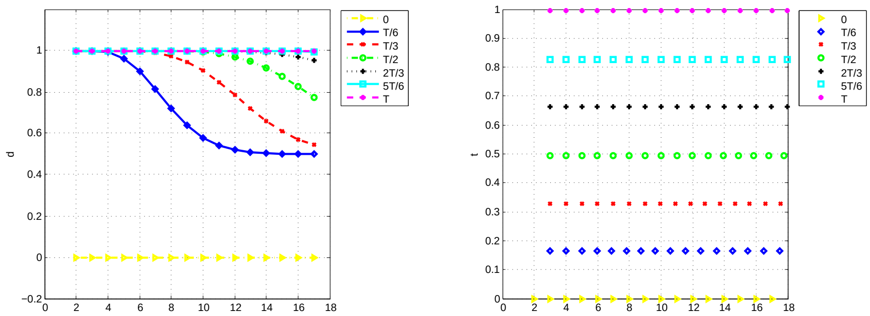
All three control strategies are decentralized such that each vehicle utilizes velocities and positions information only from its neighbors. As discussed in the previous section that the decentralized control strategies formulated in this study resemble solutions to partial differential equations, the numerical solutions to the control problems have time step stability requirements similar to the CFL conditions in the field of computational fluid dynamics. It appears that there is a CFL like restriction for the velocity as well as the position feedback. The time step stability of the numerical solution is determined by the more stringent of the two, which is usually the term involving velocity. In the subsequent analysis, we use subscript v and d to denote the velocity and displacement CFL conditions as CFL_v and CFL_d respectively.

2.1 Numerical results

The numerical results for a 17-vehicle (N=17) system subject to a step input are shown below.

2.1.1 Numerical solution using forward 3-point control

Figure 1 (a) plots the time history at 6 equal intervals, of the displacements of vehicle string. Figure 1 (b) plots the actual spatial position of each vehicle at various time instances. In the simulation, we used $R = 2$, $a = 110$ and $t_f = 1$, where t_f is the termination time.



(a) (b)

Figure 1: a) Time history of vehicle displacements b) Spatial distribution of the vehicle platoon at various time instances. Results obtained using forward 3-point control with $R = 2$, $a = 110$ and $t_f = 1$.

2.1.2 Numerical solution using symmetric 3-point control

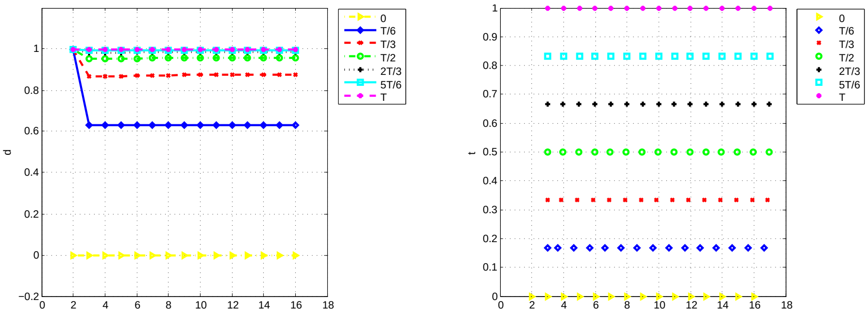
Figure 2 (a) plots the time history at 6 equal intervals, of the displacements of vehicle string. Figure 2 (b) plots the actual spatial position of each vehicle at various time instances. In the simulation, we used $R = 25$, $a = 300$ and $t_f = 1$, where t_f is the termination time.

2.1.3 Numerical solution using symmetric 5-point control

Figure 3 (a) plots the time history at 6 equal intervals, of the displacements of vehicle string. Figure 3 (b) plots the actual spatial position of each vehicle at various time instances. In the simulation, we used $R = 8$, $a = 240$ and $t_f = 1$, where t_f is the termination time.

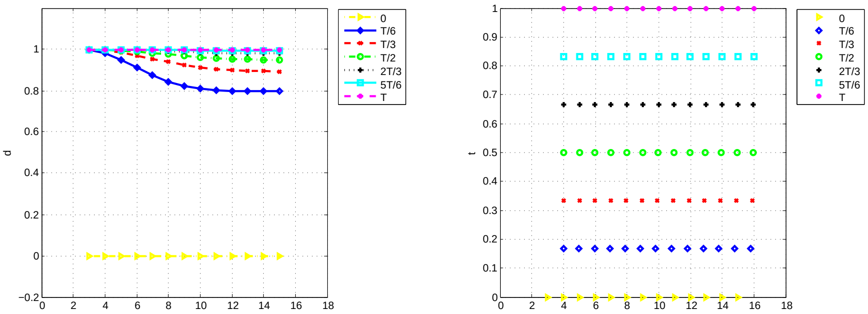
2.2 Forward 3-point control

When feedbacks are limited to at most two intervals, and with only a forward look, the numerical scheme of the control strategy previously proposed is re-



(a) (b)

Figure 2: a) Time history of vehicle displacements b) Spatial distribution of the vehicle platoon at various time instances. Results obtained using symmetric 3-point control with $R = 25$, $a = 300$ and $t_f = 1$.



(a) (b)

Figure 3: a) Time history of vehicle displacements b) Spatial distribution of the vehicle platoon at various time instances. Results obtained using symmetric 3-point control with $R = 8$, $a = 240$ and $t_f = 1$, where t_f.

stated here for completeness

$$\dot{v}_j = -2\frac{Ra}{\Delta x}(v_j - v_{j-1}) - \frac{a^2}{\Delta x^2}(d_j - 2d_{j-1} + d_{j-2})$$
$$\dot{d}_j = v_j.$$

2.2.1 Time step requirements

This system of first order equations can be integrated in time using multi-stage Runge-Kutta scheme with a time step restriction determined by CFL_v or CFL_d, whichever is more stringent. Here the CFL conditions are defined as:

$$CFL_v = \frac{Ra}{\Delta x}dt_v \ , \ CFL_d = \frac{a}{\sqrt{2}\Delta x}dt_d$$

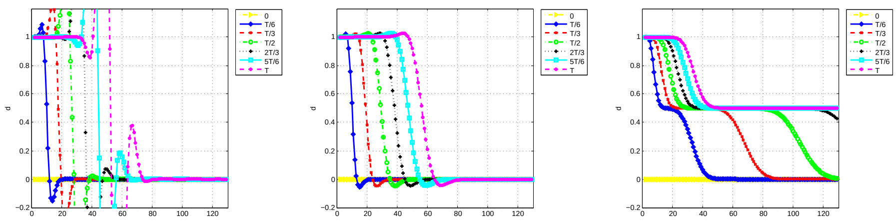
Stability requires $CFL_v \leq 1$ and $CFL_d \leq 1$. The time step of the numerical simulation dt is determined as

$$dt = \min(dt_v, dt_d)$$

It has been found, through numerical tests, that the time step related to the velocity term is usually the more stringent of the two, and hence determines the largest allowable time step.

2.2.2 System oscillation and response

While the time step stability requirement is determined by the CFL-like condition, previous analysis also indicates that, for the forward 3-point scheme, the parameter R needs to be greater than 1 for stability. This has been verified in the numerical tests. For $R < 1$, the scheme is unstable, as shown in Figure 4 (a). For $R = 1$, there is an initial oscillation, but which is not amplified but behaves more like a traveling wave. This is illustrated in Figure 4 (b). For $R > 1$, the system responds without any overshoot or oscillation.



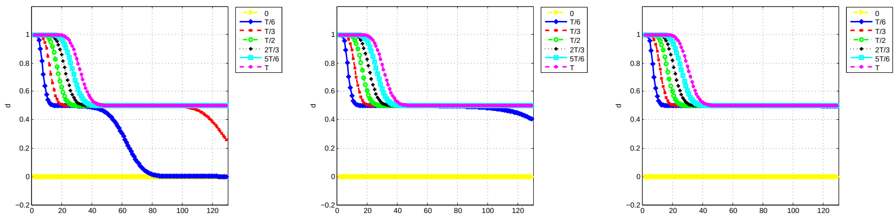
(a) (b) (c)

Figure 4: Step responses using a) R=0.95 b) R=1 c) R=1.5

The scheme has two free parameters. When the CFL conditions are satisfied and $R > 1$, the system is stable regardless the choice of R and a. However the choice of R and a determine the characteristics of the responses of the string of vehicles as follows. The choice of R determines the transient of the response, i.e. how oscillatory the string of vehicles is. Small R leads to oscillation while larger value leads to responses that are more damped. For a given R the choice of a, effectively the ratio of $\frac{a}{R}$, determines the responsiveness of the system as a whole, i.e. how fast does the entire platoon responses to the position error and adapt to the change commanded by the lead vehicle. The platoon adapts to the command in a shorter time with a larger choice of $\frac{a}{R}$. Effectively R can be treated as a damping parameter while a as the wave speed of information propagation. These are illustrated in the following numerical tests.

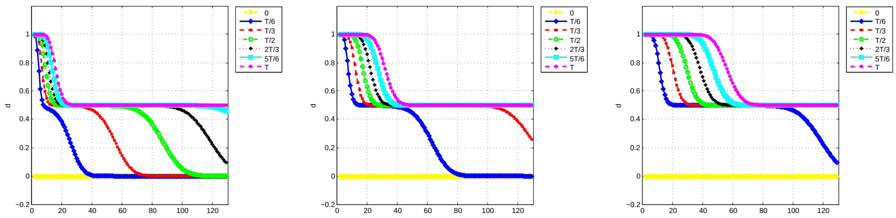
Figure 5 (a), (b) and (c) show the system responses with increasing value of R while holding $\frac{a}{R} = 55$ the same. With larger R, the transient is less oscillatory. The number of vehicles reaching the unit step remains the same.

Figure 6 (a), (b) and (c) show the system responses with increasing value of $\frac{a}{R}$ while holding $R = 2$ the same. With larger a, a greater number of vehicles adapt to the step command in a given time.



(a) (b) (c)

Figure 5: Step responses using a) R=2 b) R=3 c) R=4



(a) (b) (c)

Figure 6: Step responses using a) a=50 b) a=110 c) a=200

2.3 Symmetric 3-point control

With looks in both directions the numerical scheme using feedback only from immediate neighbors is

$$\begin{aligned}\dot{v}_j &= a_1(v_{j-1} - v_j) + a_1(v_{j+1} - v_j) \\ &\quad + b_1(d_{j-1} - d_j) + b_1(d_{j+1} - d_j) \\ \dot{d}_j &= v_j\end{aligned}$$

where

$$a_1 = \frac{Ra}{\Delta x} \; , \; b_1 = \frac{a^2}{2\Delta x^2}$$

or

$$\ddot{d}_j - \frac{Ra}{\Delta x}(\dot{d}_{j+1} - 2\dot{d}_j + \dot{d}_{j-1}) - \frac{a^2}{2\Delta x^2}(d_{j+1} - 2d_j + d_{j-1}) = 0$$

2.3.1 Time step requirements

Here the CFL conditions are defined as:

$$CFL_v = \frac{Ra}{\Delta x} dt_v \; , \; CFL_d = \frac{a}{\sqrt{2}\Delta x} dt_d$$

Stability requires $CFL_v \leq 1$ and $CFL_d \leq 1$. The time step of the numerical simulation dt is determined as

$$dt = \min(dt_v, dt_d)$$

It has been found, through numerical tests, that the time step related to the velocity term is the more stringent of the two, and hence determines the largest allowable time step.

2.3.2 System oscillation and response

Previous analysis indicates that the scheme is stable for $R > 0$. For a stable CFL condition for the time step, we find that increasing R increases the damping factor of the control system, while increasing a increases the speed of traveling wave due to the step command from the lead vehicle. The optimal choice of R and a depends on the desired response time as well as the total length (i.e. number) of the vehicle platoon, since it is reasonable to expect that the wave speed should scale with the ratio of total length scale over response time. For example, a choice of $R = 25$ and $a = 300$ leads a critically damped 17-vehicle system response with a time scale of 1 second, as shown in Figure 2. For a given a, increasing R increases the contribution of the velocity feedback and diminishes the contribution of the position feedback. While the response is less oscillatory, it takes a longer time to reach the desired state.

Consider the case where we have 65 vehicles and we wish the system to adapt to the lead vehicle command within a 1 second interval. The response of the system can be increased by increasing a, which can be written as a product of $R \times \frac{a}{R}$. To increase a, one can increase R for a given $\frac{a}{R}$, or increase $\frac{a}{R}$ for a given R. The effects are different. By picking a value of $\frac{a}{R}$, the relative importance of position and velocity feedback is determined, which in turn decides how damped or oscillatory the system behavior is. Further picking R determines the wave speed a, hence the amount of time to reach the desired state. Therefore one can pick a small $\frac{a}{R}$ to increase damping while picking a large R to improve system response. The downside is a very restrictive time step because dt_v scales with Ra or $\frac{R^2}{\frac{a}{R}}$. The alternative is to pick moderate values for both $\frac{a}{R}$ and R. The system response time is comparable, the time step certainly less restrictive, but the system will be more oscillatory. Finally, one can pick a very large a and also a very large $\frac{a}{R}$. This can lead to a large R. We have investigated the system responses under various choices of a, R, and $\frac{a}{R}$. The results are shown below.

To understand the effect of wave speed, we fix the ratio $\frac{a}{R}$ and vary a. To make the analysis more general, we further define $a = k_a \frac{L}{T}$, where $L = 65$ is the length of the string and $T = 1$ is the final time. Figure 7 shows the systems with increasing value of k_a while holding $\frac{a}{R} = 10$.

To investigate the effect of $\frac{a}{R}$, we consider first the case with $k_a = 5$. We observe that with $\frac{a}{R} = 10$ the response is oscillatory, as shown in Figure **??** (a). To increase damping, we need a smaller $\frac{a}{R}$. Figure 8 (a), (b) and (c) show the systems with increasing value of $\frac{a}{R}$ while holding $k_a = 5$ the same. We see that smaller $\frac{a}{R}$ indeed increases damping. However, the system also responses more slowly.

Next we consider the case with $k_a = 15$. Figure 9 (a), (b) and (c) show the system with increasing value of $\frac{a}{R}$ while holding $k_a = 15$ the same. Here we observe again that small $\frac{a}{R}$ leads to slower system response while large $\frac{a}{R}$ leads to more oscillation.

Finally, we can use very large $\frac{a}{R}$ and very large a to obtain fast response and a large value of R. Consider the case with $k_a = 50$.

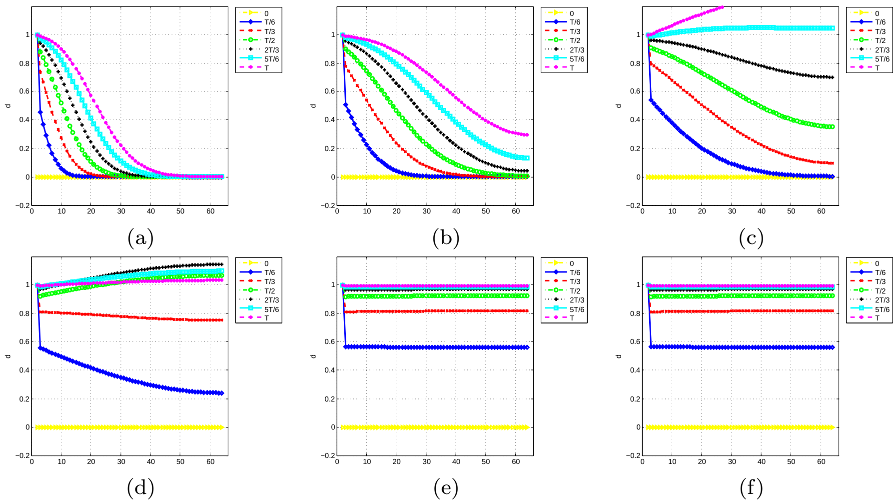


Figure 7: a) k_a=0.5 b) k_a=1 c) k_a=2 d) k_a=5 e) k_a=15 f) k_a=20

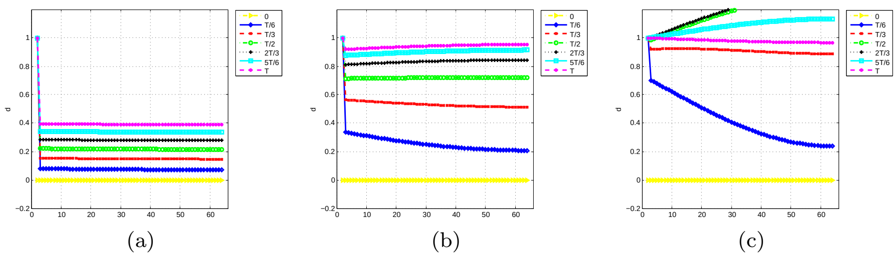


Figure 8: a) a/R=1 b) a/R=5 c) a/R=15

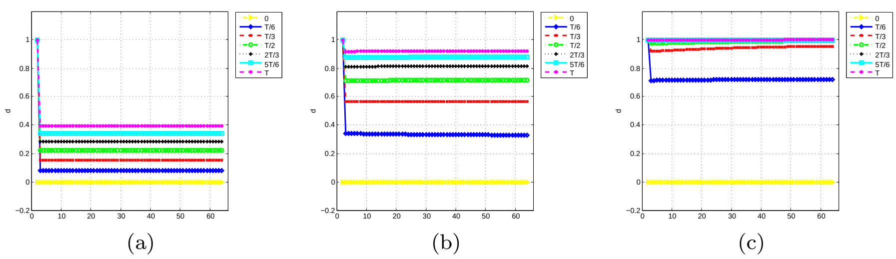
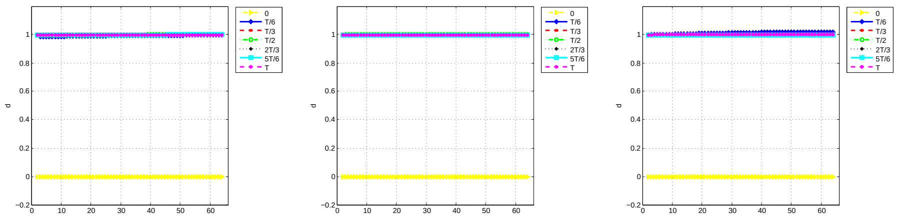


Figure 9: a) a/R=1 b) a/R=5 c) a/R=15

2.4 Symmetric 5-point scheme

With looks in both directions, and using feedback from immediate two neighbors, another possible numerical solution is

$$\begin{aligned}\dot{v}_j &= a_1(v_{j-1} - v_j) + a_1(v_{j+1} - v_j)\\ &\quad + b_1(d_{j-1} - d_j) + b_1(d_{j+1} - d_j)\\ &\quad + b_2(d_{j-2} - d_j) + b_2(d_{j+2} - d_j)\\ \dot{d}_j &= v_j\end{aligned}$$



(a) (b) (c)

Figure 10: a) a/R=50 b) a/R=100 c) a/R=150

where

$$a_1 = \frac{Ra}{\Delta x^2} \ , \ b_1 = \frac{4a^2}{\Delta x^4} \ , \ b_2 = \frac{a^2}{\Delta x^4}$$

or

$$\ddot{d}_j - \frac{Ra}{\Delta x^2}(\dot{d}_{j-1} - 2\dot{d}_j + \dot{d}_{j+1}) - \frac{a^2}{\Delta x^4}(d_{j-2} - 4d_{j-1} + 6d_j - 4d_{j+1} + d_{j+2}) = 0$$

2.4.1 Time step requirements

Here the CFL conditions are defined as:

$$CFL_v = \frac{2Ra}{\Delta x} dt_v \ , CFL_d = \frac{a}{\Delta x^2} dt_d$$

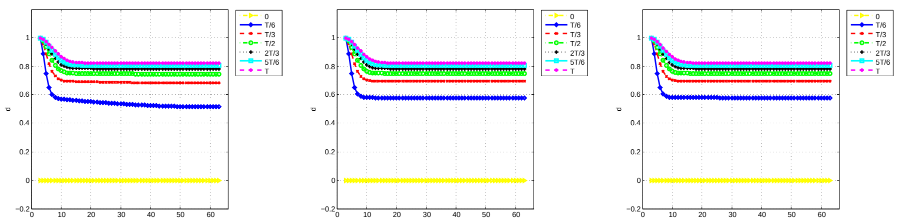
Stability requires $CFL_v \leq 1$ and $CFL_d \leq 1$. The time step of the numerical simulation dt is determined as

$$dt = \min(dt_v, dt_d)$$

It has been found, through numerical tests, that the time step related to the velocity term is the more stringent of the two, and hence determines the largest allowable time step.

2.4.2 System oscillation and response

Figure 11 plots the system responses with increasing value of k_a while holding $\frac{a}{R} = 10$.



(a) (b) (c)

Figure 11: a) ka=10 b) ka=30 c) ka=50

Figure 12 plots the system responses with increasing value of $\frac{a}{R}$ while holding $k_a = 50$.

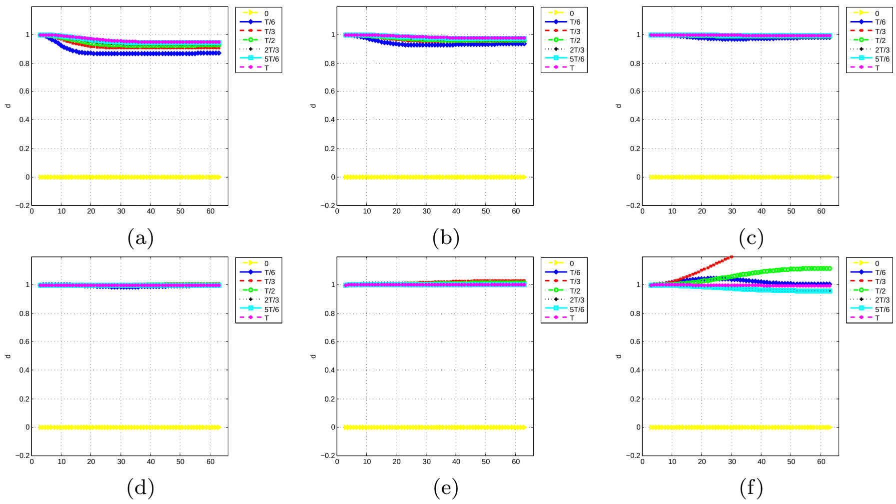


Figure 12: a) a/R=100 b) a/R=200 c) a/R=300 d) a/R=350 e) a/R=400 f) a/R=1000

3 String response with a source term

3.1 Symmetric 3-point control

We consider the forcing function with both the temporal and spatial harmonic distributions. A symmetric source term can be defined as follows:

$$f_s = 200 \sin\left(\frac{4\pi t}{t_f}\right) \sin\left(\frac{2\pi x}{L}\right)$$

The results are plotted in Figure 15 for increasing value of a for constant $\frac{a}{R}$.

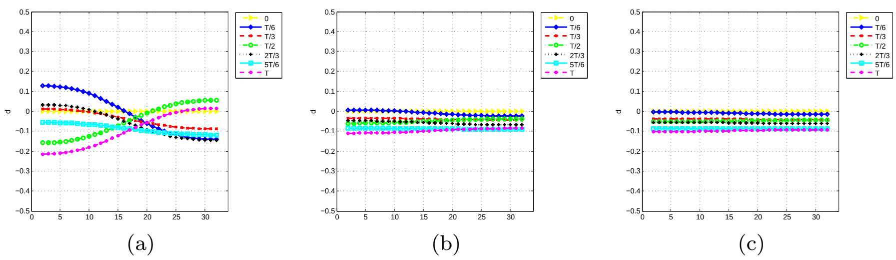
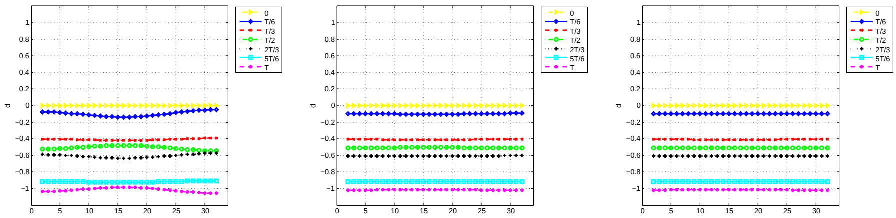


Figure 13: a) ka=10, a/R=10 b) ka=30, a/R=10 c) ka=50, a/R=10

A asymmetric source term can be defined as follows:

$$f_s = 200 \sin\left(\frac{4\pi t}{t_f}\right) \cos\left(\frac{2\pi x}{L}\right)$$

The results are plotted in Figure 16 for increasing value of a for constant $\frac{a}{R}$.



(a) (b) (c)

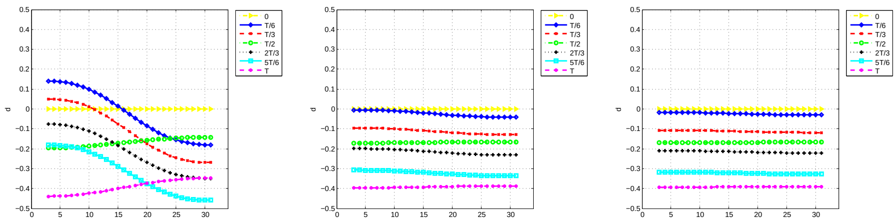
Figure 14: a) ka=10, a/R=10 b) ka=30, a/R=10 c) ka=50, a/R=10

3.2 Symmetric 5-point control

We consider the forcing function with both the temporal and spatial harmonic distributions. A symmetric source term can be defined as follows:

$$f_s = 200 \sin\left(\frac{4\pi t}{t_f}\right) \sin\left(\frac{2\pi x}{L}\right)$$

The results are plotted in Figure 15 for increasing value of a for constant $\frac{a}{R}$.



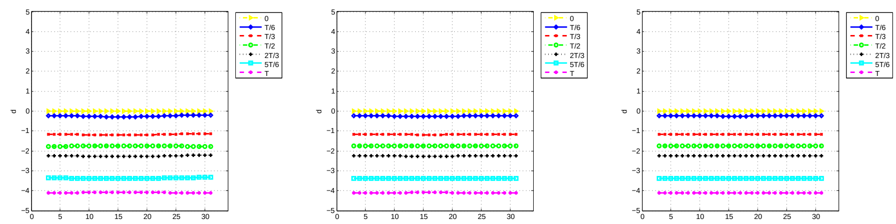
(a) (b) (c)

Figure 15: a) ka=10, a/R=10 b) ka=30, a/R=10 c) ka=50, a/R=10

A asymmetric source term can be defined as follows:

$$f_s = 200 \sin\left(\frac{4\pi t}{t_f}\right) \cos\left(\frac{2\pi x}{L}\right)$$

The results are plotted in Figure 16 for increasing value of a for constant $\frac{a}{R}$.



(a) (b) (c)

Figure 16: a) ka=10, a/R=10 b) ka=30, a/R=10 c) ka=50, a/R=10

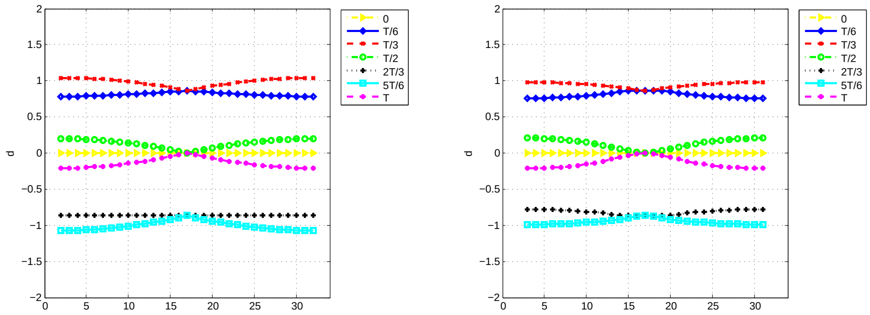
4 String response with sinusoidal input

Consider the case when the string is following the command of a vehicle in the center of the platoon. The center vehicle is undergoing a sinusoidal profile in its displacement according to

$$d = sin(\frac{2\pi t}{T})$$

where T is the period of the oscillation.

The resulting responses of the string using the symmetric 3-point scheme and 5-point scheme are shown in Figure 17(a) and (b) respectively.



(a) (b)

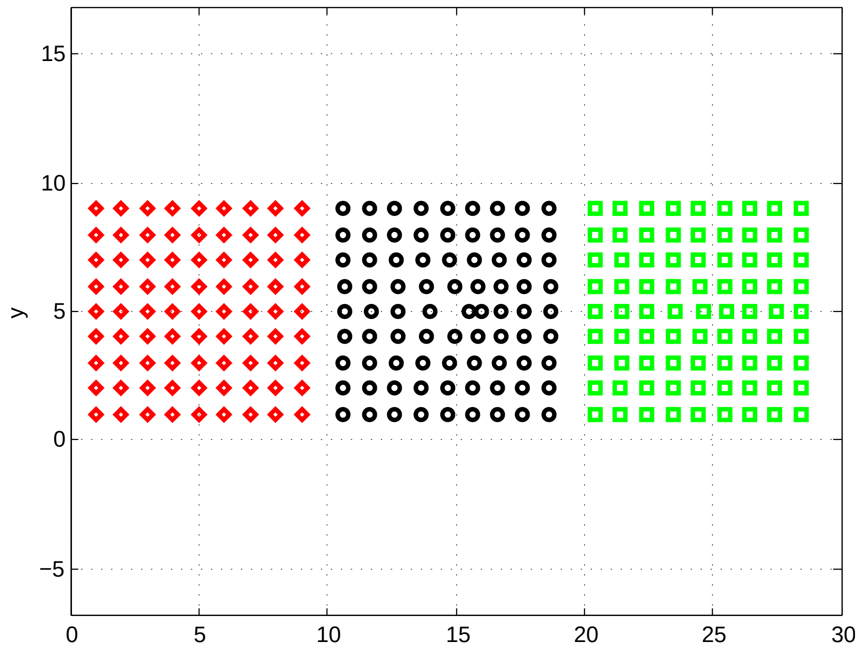
Figure 17: a) ka=5, a/R=10 b) ka=5, a/R=10

5 2D platoon response with sinusoidal input

The numerical scheme has also been extended to two dimensional. A 2D platoon following the sinusoidal displacement of a lead vehicle at the very center of the platoon was simulated. The lead vehicle in the center of the platoon is moving according to

$$d = 20 \times sin(\frac{2\pi t}{T})$$

The snapshots of the platoon distribution at certain instances in time are captured in Figure 18. This was obtained using the symmetric 3-p scheme.



(a)

Figure 18: a) ka=10, a/R=10

6 Solutions to partial differential equations

6.1 Solution to $u_t = \sigma u_{xx}$

Consider the following partial differential equation

$$\frac{\partial u}{\partial t} = \sigma \frac{\partial^2 u}{\partial x^2}$$

A possible solution assumes the following form

$$u = e^{-\alpha t} U(x)$$

Upon substitution we have

$$-\alpha U = \sigma \frac{\partial^2 u}{\partial x^2}$$

This can be satisfied if

$$U = \cos nx, n^2 = \frac{\alpha}{\sigma}, \alpha = \sigma n^2$$

A general form of the solution is

$$u = \sum a_n e^{-\sigma n^2 t} \cos nx$$

If the partial differential equation has a forcing term

$$\frac{\partial u}{\partial t} = \sigma \frac{\partial^2 u}{\partial x^2} = f(x,t)$$

Let

$$f = \cos nx e^{ikt}$$

then

$$u = A\cos nx e^{ikt}$$

where A is

$$(ik + \sigma n^2)A = 1$$

$$A = \frac{1}{ik + \sigma n^2}$$

$$|A| = \frac{1}{\sqrt{\sigma^2 n^4 + k^2}}$$

6.2 Solution to $u_{tt} = a^2 u_{xx}$

Consider the following advection like partial differential equation

$$\frac{\partial^2 u}{\partial t^2} = a^2 \frac{\partial^2 u}{\partial x^2}$$

A possible solution assumes the following form

$$u = e^{ikt}U(x)$$

Upon substitution we have

$$a^2 U_{xx} + k^2 U = 0$$

This can be satisfied if

$$U = \cos nx, n^2 a^2 = k^2, k = an$$

A general form of the solution is

$$u = \sum a_n e^{iant} \cos nx$$

If the partial differential equation has a forcing term

$$\frac{\partial^2 u}{\partial t^2} - a^2 \frac{\partial^2 u}{\partial x^2} = f(x,t)$$

Let

$$f = e^{ikt}\cos nx$$

then

$$u = Ae^{ikt}\cos nx$$

where

$$(-k^2 + a^2 n^2)A = 1$$

Resonance occurs if $k = an$

$$A = \frac{1}{a^2 n^2 - k^2} \to \infty \text{ if } k = an$$

6.3 Solution to $u_{tt} - Ra^2u_{xxt} - a^2u_{xx} = f(x,t)$

Consider the partial differential equation

$$\frac{\partial^2 u}{\partial t^2} - 2Ra\frac{\partial^3 u}{\partial x^2 \partial t} + a^2\frac{\partial^4 u}{\partial x^4} = f(x,t)$$

where

$$f(x,t) = e^{ikt}\cos nx$$

Let

$$u = Ae^{ikt}\cos nx$$

then

$$\left(-k^2 + a^2n^2(1+iRk)\right)A = 1$$

$$A = \frac{1}{a^2n^2 - k^2 + iRka^2n^2}$$

$$|A| = \frac{1}{\sqrt{(a^2n^2-k^2)^2 + R^2k^2a^4n^4}}$$

If $k = an$

$$|A| = \frac{1}{Rka^2n^2} = \frac{1}{Ra^3n^3}$$

6.4 Solution to $u_{tt} - 2Ra^2u_{xxt} + a^2u_{xxxx} = f(x,t)$

Consider the partial differential equation

$$\frac{\partial^2}{\partial t^2} - 2Ra\frac{\partial^3 u}{\partial t \partial^2 x} + a^2\frac{\partial^4}{\partial x^4} = f(x,t)$$

where

$$f(x,t) = e^{ikt}\cos nx$$

If $R = 1$ the partial differential equation becomes

$$\left(\frac{\partial}{\partial t} - a\frac{\partial^2}{\partial x^2}\right)^2 u = f$$

Let

$$u = Ae^{ikt}\cos nx$$

then

$$(-k^2 + iRkan^2 + a^2n^4)A = 1$$

$$A = \frac{1}{a^2n^4 - k^2 + iRkan^2}$$

$$|A| = \frac{1}{\sqrt{(a^2n^4)^2 + R^2k^2a^2n^4}}$$

If $k = an^2$

$$|A| = \frac{1}{Rkan^2} = \frac{1}{Ra^2n^4}$$