Then to Now, the CFD Revolution, a Personal and Historical Perspective

Antony Jameson

Aerospace Computing Laboratory
Department of Aeronautics and Astronautics
Stanford University

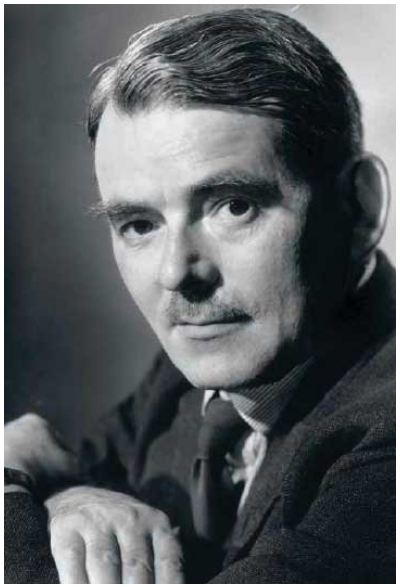
University of Toronto,
June 4, 2014

Outline of the Talk

1. Introduction
   - Biography
2. Part 1: The development of CFD for aerodynamic analysis
   - Emergence of CFD
   - Personal position
   - Personal contributions
   - Reflections on the JST Scheme
   - Reflections on the AIRPLANE code
   - The quest for a fast solver
   - Impact of bugs
3. Part 2: The development of computational design methods
   - Early efforts – Inverse design Lighthill-James-Jameson method (SYN1)
   - Aerodynamic Design & Shape Optimization via Control Theory
4. Part 3: The current status of CFD
   - High Order Methods
5. Summary and Conclusions

Genealogy and biography

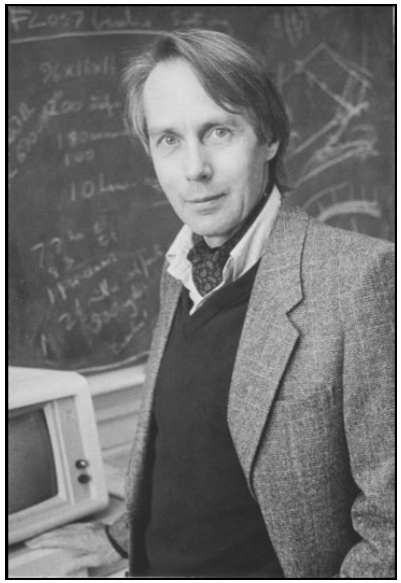
Genealogy



Frank Whittle

William Hawthorne

Arthur Shercliff



Antony Jameson

Influential People in My Life and Sponsors

Father Jacques	St. Edmund's School	Shillong, India
Sandy (Mr. Sanderson)	Mowden School	England
Dr. Hutton	Winchester College	England
Walter Oakeshott	Winchester College	England
Ernest Franke	Trinity Hall, Cambridge	England
Arthur Shercliff	Cambridge Engineering	England
Sir William Hawthorne	Cambridge Engineering	England
Len Murray	Trades Union Congress	England
Rudy Meyer	Grumman Aerospace	US
Grant Hedrick	Grumman Aerospace	US
Paul Garabedian	Courant Institute	US

Acknowledgments

I would like to thank my sponsors who have supported my various projects over the years. Most notably, they are:

- **Jerry C. South, Jr., NASA**
- **Morton Cooper, ONR**
- **Spiro Lekoudis, ONR**
- **Charles Holland, AFOSR**
- **Fariba Fahroo, AFOSR**
- **Leland Jameson, NSF**

Aside from the contributions of my students to the research of our group, I am specially indebted to them for their assistance in creating papers and presentations in electronic form. In recent years I have been particularly helped in this regard by

- **Kasidit Leoviriyakit**
- **Nawee Butsuntorn**
- **Kui Ou**
- **Andre Chan**

Biographical elements

Experiences in the British Army

I served in the British Army for two years, 1955-58, directly after leaving 'High School' (Winchester) before going to Cambridge.

Following family tradition I was in the Royal Engineers (Sappers)

After six months of basic training I passed a selection board and spent a further six months in Officer Training.

Then I was sent to Malaya as a 2nd Lieutenant, age 19, where I commanded parties of about 70 men, so I have been going downhill ever since then.

A particular piece of military training

We were given instruction in how to plan and execute in 4 stages, which I have found useful throughout my life !

1. Decide on a goal
2. Make an 'appreciation' of the situation

   The enemy has 1000 troops, you have 50
   – the goal may be unrealistic so one may have to iterate between (1) and (2)

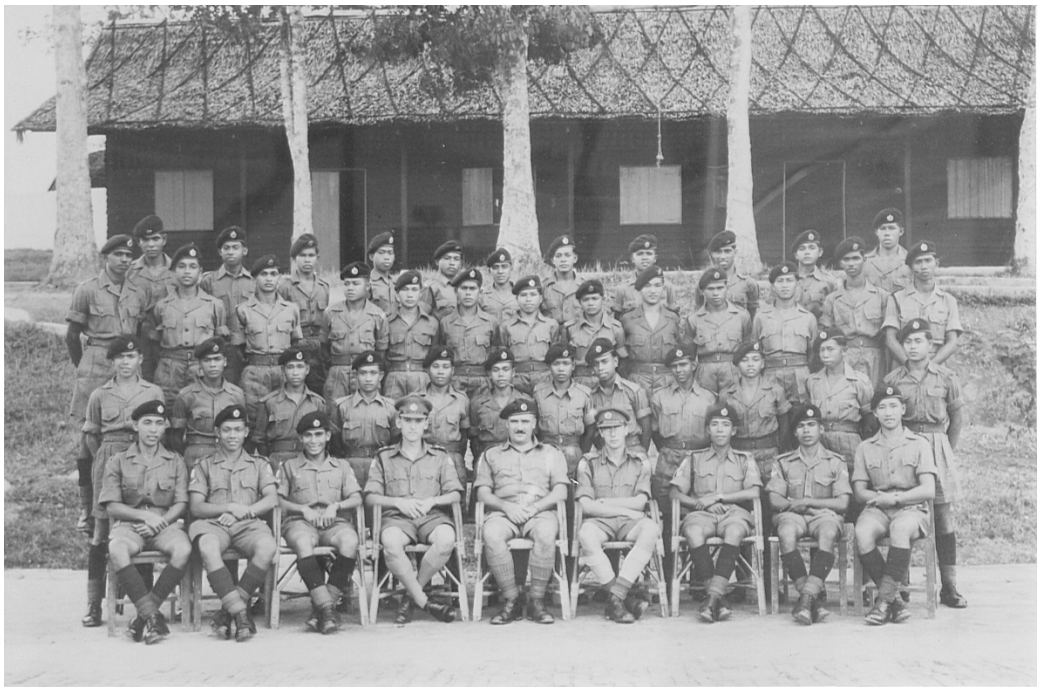
3. Formulate a plan
4. Carry it out !

My time in Malaya

I was training Malaysian soldiers in all aspects of military engineering, including mine warfare, watermanship, bridge building, etc.

One exercise was to build a bridge with wooden poles bound together with rope bindings (similar to the 'Bridge over the River Kwai').

Our bridge was about 70 feet long over a dry gap of about 15 feet deep created by a bulldozer.



My time in Malaya

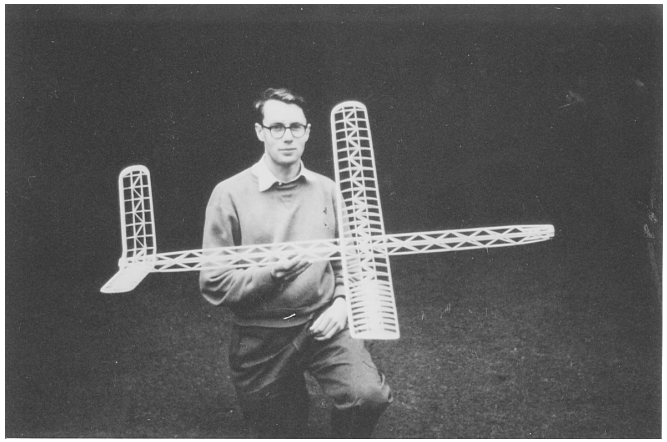
According to the military handbook, the horizontal poles would not slip over the vertical poles if the rope bindings were tight enough, with no need to hammer in nails to prevent the bindings from slipping. Personally, I did not believe this!

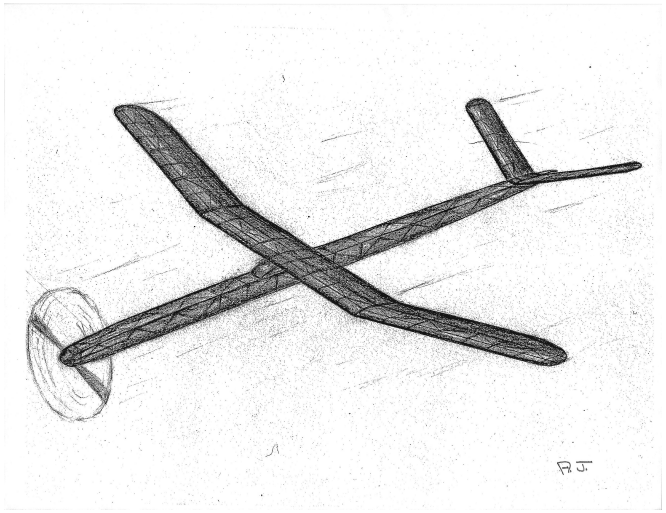
After we had completed the bridge my immediate commander, Major ”Pinky” Parker arrived in his Land Rover. He liked the bridge and wanted to drive over it.

I told him this was a bad idea, but he kept insisting. Finally, I suggested we test the bridge by having the squad march onto it. As I expected, the deck slipped on one side under the load, finishing at an angle of about 30 degrees and my men were left clinging to it – fortunately, no one was hurt. But if Major Parker had driven onto the bridge, it would have gone off the side, probably turning over and landing on him.

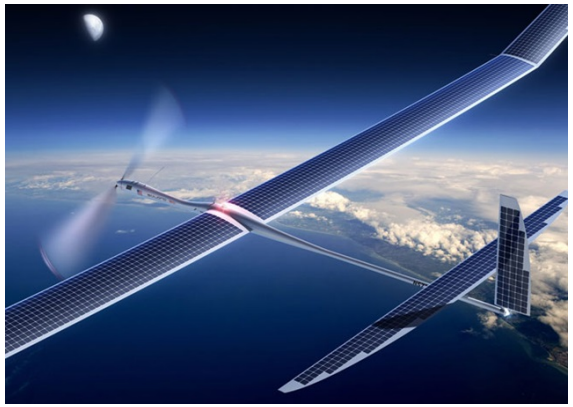
Well, I have more military tales to amuse you, but to return to the main topic....

Some designs just never get old...





My drone design, circa 1955



Google / Titan Aerospace Solera 50 drone

How I got started in CFD

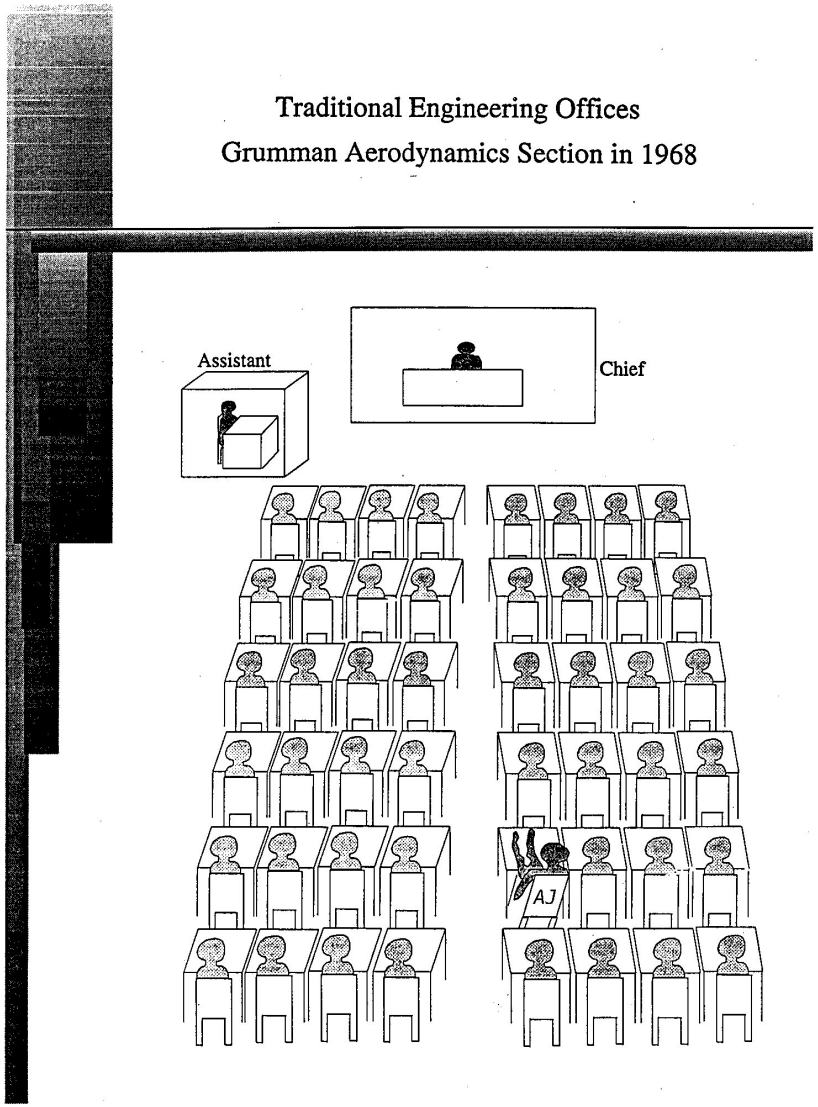
Then he handed me off to his Group Leader for Stability and Control, Rudy Meyer. When I joined Grumman in 1966 I was placed in the Aerodynamics Section. On my first day, I was interviewed by the Section Chief (Bill Murphy) who told me it had been a mistake to hire me! He had no need for anybody with a Ph.D., and the previous year he had had one who was 'stuck up' and useless.

I wound up spending the next two years working on optimal control techniques, in particular what is now called "sparse" control where one tries to minimize the number of feedback interconnections.

After two years I was promoted to a staff position, reporting directly to the Manager of Flight Sciences, (Stu Harvie).

Can you guess what? Stu wasn't sure what to do with me !

At Grumman


Traditional Engineering Offices
Grumman Aerodynamics Section in 1968
Assistant
Chief
AJ

Whitcomb's file

At first he had me work on the design of a wind tunnel. This was not a very good idea, as I had no knowledge of how to design a wind tunnel, and in any event Grumman would never have invested the money needed to build one.

In November 1969, Stu suddenly called me to his office, and told me we needed to do something about supercritical wings. I didn't know what he meant by a 'supercritical wing' and asked him. He explained that Whitcomb had recently developed wing sections which worked well in transonic flow. This was an important advance, but Whitcomb was an experimentalist who relied on his intuition to make changes to his models, and we couldn't rely on "Whitcomb's File."

I suggested that I should maybe to the AIAA Aerospace Science Meeting which was being held in New York that winter and try to find out what was going on in transonic flow research. This was the meeting where Earll Murman first presented the Murman-Cole scheme for numerically solving the transonic small disturbance equation.

I reported back that this was a good idea, which we should try, but no one at Grumman seemed interested. So, I decided to try myself. At that time I had no experience of numerical methods nor of transonic flow. *What I could see was the opportunity to do something interesting .....*

A long term plan (or, the long road to Toronto???)

I think that my military training helped me at this point!

1. Decide on a goal
   In the long term, this was to calculate transonic flow over a complete aircraft.
2. Make an appreciation of the situation
   (a) It could only be done numerically.
   (b) Existing computers were powerful enough only to attempt two dimensional calculations, but with the rapid rate of improvement three dimensional calculations should be feasible within a few years.
   (c) New numerical algorithms would be needed, but one could build on the idea of Murman and Cole.
3. Formulate a plan
   Write a progression of solvers with incremental increases in the mathematical model complexity and also in the geometric complexity of the configurations to be analyzed:
   - transonic potential flow
   - Euler equations
   - Navier Stokes equations
   - 2D airfoils
   - 3D wing, wingbody combinations
   - complete aircraft

   Hence FLO XXX Series
4. Carry out the plan
   (I knew I probably couldn't if I stayed at Grumman.)

Part 1:
The development of CFD for aerodynamic analysis

Emergence of CFD – Its multi-disciplinary nature

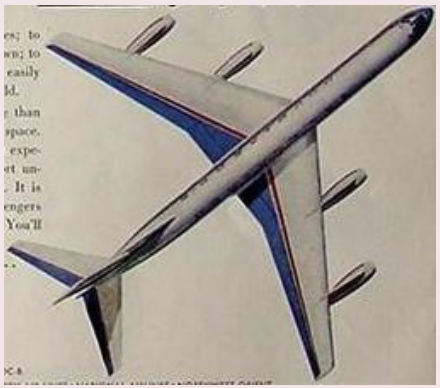
History of CFD in Van Leer's view

Emergence of CFD

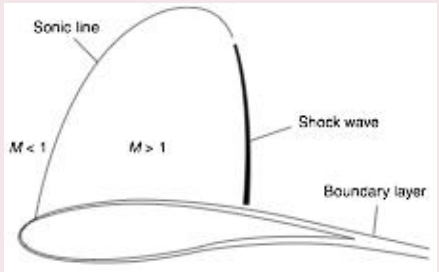
- In 1960 the underlying principles of fluid dynamics and the formulation of the governing equations (potential flow, Euler, RANS) were well established.
- The new element was the emergence of powerful enough computers to make numerical solution possible – to carry this out required new algorithms.
- The emergence of CFD in the 1965 – 2005 period depended on a combination of advances in computer power and algorithms.

Some significant developments in the 60s:

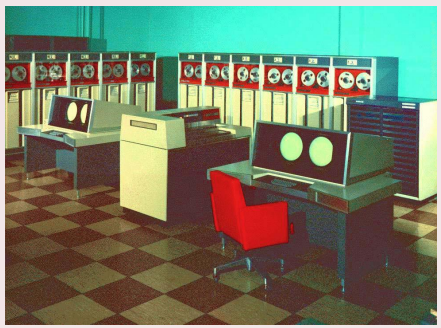
- Birth of commercial jet transport – B707 & DC-8
- Intense interest in transonic drag rise phenomena
- Lack of analytical treatment of transonic aerodynamics
- Birth of supercomputers – CDC6600



DC 8

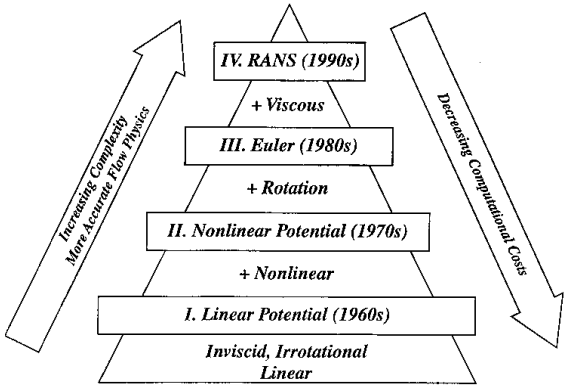


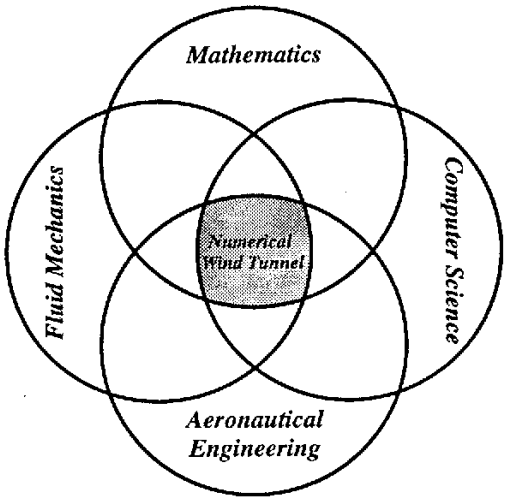
Transonic Flow



CDC 6600

Multidisciplinary nature of CFD





- In 1960 the underlying principles of fluid dynamics and the formulation of the governing equations (potential flow, Euler, RANS) were well established.
- The new element was the emergence of powerful enough computers to make numerical solution possible – to carry this out required new algorithms.
- The emergence of CFD in the 1965 – 2005 period depended on a combination of advances in computer power and algorithms.

Personal perspective

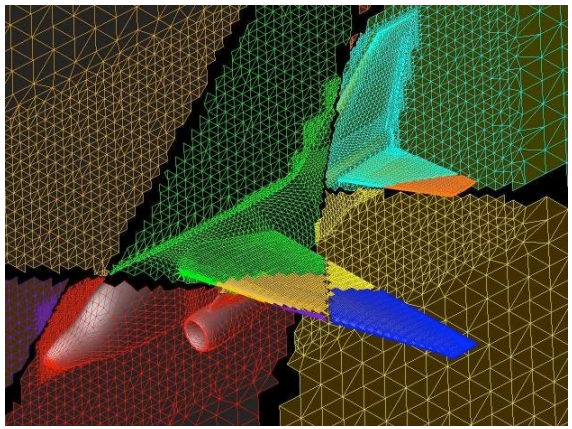
A foot in 3 camps

- Applied and computational mathematics
- Aerodynamics
- Airplane design

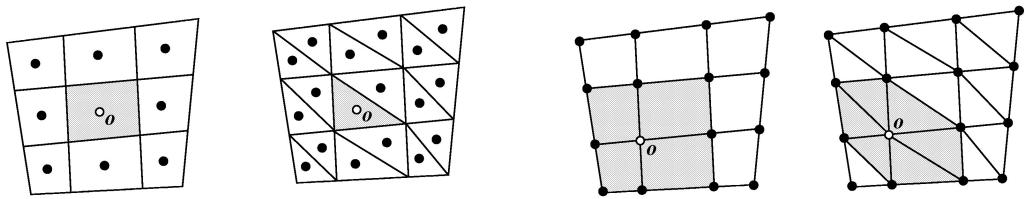
. . . But I only have 2 feet.

Basic principle of finite volume schemes for CFD

1. Divide the domain into a grid of computational cells.



2. Apply the conservation laws of mass, momentum and energy in integral form separately for each cell.



3. $5N$ equations for $5N$ unknowns on a grid of N cells

Highlights in CFD algorithm development

Mathematical Forerunners		
1959	Godunov	Shock capturing scheme for gas dynamics using piecewise constant state and Riemann solvers for interface fluxes
1960	Lax-Wendroff	Second order accurate scheme for hyperbolic equations

Potential Flow		
1962	Hess & Smith	Panel method: solution of Laplaces equation for general geometry
1970	Murman & Cole	Type dependent shock capturing scheme for the transonic small disturbance equation
1973	Jameson	Rotated difference scheme for transonic potential flow
1976 – 78	Jameson & Caughey	Flo22, Flo27: First accurate solution for transonic flow over a swept wing $\Longrightarrow$ Boeing A488
1982	Glowinsky *et al.*	Finite element solution of transonic potential flow for Falcon 50
1990	Forrester, Young *et al.*	Boeing TRANAIR

Highlights in CFD algorithm development

Euler and RANS equations		
1969	MacCormack	Two stage scheme for hyperbolic equations
1973	Boris & Book	Flux corrected transport (FCT) oscillation control via slope limiters
1974	van Leer	Higher order Godunov scheme – MUSCL
1981	Steger & Warming	Flux splitting
1981	Jameson, Schmidt & Turkel (JST)	Shock capturing via controlled diffusion – full convergence to steady state
1981	Ni	Multigrid Euler solver
1981	Roe	Approximate Riemann solver
1983	Harten	Theory of total variation diminishing (TVD) schemes
1983	Jameson	Agglomeration multigrid FAS scheme for the Euler equations
1985 – 86	Jameson, Baker & Weatherill	AIRPLANE code: first solution of 3D Euler equations on unstructured mesh – edge based data structure
1986 – 88	Yoon & Jameson	LUSGS scheme
1987	Harten, Engquist, Osher & Chakravarthy	Essentiall non-oscillatory (ENO) scheme
1990	Cockburn & Shu	Local discontrinuous Galerkin (LDG) method
1991	Jameson	Multigrid dual time stepping scheme for unsteady flow
1993	Liou	AUSM scheme
1994	Jameson	Theory of local extremum diminishing, (LED) scheme, SLIP, CUSP
1994 – 96	Liu, Osher, Chan & Shu	Weighted ENO (WENO) scheme
2001	Jameson & Caughey	Nonlinear SGS multigrid – Euler solution in 5 steps

Highlights in CFD algorithm development

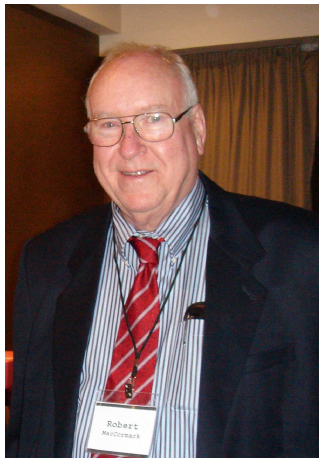
Aerodynamic shape design		
1945	Lighthill	Inverse design of airfoils by conformal mapping
1971	James & Jameson	Improved Lighthill method
1971	Lions	Methematical foundations of control theory for PDEs
1975	Bryson & Ho	Applied optimal control – Trajectory optimization
1979	Hicks & Henne	Wing design by numerical optimization using bump function
1984	Pironneau	Shape design for subsonic potential flow
1988	Jameson	Aerodynamic design via control theory (adjoint method)
1994	Jameson & Reuther	Optimum design using the Euler equations
1998	Jameson, Martinelli & Pierce	Optimum design using the Navier-Stokes equations

Some early contributors to CFD

Lax

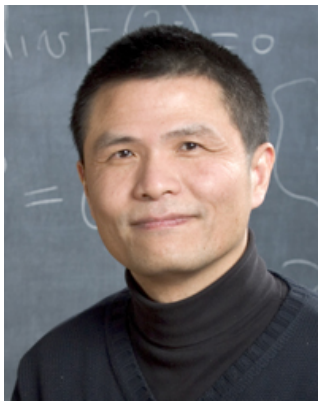
Godunov

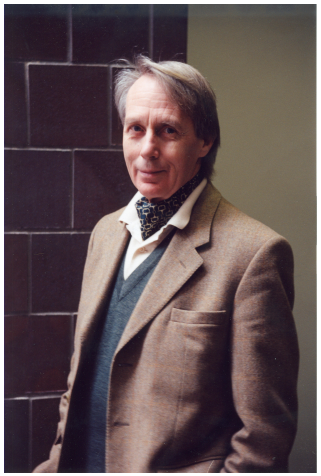
Gustafsson

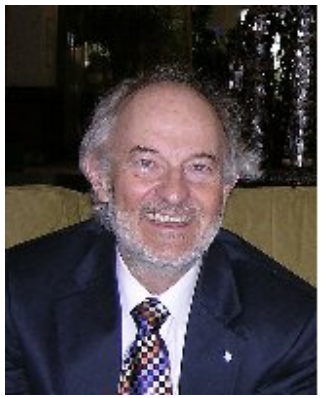

MacCormack

Harten

Osher


Shu


Jameson

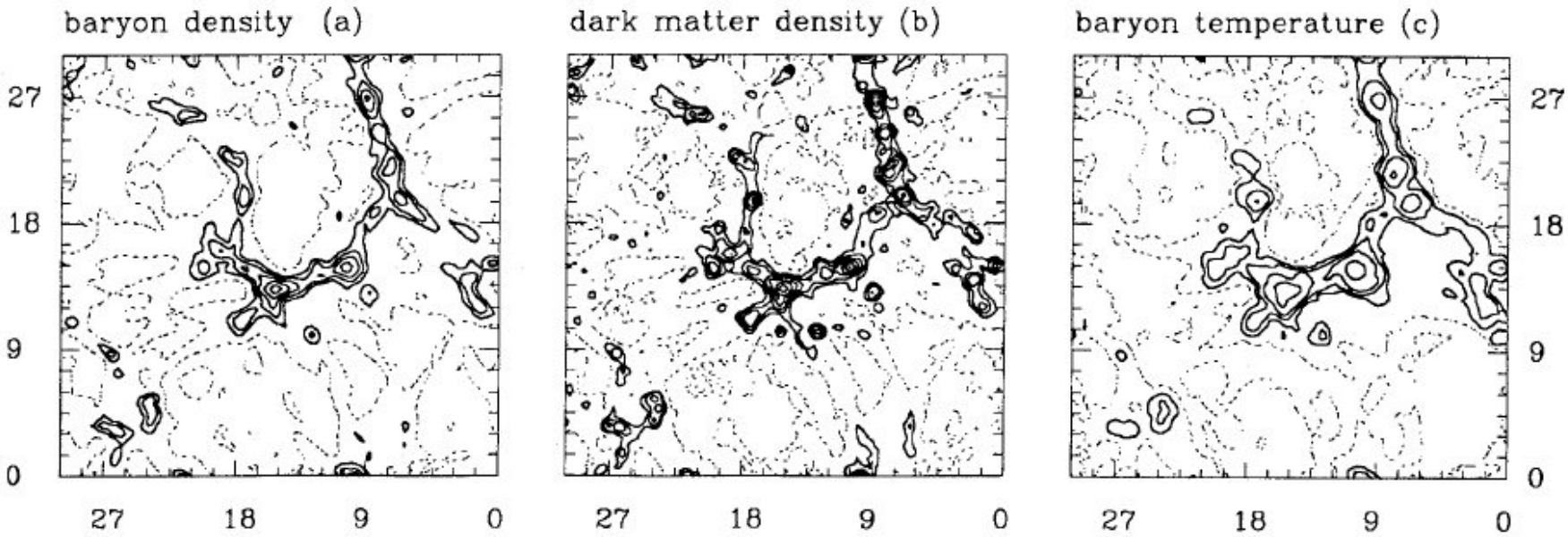

van Leer

Roe

Some personal contributions:

- Rotated difference scheme for transonic potential flow (incorporated in *FLO22*)
- Finite volume / element scheme for transonic potential flow (incorporated in *FLO27*)
- JST scheme for the Euler equations (incorporated in *FLO57*)
- Multigrid solution of the Euler equations (incorporated in *FLO82, FLO87*)
- LUSGS scheme (Yoon & Jameson, Rieger & Jameson)
- Dual time stepping (incorporated in *UFLO103, UFLO107*)
- Simulation of the evolution of the universe (R. Y. Cen, A. Jameson, F. Liu & J. P. Ostriker)
- Fast calculation of free surface flows and ship wave resistance (J. Farmer, L. Martinelli & A. Jameson)

Universe in a box



Reflections on the JST Scheme

Origins of the JST Scheme

The original JST scheme was developed in 1980-81 starting from a code that had been developed at Dornier by Rizzi and Schmidt to solve the Euler equations

This code implemented the MacCormack scheme in finite volume form with additional artificial dissipation to limit oscillations near shocks. It could not converge to a steady state and it appeared from the Stockholm Workshop in 1979 that none of the existing Euler solvers could reach a steady state.

The primary objective of the JST scheme was to solve the steady state problem. This objective was achieved through the use of blended low and high order artificial dissipation and modified Runge-Kutta time stepping with variable local time steps at a fixed CFL number.

Note: The author had been experimenting with Euler solvers since 1976 and had achieved steady state solutions for some simple geometries with the Z scheme. The code EUL1 still exists.

Original JST Scheme (1980)

The Dornier code (Rizzi-Schmidt) solved for w vol with MacCormack scheme + added diffusion

$$\sim \delta_x \epsilon \delta_x w \text{ vol}, \quad \epsilon \sim \left| \frac{p_{\jmath+1} - 2p_\jmath + p_{\jmath-1}}{p_{\jmath+1} + 2p_\jmath + p_{\jmath-1}} \right|$$

It did not preserve uniform flow on a curvilinear grid.

In order to fix this, move vol outside δ_x.

Then

$$w^{n+1} = w^n - \frac{\Delta t}{\text{vol}}(Q - D), \quad Q = \text{convective terms}$$

For dimensional consistency,

$$D \sim \delta_x \frac{\text{vol}}{\Delta t^*} \delta_x w$$

where Δt^* is nominal time step

$$\Delta t^* = \frac{\text{vol}}{(Q + cS)_\imath + (Q + cS)_\jmath}, \quad Q = \vec{q} \cdot \vec{S}$$

Higher order background diffusion was needed for convergence to a steady state. This had to be switched off in the vicinity of a shock to prevent oscillations.

Design Principles for the JST Scheme

Conservation: integral form

$\Longrightarrow$finite volume scheme

Exact for uniform flow on a curvilinear grid

$\Longrightarrow$constrains discretization, form of diffusion

Steady state independent of Δt

Eliminates Lax-Wendroff, MacCormack schemes

Concurrent computation

Eliminates LU-SGS schemes $\Longrightarrow$ RK schemes

Non-oscillatory shock capturing

$\Longrightarrow$by blended artificial diffusion

At least second order accurate

$\Longrightarrow$artificial diffusion of order Δx^3

Constant total enthalpy in steady flow

Eliminates Steger-Warming and other splittings

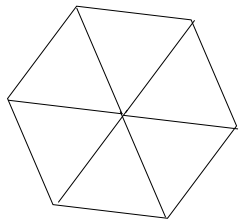
$\Longrightarrow$diffusion for energy equation $\sim \dfrac{\partial}{\partial x} \epsilon \dfrac{\partial}{\partial x} \rho H$

Simplicity

Mathematical Foundations for the JST Scheme: LED Scheme

A semi-discrete scheme is LOCAL EXTREMUM DIMINISHING (LED) if local maxima cannot increase and local minima cannot decrease.
A scheme in the form

$$\frac{dv_i}{dt} = \sum_{j \neq i} a_{ij}(v_j - v_i)$$



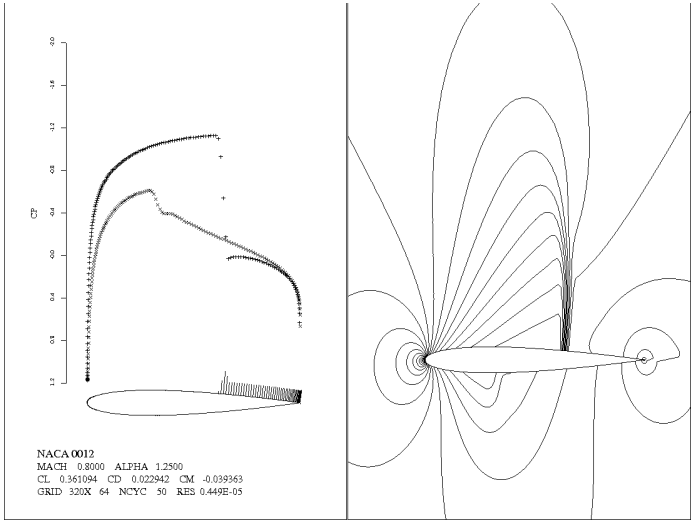
is LED if

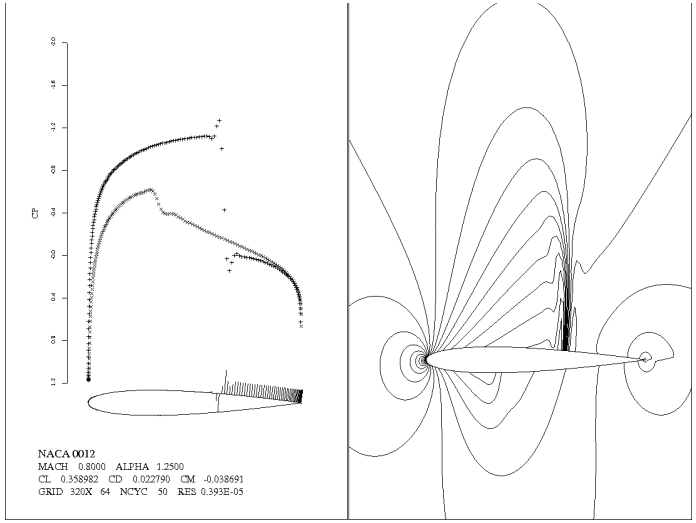
$a_{ij} \geq 0, a_{ij} = 0$ if i and j are not neighbors.
(compact stencil)

In one dimension an LED scheme is total variation diminishing (TVD). With the right switching strategy the JST scheme is LED for scalar conservation laws.

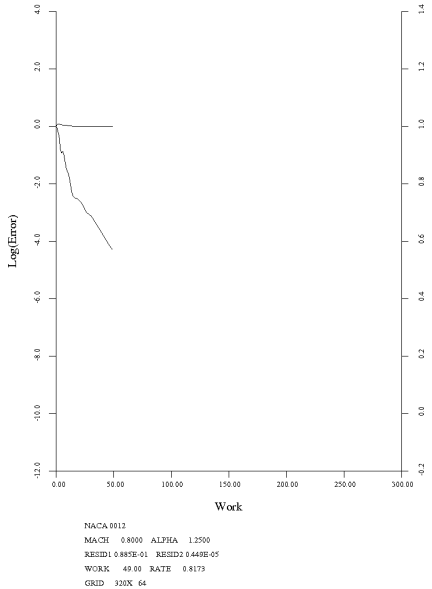
JST Results

JST Results for NACA 0012

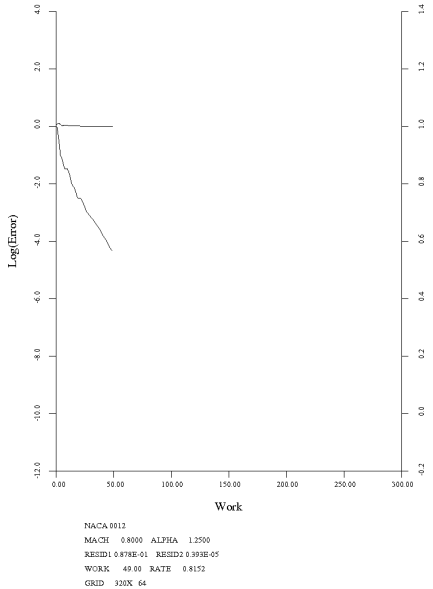




VIS2=1

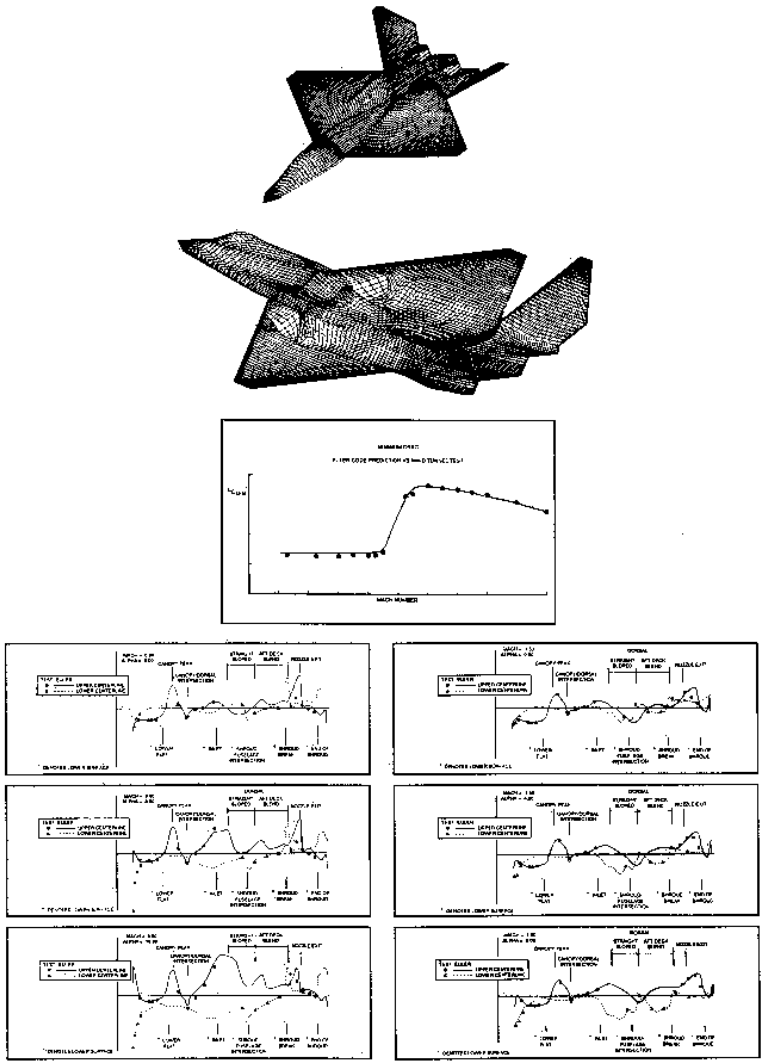


VIS2=0

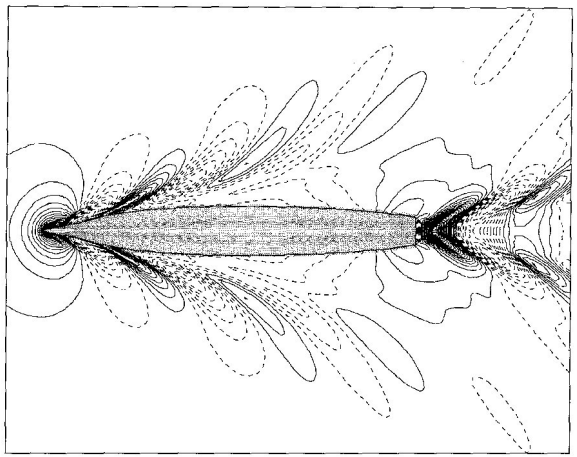


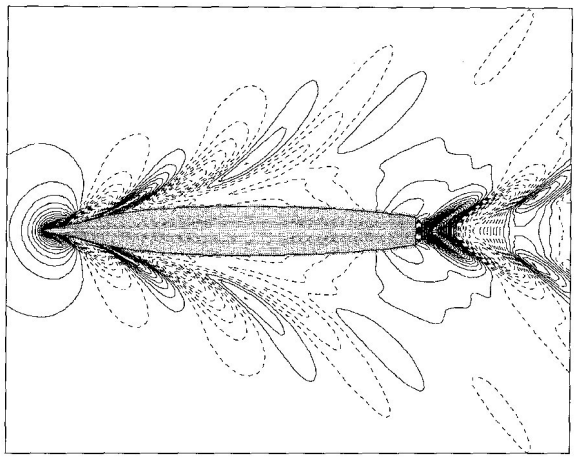
Northrup YF23, Euler solutions around 1985 – 90 (by R. J. Busch, Jr.)

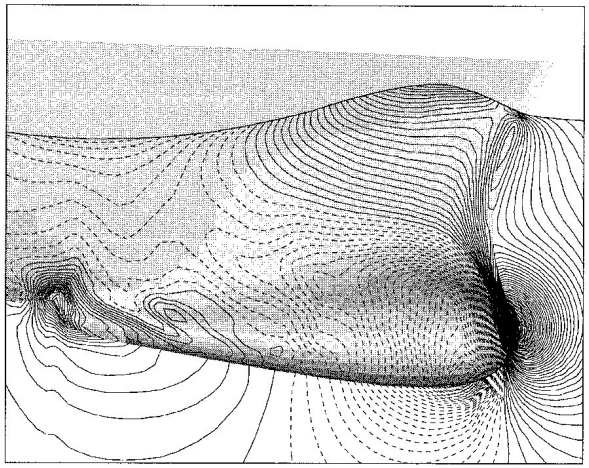
FLO57



Wave patterns of naval ship

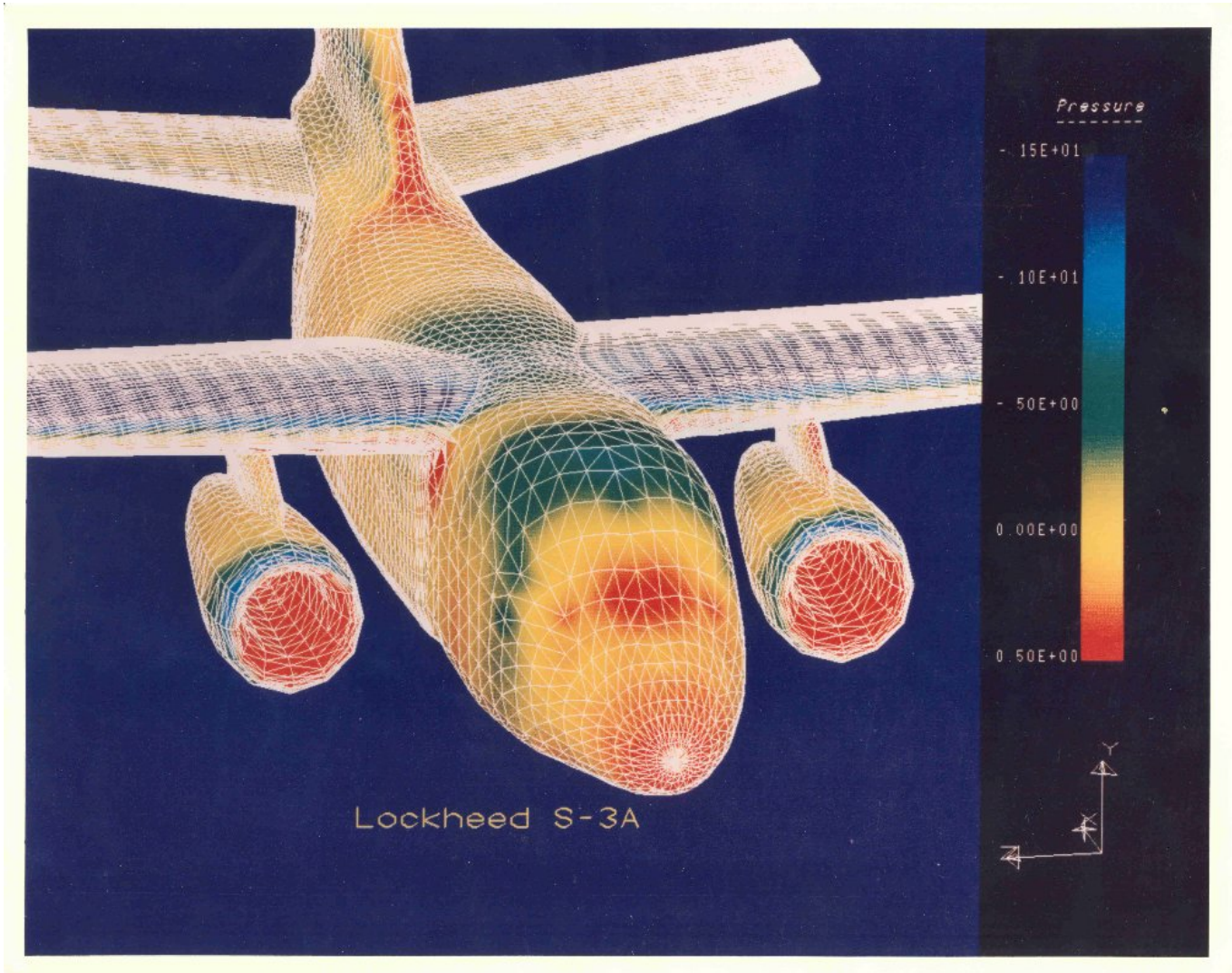




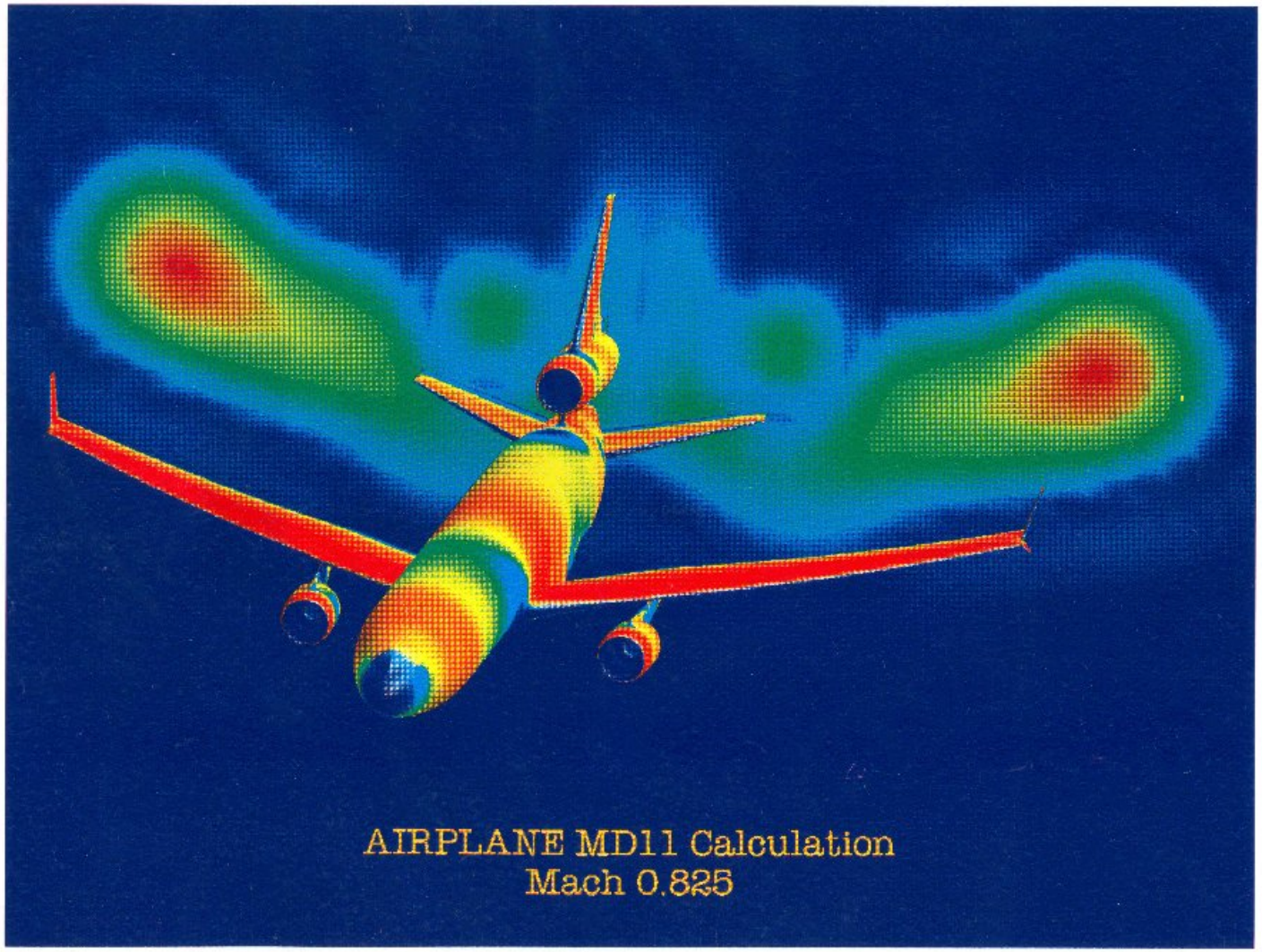


Calculations using the AIRPLANE code 1987 – 2005

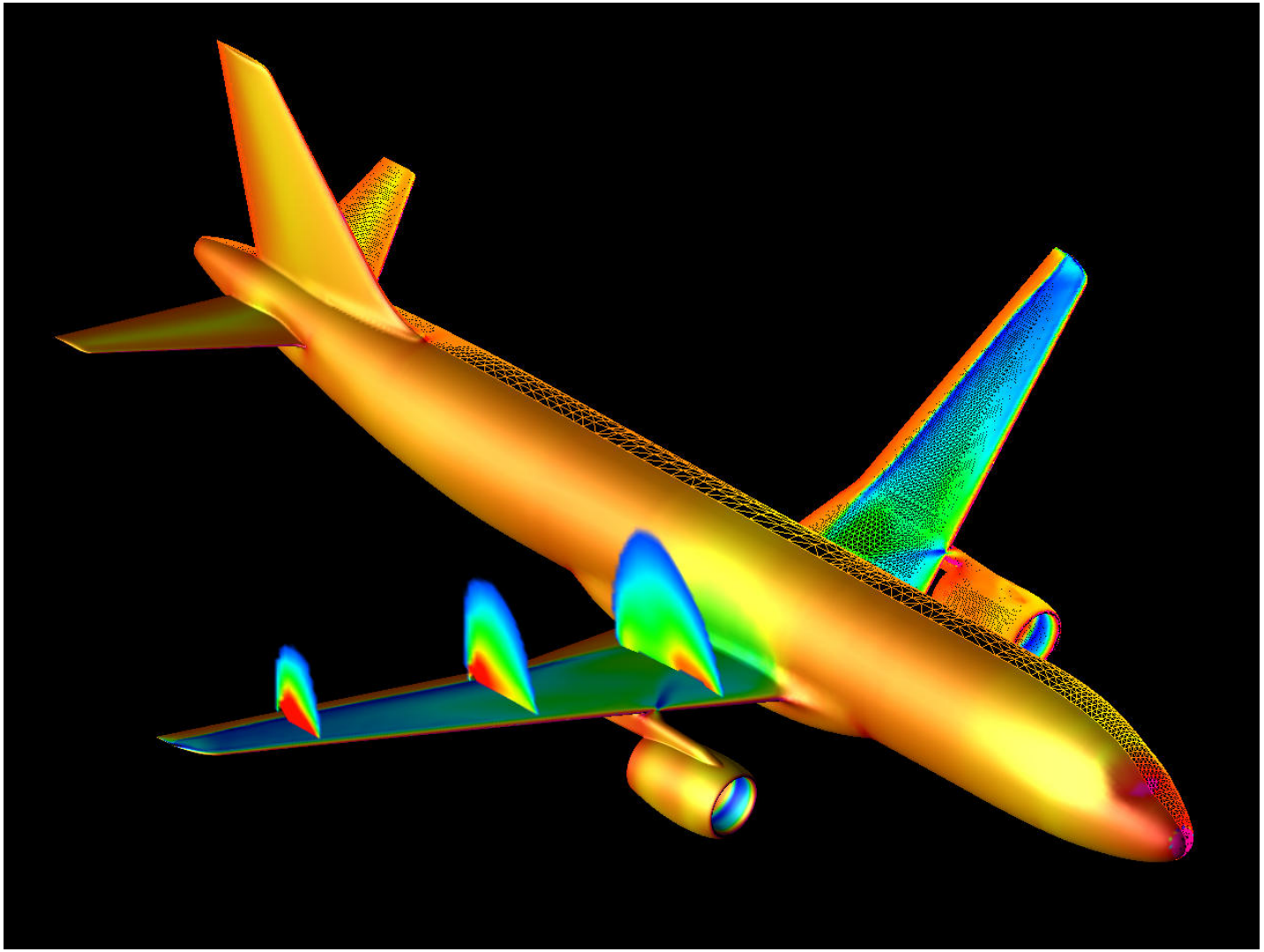
Lockheed S–3A



McDonnell Douglas MD–11



Airbus A320



The Quest for a Fast Solver

The Quest for a Fast Solver

Major aspects of aircraft design such as wing design require solutions of steady state problems.

A fast steady state solver may also be an important ingredient of an implicit scheme for unsteady flow.

Steady state Solutions and Implicit Schemes

Consider the semi-discrete system

$$\frac{dw}{dt} + R(w) = 0$$

where $R(w)$ is the space residual which results from spatial discretization of the flow equations.

Any implicit scheme, for example the backward Euler scheme

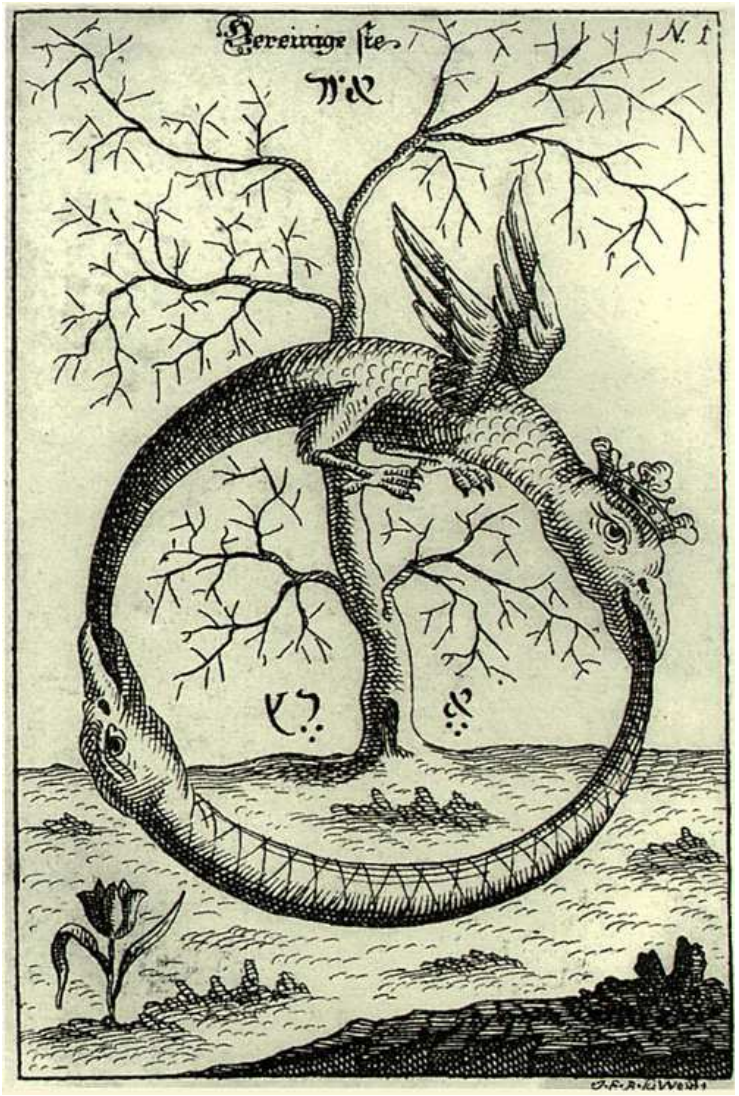
$$w^{n+1} = w^n - \Delta t R\left(w^{n+1}\right)$$

requires the solution of a very large number of coupled nonlinear equations which have the same complexity as the steady state problem

$$R(w) = 0.$$

Accordingly a fast steady state solver is an essential building block for an implicit scheme.

Paradox



RANS Example

- In current practice a steady flow over a wing is typically simulated with the Reynolds Averaged Navier-Stokes (RANS) equations on a grid with 10 million cells.
- Using a two-equation turbulence model, this requires the solution of a system of nonlinear equations with $N = 70$ million unknowns.
- Even a linear problem of this size would require iterative solution, considering that direct inversion by Gaussian elimination would require order (N^3) operations.
- By taking advantage of sparsity this might be reduced to order (N^2) with a sophisticated direct solver, but a Newton iteration requiring the solution of a sequence of linear problems of this size would still be very expensive.
- No lower bound for the cost of solving steady state problems has been established, but the author believes we should not be satisfied until they can be solved with no more than 100 iterations, each with a cost of order N operations.

Multigrid Time Stepping Scheme

Towards this goal the author has focused on multigrid time stepping in a full approximation scheme (Jameson 1983)

For the Euler equations this approach has proved successful using

1. Additive Runge-Kutta schemes designed to act as low pass filters (Jameson 1983, 1985)
2. Variations of LU-SGS schemes (Yoon and Jameson 1988, Rieger and Jameson 1988, Jameson and Caughey 2001)

Euler solutions with engineering accuracy can be obtained in about 25 steps with RK schemes, and as few as 5 steps with SGS schemes.

RK-SGS Scheme

RK multigrid schemes are typically augmented by residual averaging (Jameson and Baker 1983) where at each stage the correction Δw is smoothed implicitly.

In 1-D

$$-\epsilon\Delta\overline{w}_{i+1} + (1+2\epsilon)\Delta\overline{w}_i - \epsilon\Delta\overline{w}_{i-1} = \Delta w_i$$

and $\Delta\overline{w}$ is used for the stage update.

Rossow (2006) proposed substituting LU-SGS preconditioning sweeps to modify the correction. This concept was further developed by Rossow, Swanson and Turkel (2007). They presented results obtained with 3 and 5 stage RK schemes using 3 LU-SGS sweeps at each stage.

Accordingly the cost of each time step is much greater than that of a standard RK scheme.

RK-SGS Scheme

During the last year the present author has systematically investigated RK-SGS schemes using an alternate formulation of the LU-SGS preconditioner while exchanging results with Swanson. Two schemes have emerged as best.

1. 2-Stage Additive RK-SGS Scheme

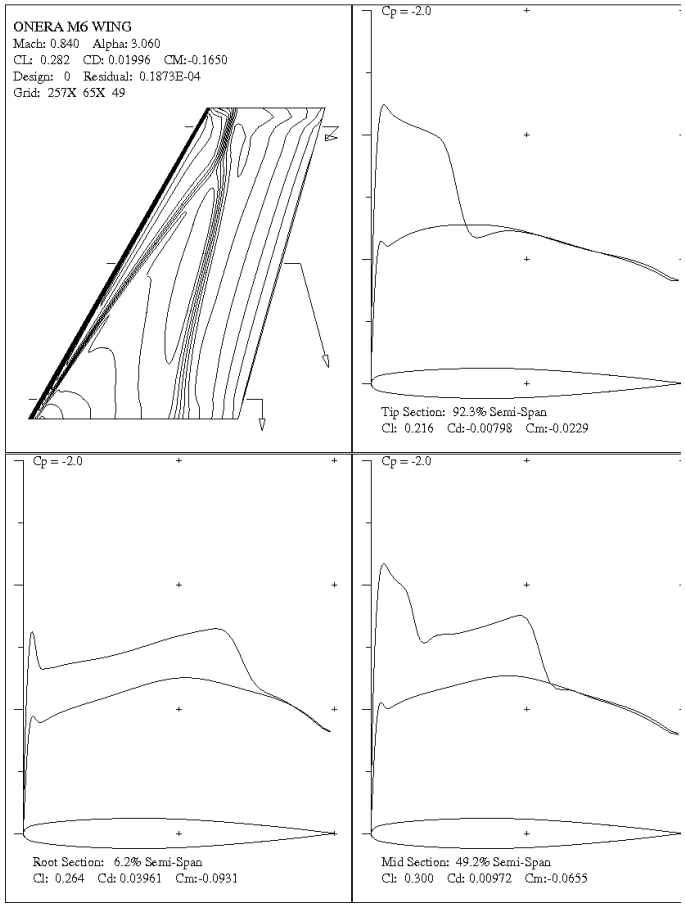
$$\begin{aligned} \alpha_1 &= 0.24 \quad & \beta_1 &= 1.00 \\ \alpha_2 &= 1.0 \quad & \beta_2 &= \tfrac{2}{3} \end{aligned} \tag{1}$$

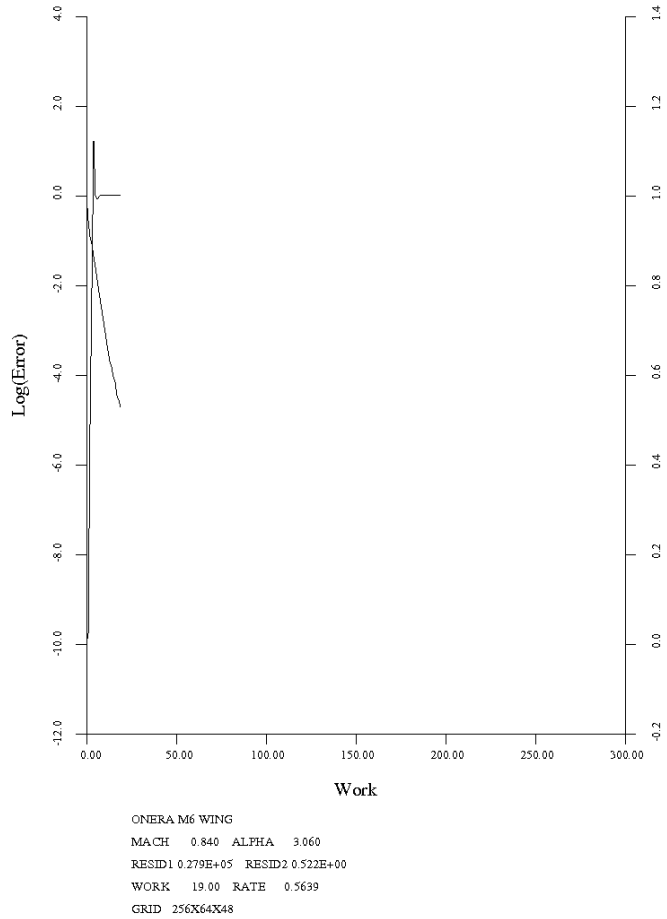
2. 3-Stage Additive RK-SGS Scheme

$$\begin{aligned} \alpha_1 &= 0.15 \quad & \beta_1 &= 1.00 \\ \alpha_2 &= 0.4 \quad & \beta_2 &= 0.5 \\ \alpha_3 &= 1.0 \quad & \beta_3 &= 0.5 \end{aligned} \tag{2}$$

Both schemes have proved robust with a single LU-SGS sweep at each stage, provided that the absolute eigenvalues used in the preconditioner are appropriately bounded away from zero. Hence the computational cost of each time step is quite low.

Results of RK-SGS Scheme Combined with JST Scheme





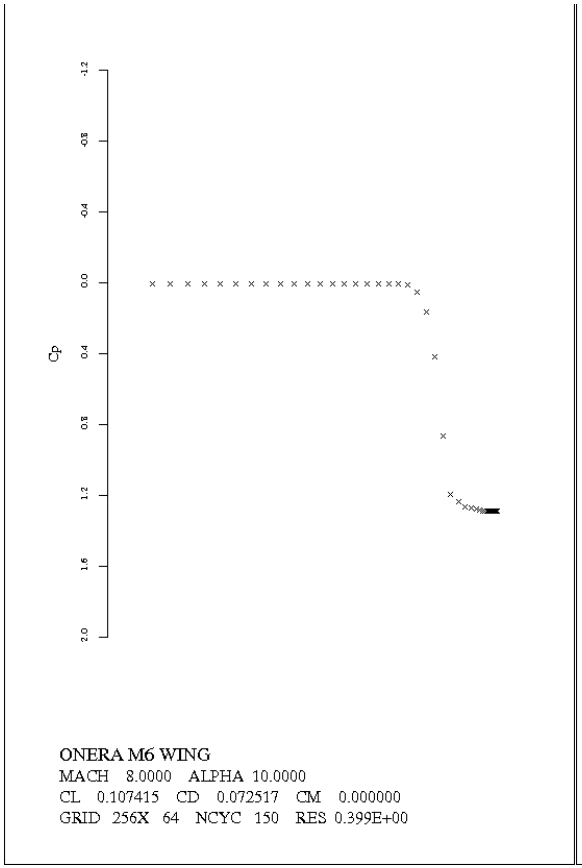
ONERA M6 Wing

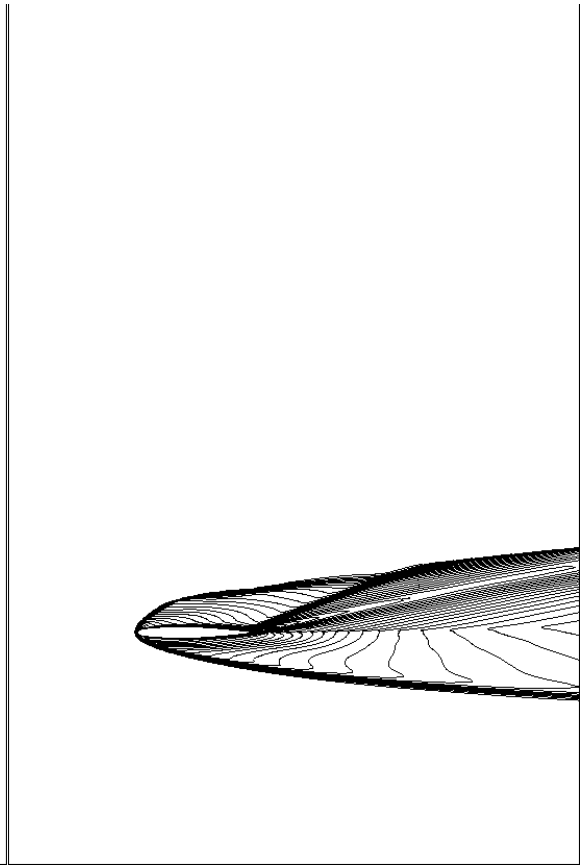
$M = 0.84, \alpha = 3.06, Re = 6 \times 10^6$

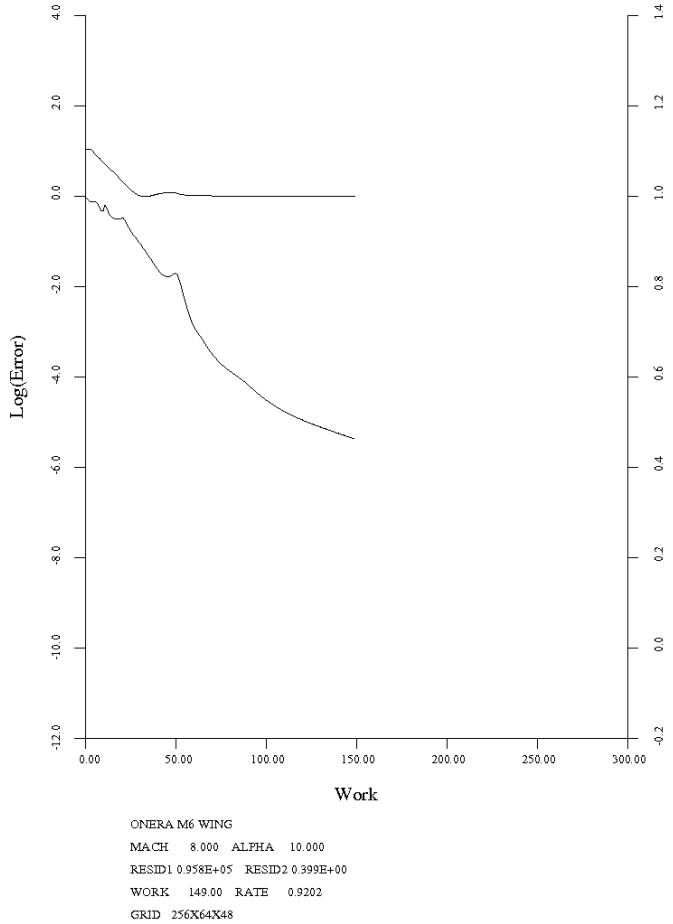
5 Digit accuracy of C_L and C_D in 20 steps (Convergence Rate = 0.56)

15 Orders of magnitude reduction of residuals to machine zero in 130 steps (Convergence Rate = 0.77)

Results of RK-SGS Scheme Combined with JST Scheme







ONERA M6 Wing

$M = 8.0, \alpha = 10.0, Re = 6 \times 10^6$

Solution in 150 steps (Convergence Rate = 0.92)

Needs extra dissipation during the first 80 steps to avoid negative pressure near the wing tip.

Bugs, etc.

Note: This talk is off the record.
I may find it necessary to disavow any remarks that may be attributed to me.

(Originally presented at the University of Tennessee Space Institute, Tullahoma, in 1984.)

Estimated frequency of bugs

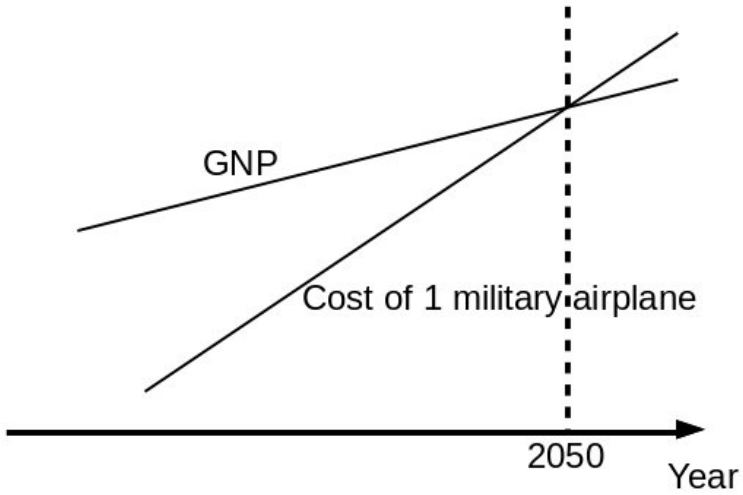
1 per 50 lines
(early phase)

1 per 1000 lines
(after intensive effort)

Theorem:
If the code has more than 5000 lines,
it gives the WRONG ANSWER
with PROBABILITY ONE.

Pressures for complexity

Cost projection



GNP will be sufficient for 1 airplane in 2050.
Pressures for complexity:

1. DOD has departments to generate requirements. It can be estimated that the number of requirements will be roughly proportional to the number of people employed to think of them.
2. 2000 subcontractors each have an interest in introducing more complex subsystems.
3. A design staff of 1000 have to occupy themselves somehow (35 designed the Mirage 3).

Complexity in computer programs

Is there a danger that 1 computer program will require the GNP say by the year 2100?

Pressures for complexity:

1. One might imagine that the purpose of computers is to replace HUMAN effort by MACHINE effort. The GOAL of numerical analysts is precisely the OPPOSITE!
2. The objective of professors and research scientists is generally to show how clever they are. This objective is not realized by a SIMPLE SOLUTION
   – that merely suggests that the problem was easy.
3. A solution terminates funding
   – the secret of funding is the LIGHT AT THE END OF THE TUNNEL.

Case histories of bugs

Bugs

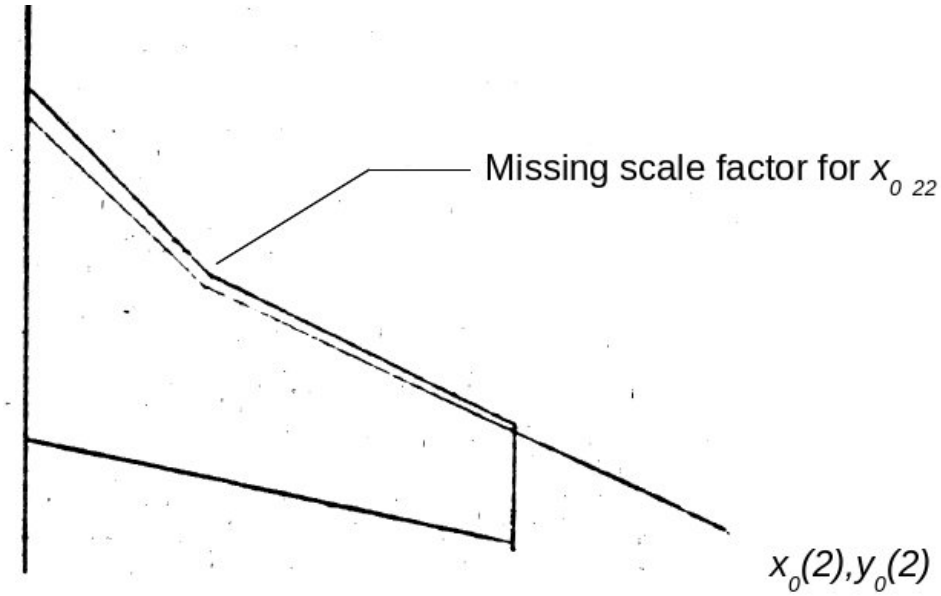
Cranked wing	*FLO22*	1976 – 1983
Swept forward wing	*FLO27*	1977 (6 weeks)
Extended wing tip	*FLO57*	1981 – 1982
Missing tailplane	*FLO59*	1983

(Multigrid)

Wrong calling sequence	*FLO52*	April 1981 – February 1983
Missing DO statement	*FLO57*	1983 (3 months)

Cranked wing bug

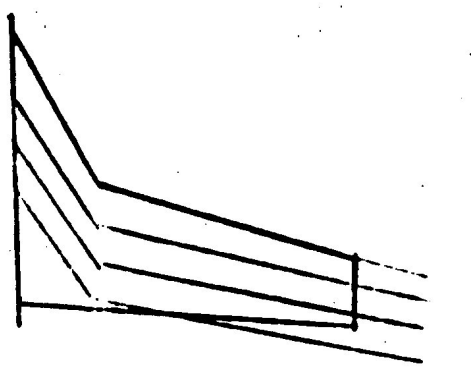
FLO22 1976 – 1983



Found by I-Chung Chang / D. Caughey

The cranked wing failure

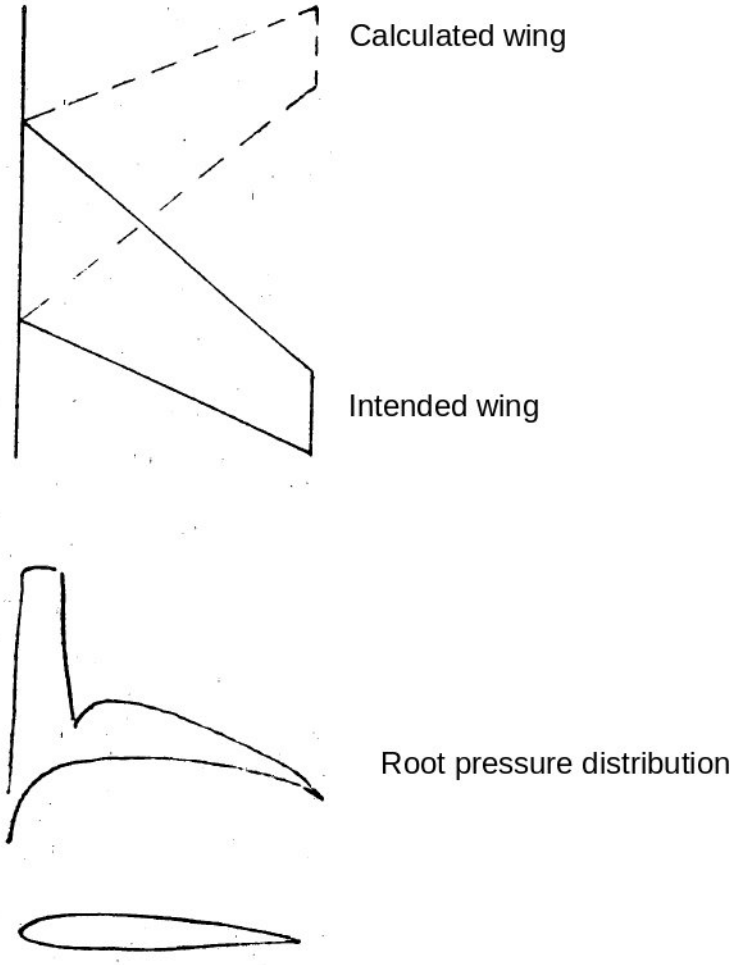
1. I tried a change of scale (GOOD), but I did it with a straight leading edge (BAD – not exercising all the options).
2. I expected FAILURE – I assumed a KINK would invalidate the assumptions of the coordinate transformation.
3. I attributed bad results for TACT wing to
   1) Singularity in the transformation.
   2) Sparse mesh on outer wing.



There were WRONG PLAUSIBLE EXPLANATIONS.

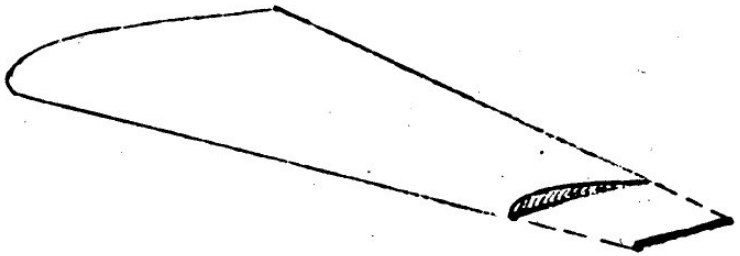
Swept forward wing

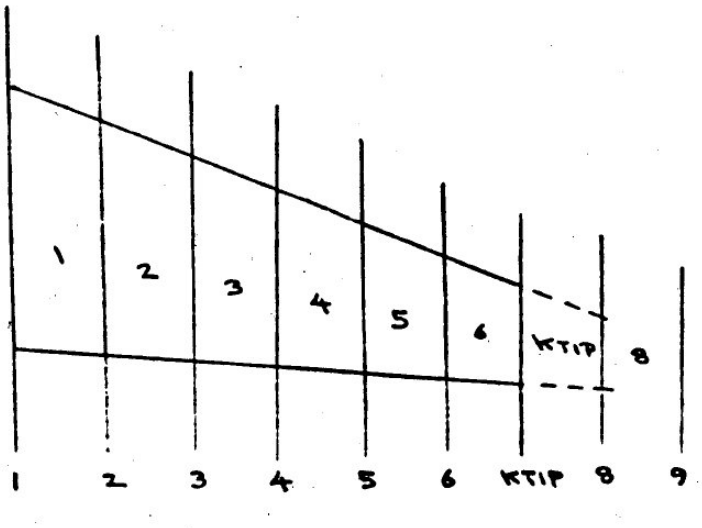
FLO27 1977



Extended wing tip

FLO57 1981 – 1982

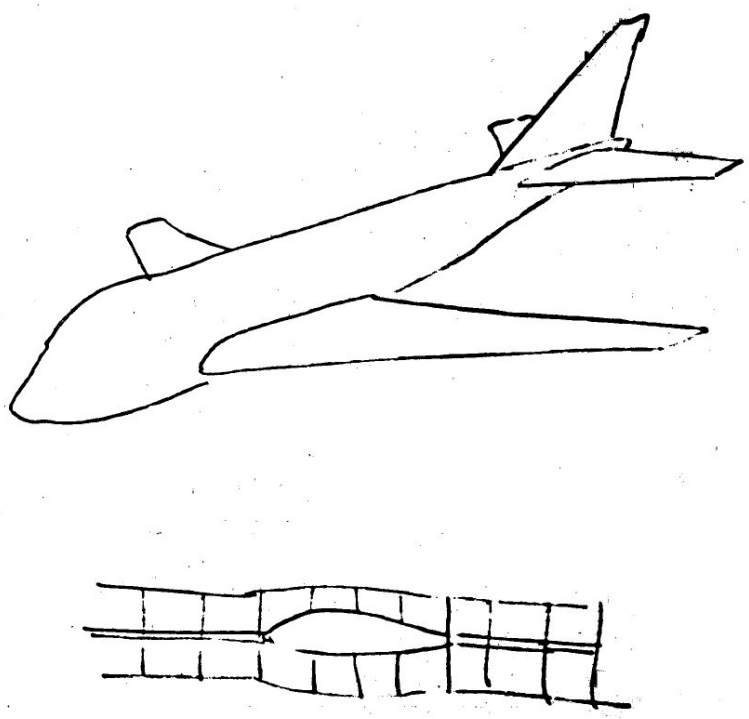




On the wing IF (K .LT. KTIP)
not IF (K .LE. KTIP).

Missing tailplane

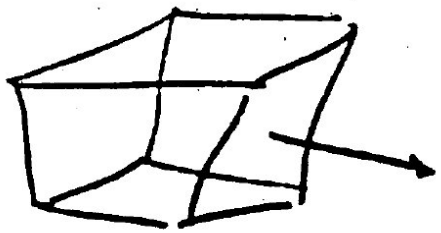
FLO59 1983



Fluxes recalculated for tailplane cells Δt not transmitted to this subroutine.

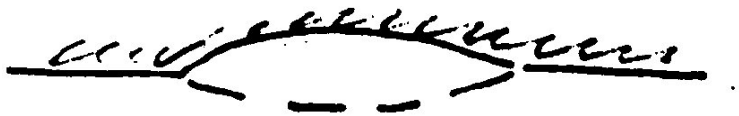
Inside out problem

FLO57, FLO59 What is sign of projected face area (surface normal)?



Wrong sign on all faces.
April 1981 24 hours

Inside out tail
1983 6 months



The sign problem

Korn's Lemma

The sign is either POSITIVE,
or else it is NEGATIVE.

When in doubt, TRY BOTH.

Debugs (1)

(Frequently not performed by me when they should have been.)

Consistency checks

- Double the scale of the profile
- Insert uniform flow
- Check symmetry with symmetric flow
- Check convergence with decreasing mesh size

Programming checks

- Exercise all options
- Set core to indefinite

Debugs (2)

External checks

- Check against known exact solutions
- Check plausibility of result
- Check against experimental data
- Plot everything

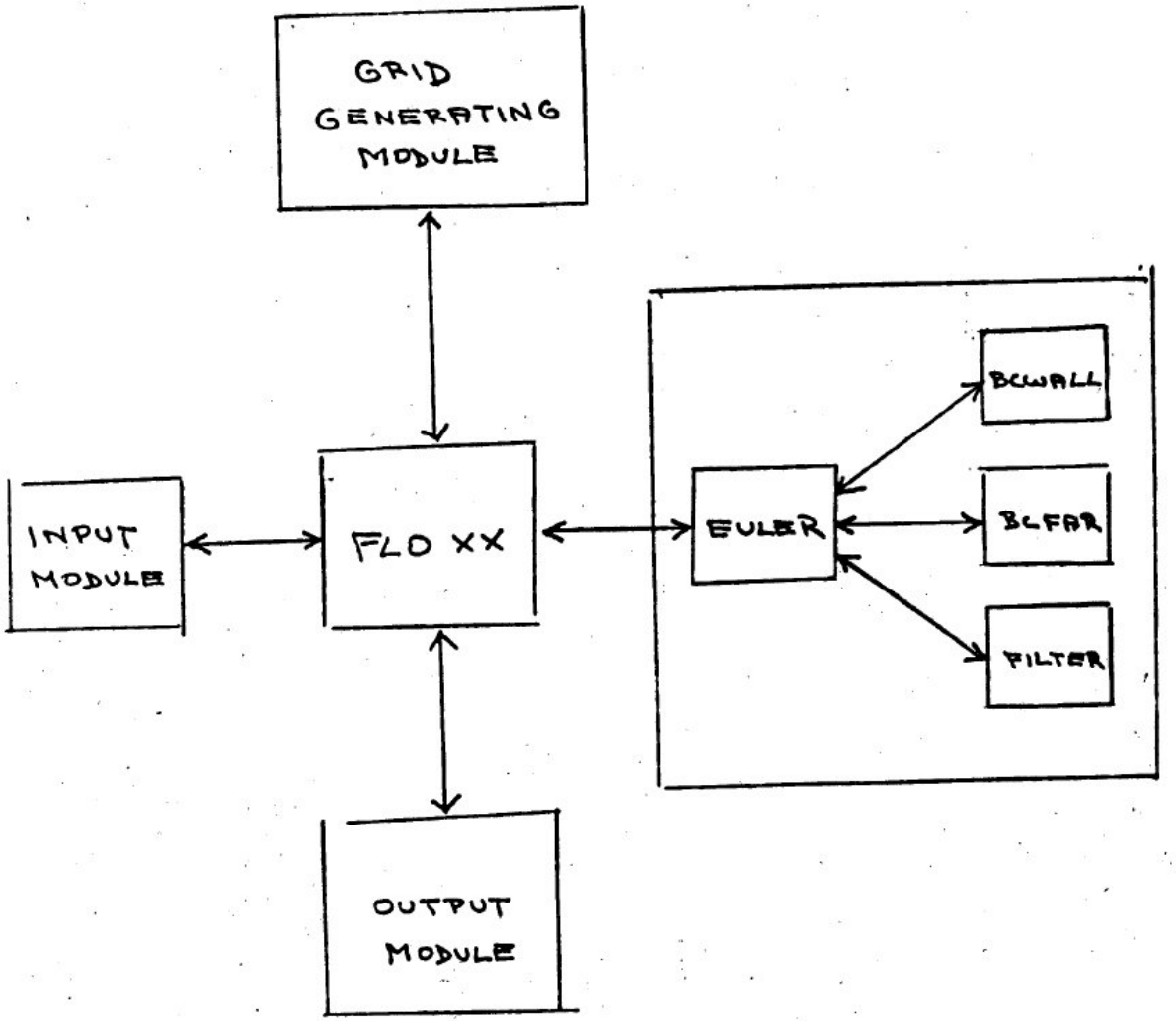
Accidental discovery

- When coding new versions
- By third parties
  - release pilot code to carefully selected friendly users
- By concentrated thinking
  - it pays to carry the whole code in your head

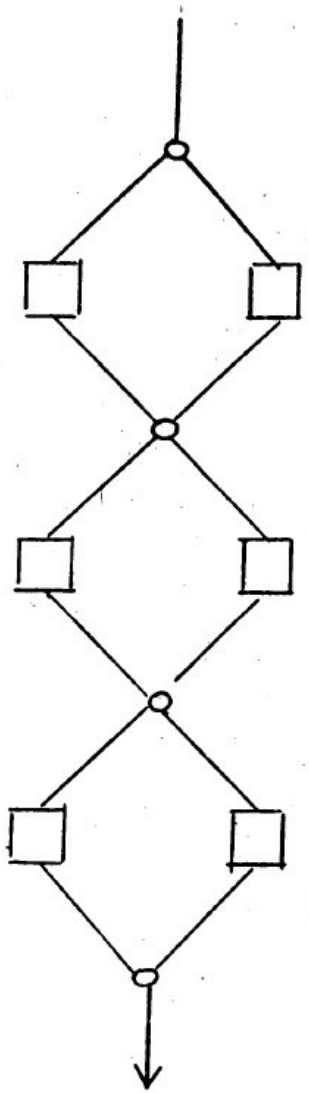
Defensive programming

1. Modular structure
   – change one thing at a time
2. Dual path programming
   – interchangeable subroutines in each segment
3. Simplicity: avoid
   (a) AICS (acquired if contamination syndrome)
   (b) RSNS (random statement number syndrome)
4. Rigid protocol

Modular programming

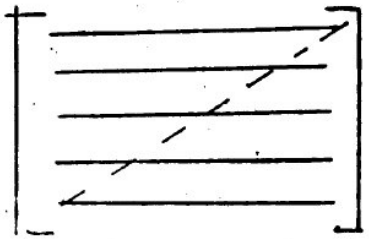


Dual path programming

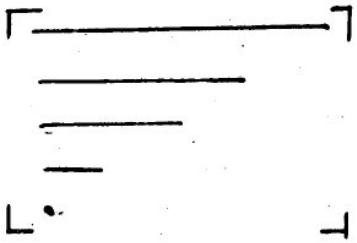


Efficiency simplicity trade-off

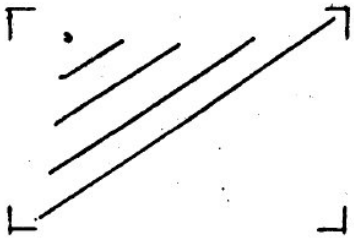
Example: Symmetric matrices



Complete storage



Reduced storage (1)



Reduced storage (2)

One certain result: schemes (1) and (2) lead to INCOMPREHENSIBLE CODE

Part 2:
The development of computational design methods

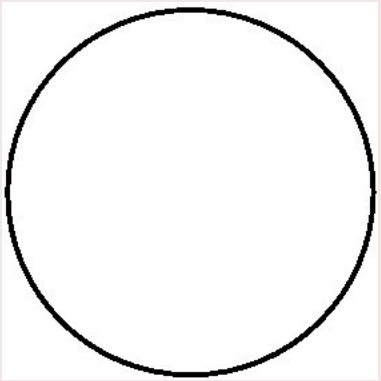
Early efforts – Inverse design
Lighthill-James-Jameson method (SYN1)

Lighthill's method

Profile P on z plane

⇓

Profile C on σ plane



- Let the profile P be conformally mapped to an unit circle C
- The surface velocity is $q = \frac{1}{h} |\nabla \phi|$ where ϕ is the potential in the circle plane, and h is the mapping modulus $h = \left| \frac{dz}{d\sigma} \right| = \frac{ds}{d\theta}$
- Choose $q = q_T$
- Solve for the maping modulus $h = \frac{1}{q_T} |\nabla \phi|$

Implementation of Lighthill's Method

Design Profile C for Specified Surface speed q_t. Let a profile C be conformally mapped to a circle by

$$\log \frac{dz}{d\sigma} = \sum \frac{C_n}{\sigma^n}$$

$$\log \frac{ds}{d\theta} + i(\alpha - \theta - \frac{\pi}{2}) = \sum (a_n \cos(n\theta) + b_n \sin(n\theta)) + i \sum (b_n \cos(n\theta) - a_n \sin(n\theta))$$

where

$$q = \frac{\nabla \Phi}{h}, \; h = \left| \frac{dz}{d\sigma} \right|$$

and

$$\Phi = (r + \frac{1}{r}) \cos\theta + \frac{\Gamma}{2\pi}\theta \text{ is known}$$

On C set $q = q_t$

$$\rightarrow \frac{ds}{d\theta} = \frac{\Phi_\theta}{q_t} \rightarrow a_n, \; b_b$$

Constraints with Lighthill's Method

To preserve q_∞

$$c_0 = 0$$

Also, integration around a circuit gives

$$\Delta z = \oint \frac{dz}{d\sigma} d\sigma = 2\pi i c_1$$

Closure $\longrightarrow c_1 = 0$ Thus,

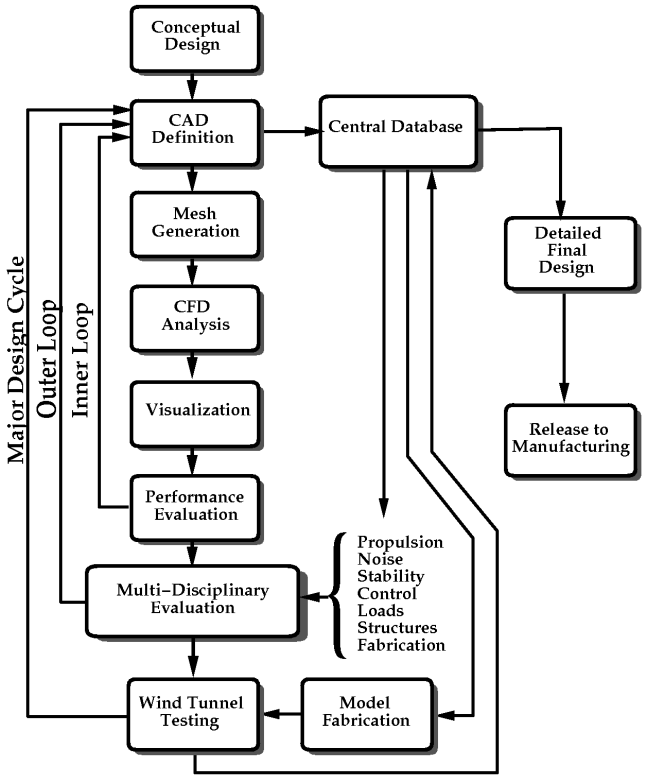
$$\int \log(q_t) d\theta = 0$$

$$\int \log(q_t) \cos(\theta) d\theta = 0$$

$$\int \log(q_t) \sin(\theta) d\theta = 0$$

Aerodynamic Design & Shape Optimization via Control Theory

Aerodynamic Design Process



The Aerodynamic Design Process

Aerodynamic Design Based on Control Theory

- Regard the wing as a device to generate lift (with minimum drag) by controlling the flow
- Apply theory of optimal control of systems governed by PDEs (Lions) with boundary control (the wing shape)
- Merge control theory and CFD
- Find the Frechet derivative (infinite dimensional gradient) of a cost function (performance measure) with respect to the shape by solving the adjoint equation in addition to the flow equation
- Modify the shape in the sense defined by the smoothed gradient
- Repeat until the performance value approaches an optimum

Aerodynamic Shape Optimization: Gradient Calculation

For the class of aerodynamic optimization problems under consideration, the design space is essentially infinitely dimensional. Suppose that the performance of a system design can be measured by a cost function I which depends on a function $\mathcal{F}(x)$ that describes the shape,where under a variation of the design $\delta\mathcal{F}(x)$, the variation of the cost is δI. Now suppose that δI can be expressed to first order as

$$\delta I = \int \mathcal{G}(x)\delta\mathcal{F}(x)dx$$

where $\mathcal{G}(x)$ is the gradient. Then by setting

$$\delta\mathcal{F}(x) = -\lambda\mathcal{G}(x)$$

one obtains an improvement

$$\delta I = -\lambda \int \mathcal{G}^2(x)dx$$

unless $\mathcal{G}(x) = 0$. Thus the vanishing of the gradient is a necessary condition for a local minimum.

Aerodynamic Shape Optimization: Gradient Calculation

Computing the gradient of a cost function for a complex system can be a numerically intensive task, especially if the number of design parameters is large and the cost function is an expensive evaluation. The simplest approach to optimization is to define the geometry through a set of design parameters, which may, for example, be the weights α_i applied to a set of shape functions $\mathcal{B}_i(x)$ so that the shape is represented as

$$\mathcal{F}(x) = \sum \alpha_i \mathcal{B}_i(x).$$

Then a cost function I is selected which might be the drag coefficient or the lift to drag ratio; I is regarded as a function of the parameters α_i. The sensitivities $\frac{\partial I}{\partial \alpha_i}$ may now be estimated by making a small variation $\delta\alpha_i$ in each design parameter in turn and recalculating the flow to obtain the change in I. Then

$$\frac{\partial I}{\partial \alpha_i} \approx \frac{I(\alpha_i + \delta\alpha_i) - I(\alpha_i)}{\delta\alpha_i}.$$

Symbolic Development of the Adjoint Method

Let I be the **cost** (or **objective**) function

$$I = I(w, \mathcal{F})$$

where

$$\begin{aligned} w &= \text{flow field variables} \\ \mathcal{F} &= \text{grid variables} \end{aligned}$$

The **first variation** of the cost function is

$$\delta I = \frac{\partial I}{\partial w}^T \delta w + \frac{\partial I}{\partial \mathcal{F}}^T \delta \mathcal{F}$$

The **flow field equation** and its **first variation** are

$$R(w, \mathcal{F}) = 0$$

$$\delta R = 0 = \left[\frac{\partial R}{\partial w}\right] \delta w + \left[\frac{\partial R}{\partial \mathcal{F}}\right] \delta \mathcal{F}$$

Symbolic Development of the Adjoint Method (cont.)

Introducing a **Lagrange Multiplier**, ψ, and using the **flow field equation** as a **constraint**

$$\delta I = \frac{\partial I}{\partial w}^T \delta w + \frac{\partial I}{\partial \mathcal{F}}^T \delta \mathcal{F} - \psi^T \left\{ \left[\frac{\partial R}{\partial w} \right] \delta w + \left[\frac{\partial R}{\partial \mathcal{F}} \right] \delta \mathcal{F} \right\}$$

$$= \left\{ \frac{\partial I}{\partial w}^T - \psi^T \left[\frac{\partial R}{\partial w} \right] \right\} \delta w + \left\{ \frac{\partial I}{\partial \mathcal{F}}^T - \psi^T \left[\frac{\partial R}{\partial \mathcal{F}} \right] \right\} \delta \mathcal{F}$$

By choosing ψ such that it satisfies the **adjoint equation**

$$\left[\frac{\partial R}{\partial w} \right]^T \psi = \frac{\partial I}{\partial w},$$

we have

$$\delta I = \left\{ \frac{\partial I}{\partial \mathcal{F}}^T - \psi^T \left[\frac{\partial R}{\partial \mathcal{F}} \right] \right\} \delta \mathcal{F}$$

This reduces the **gradient** calculation for an arbitrarily large number of design variables at a **single design point** to

$\Longrightarrow$ One Flow Solution + One Adjoint Solution

Gradient Smoothing

Consider a shape change $f(x) \quad \Longrightarrow \quad f + \delta f$

Set $\delta f = -\lambda g$ to obtain

$$\delta I = \int g \, \delta f \, dx = -\lambda \int g^2 dx$$

A smoothed gradient $\overline{g}$ is defined by

$$\overline{g} - \frac{\partial}{\partial x} \epsilon \frac{\partial \overline{g}}{\partial x} = g$$

and $\overline{g} = 0$ at the end points.

Now set

$$\delta f = -\lambda \overline{g}$$

Then

$$\begin{aligned} \delta I &= -\lambda \int \left(\overline{g} - \frac{\partial}{\partial x} \epsilon \frac{\partial \overline{g}}{\partial x} \right) \overline{g} dx \\ &= -\lambda \int \left(\overline{g}^2 + \epsilon \left(\frac{\partial \overline{g}}{\partial x} \right)^2 \right) \overline{g} dx \end{aligned}$$

Note:

1. If $\overline{g} = 0$ then $g = 0$.
2. The smoothed gradient is the gradient with respect to a Sovolev inner product.

$$< u, v >= \int \left(uv + \epsilon u' v' \right) dx$$

Constraints

- Fixed C_L.
- Fixed span load distribution to present too large C_L on the outboard wing which can lower the buffet margin.
- Fixed wing thickness to prevent an increase in structure weight.

  - - Design changes can be limited to a specific spanwise range of the wing.
  - - Section changes can be limited to a specific chordwise range.

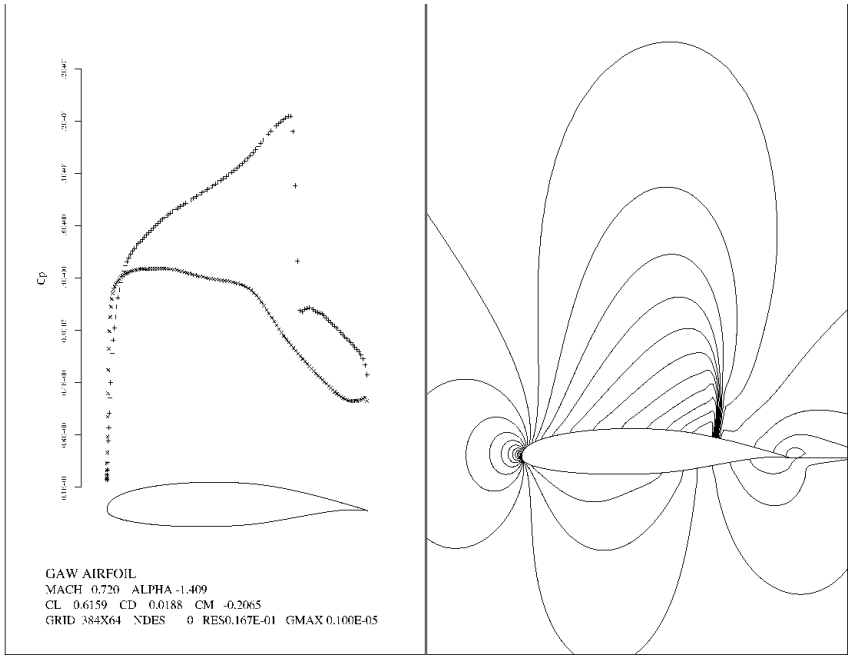
- Smooth curvature variations via the use of Sobolev gradient.

Shock free airfoil design

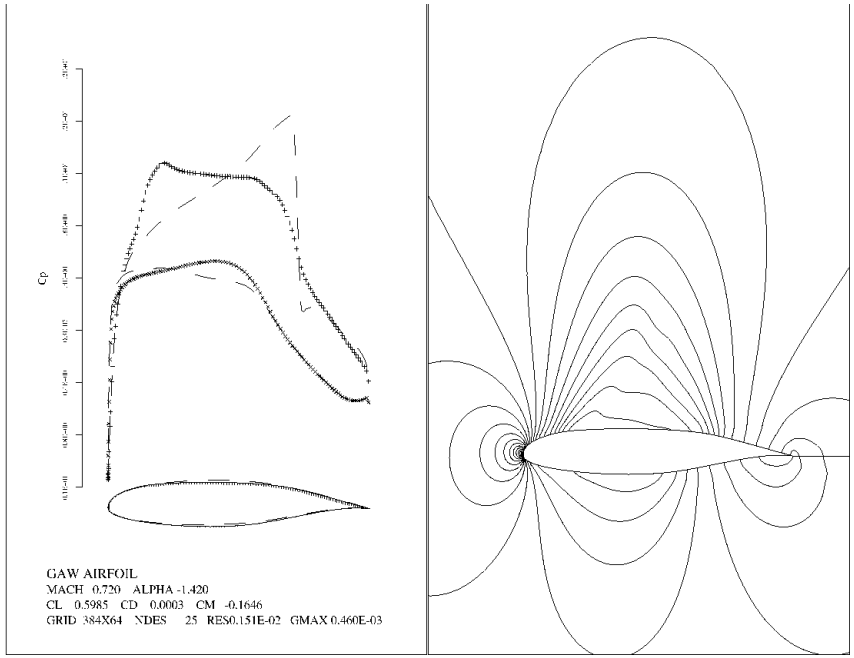
- The search for profiles which give shock free transonic flows was the subject of intensive study in the 1965-70 period.
- Morawetz' theorem (1954) states that a shock free transonic flow is an isolated point. Any small perturbation in Mach number, angle of attack, or shape causes a shock to appear in the flow.
- Nieuwland generated shock free profiles by developing solutions in the hodograph plane. The most successful method was that developed by Garabedian and his co-workers. This used complex characteristics to develop solution in the hodograph plane, which was then mapped to the physical plane. It was hard to find hodograph solutions which mapped to physical realizable closed profiles. It generally took one or two months to produce an acceptable solution.
- By using shape optimization to minimize the drag coefficient at a fixed lift, shock free solutions can be found in less than one minute.

Two dimensional studies of transonic airfoil design(cont'd)

Pressure distribution and Mach contours for the GAW airfoil



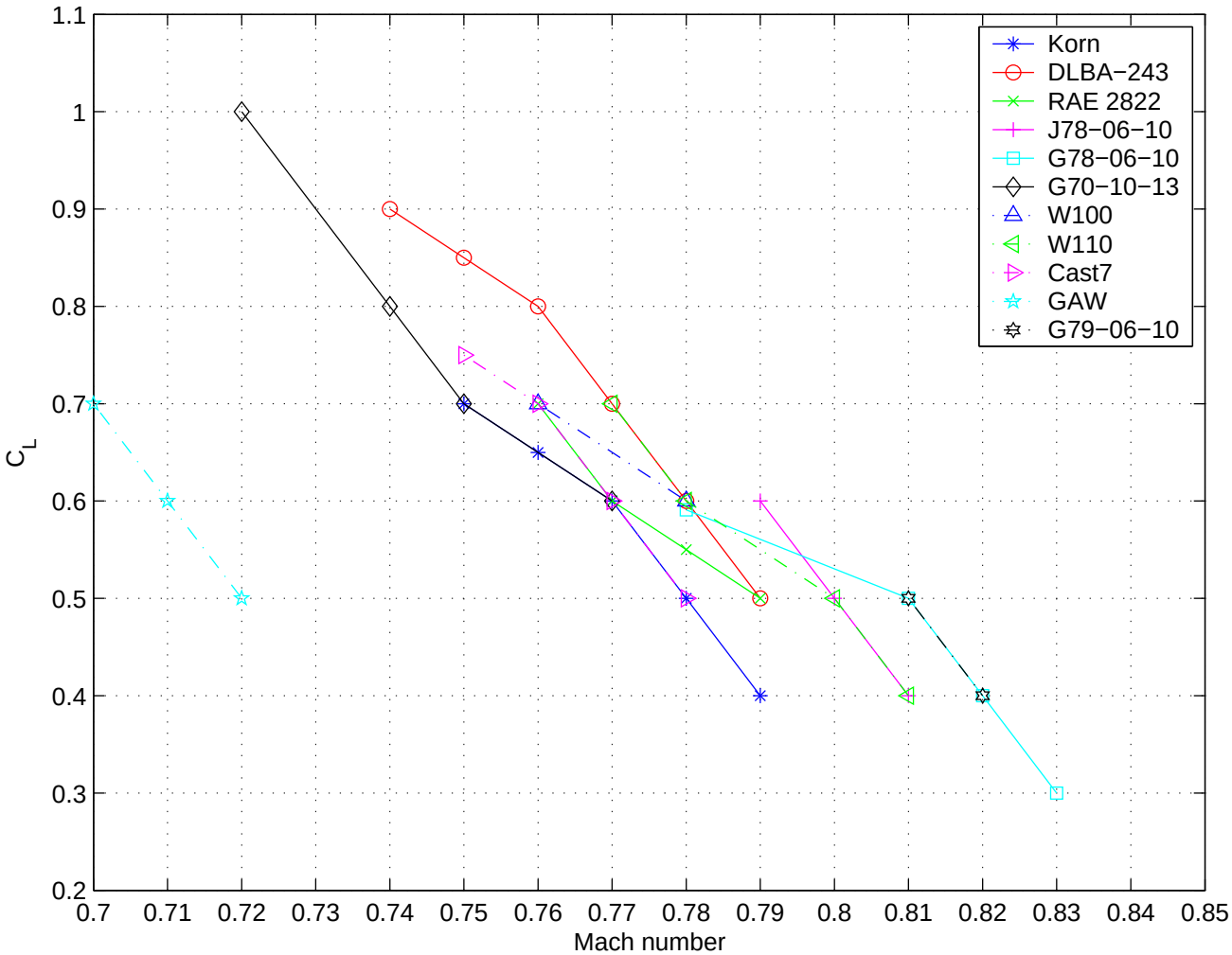
Before the redesign



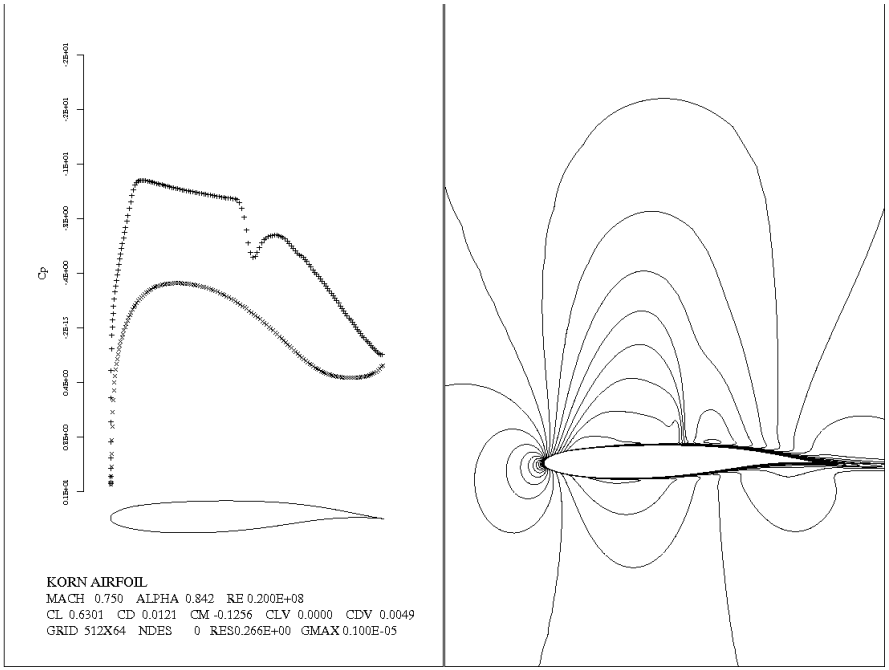
After the redesign

Two dimensional studies of transonic airfoil design

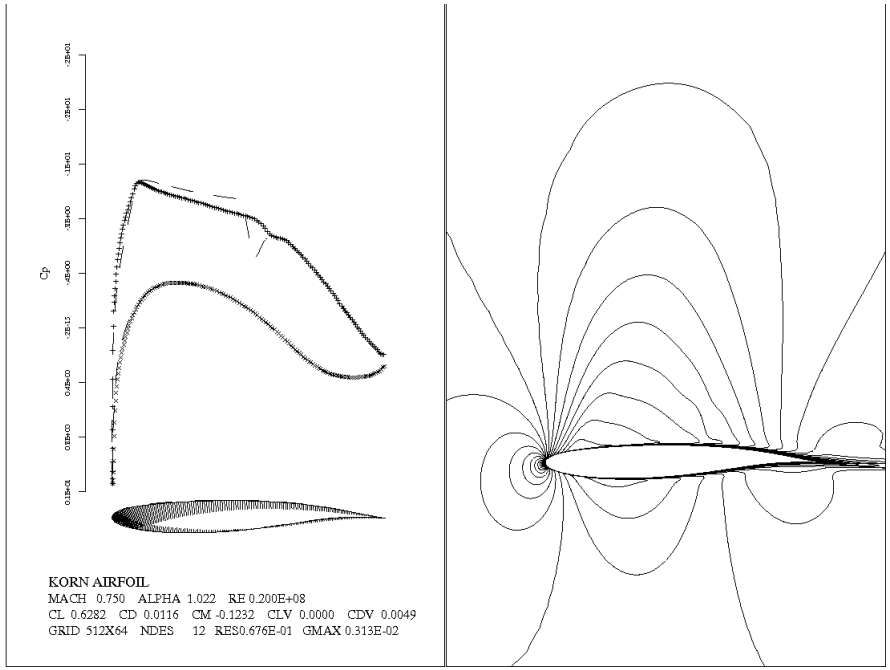
Attainable shock-free solutions for various shape optimized airfoils



Viscous Korn Airfoil Design

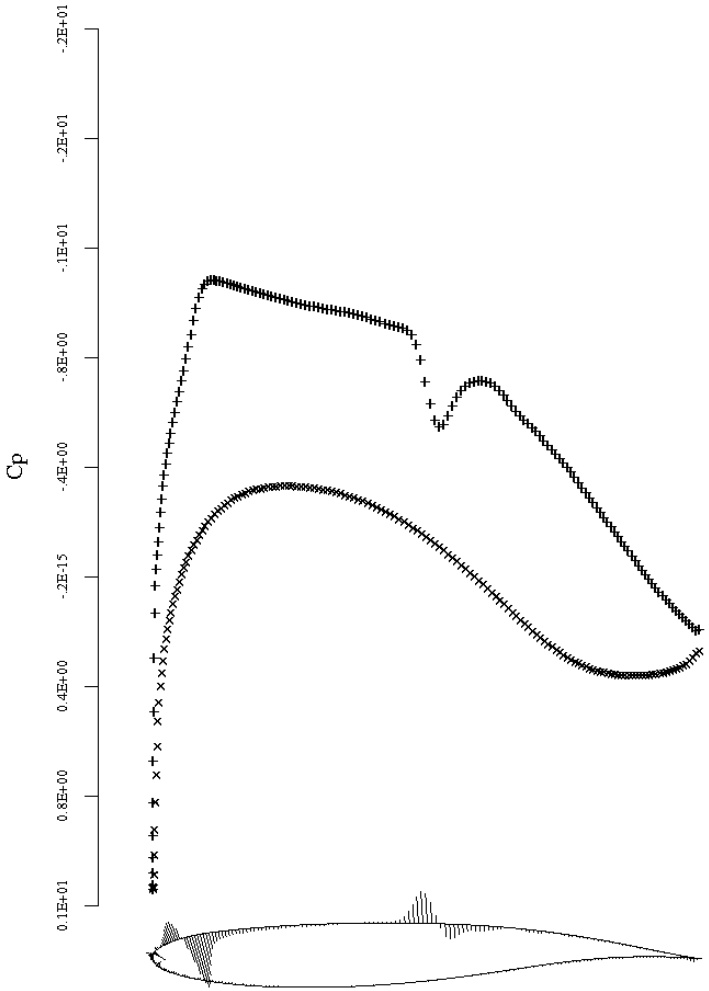


Initial



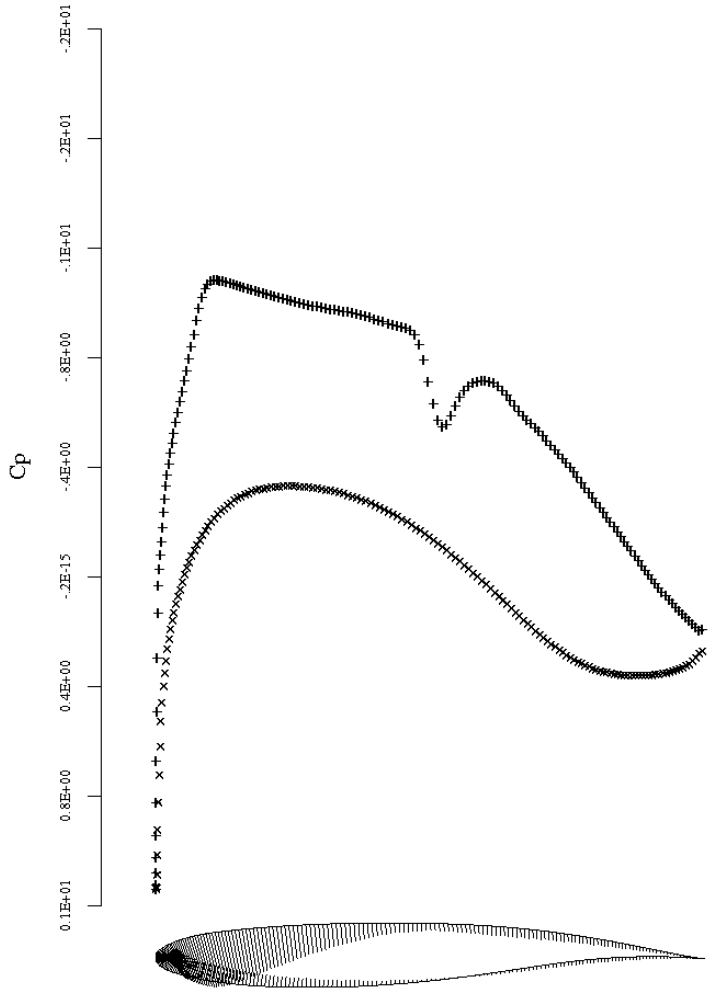
Final

Viscous Korn Airfoil Design



KORN AIRFOIL
MACH 0.750 ALPHA 0.853 RE 0.200E+08
CL 0.6282 CD 0.0118 CM -0.1257 CLV 0.0000 CDV 0.0049
GRID 512X64 NDES 1 RES0.284E+00 GMAX 0.535E-01

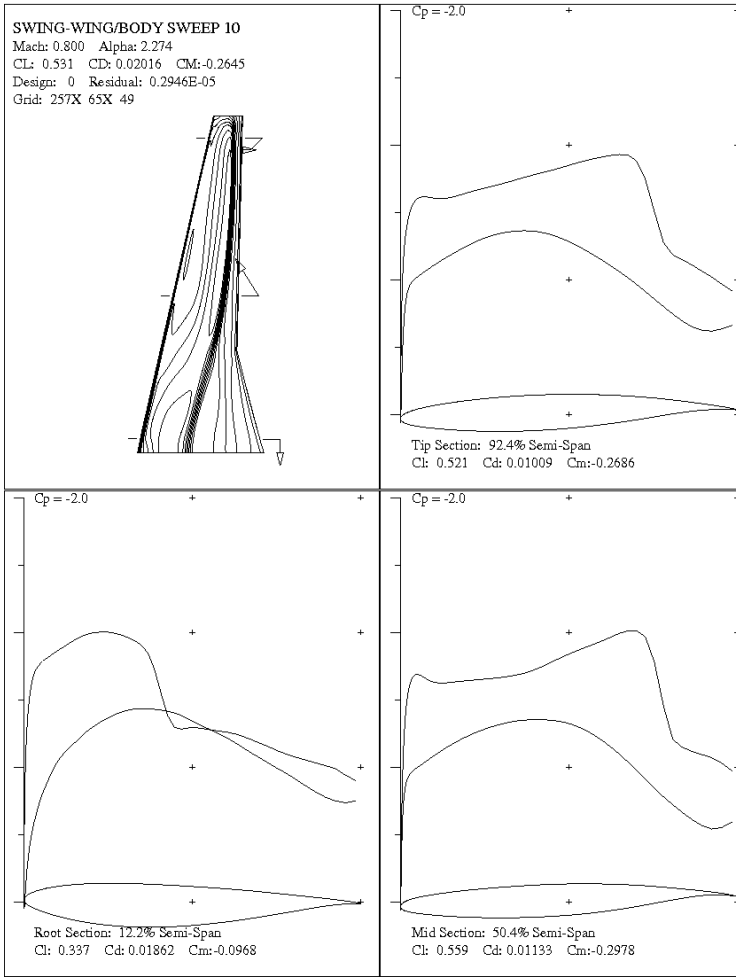
Unsmoothed



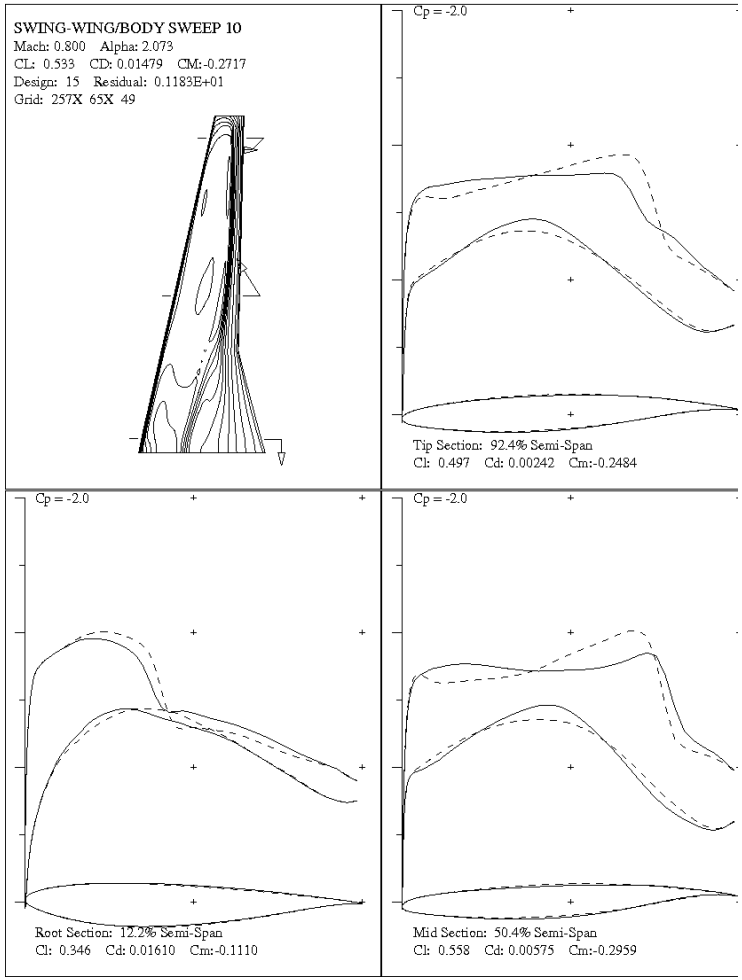
KORN AIRFOIL
MACH 0.750 ALPHA 0.853 RE 0.200E+08
CL 0.6282 CD 0.0118 CM -0.1257 CLV 0.0000 CDV 0.0049
GRID 512X64 NDES 1 RES0.284E+00 GMAX 0.396E-02

Smoothed

3D Redesign of a Deswept Wing Using the New Fast Solver



Initial



Final

Part 3:
The current status of CFD
– a guess at future directions

The Current Status of CFD

- Worldwide commercial and government codes are based on algorithms developed in the 80s and 90s.
- These codes can handle complex geometry but are generally limited to 2nd order accuracy.
- They cannot handle turbulence without modeling.
- Unsteady simulations are very expensive, and questions over accuracy remain.

CFD Contributions to B787

CFD Contributions to A380

The Future of CFD

CFD has been on a plateau for the past 15 years.

- Representations of current state of the art:
  - Formula 1 cars
  - Complete aircrafts
- The majority of current CFD methods are not adequate for vortex dominated and transitional flows:
  - Rotorcraft
  - High-lift systems
  - Formation flying
- In order to address these currently intractable problems we need to move towards higher fidelity simulations with large eddy simulation (LES), or ultimately direct numerical simulation (DNS).

Large Eddy Simulation

The number of DoF for an LES of turbulent flow over an airfoil scales as $Re_c^{1.8}$ **(resp.** $Re_c^{0.4}$**) if the inner layer is resolved (resp. modeled)**

Rapid advances in computer hardware should make LES feasible within the foreseeable future for industrial problems at high Reynolds numbers. To realize this goal requires

- High-order algorithms for unstructured meshes (complex geometries)
- Sub-Grid Scale models applicable to wall bounded flows
- Massively parallel implementation

High Order Methods

At the Stanford Aerospace Computing Laboratory we have been focusing on the flux reconstruction mehtod first proposed by H. T. Huynh (2007), which provides a unifying framework for a variety of methods.

Recent Publications from the Stanford Aerospace Computing Laboratory on High Order Methods

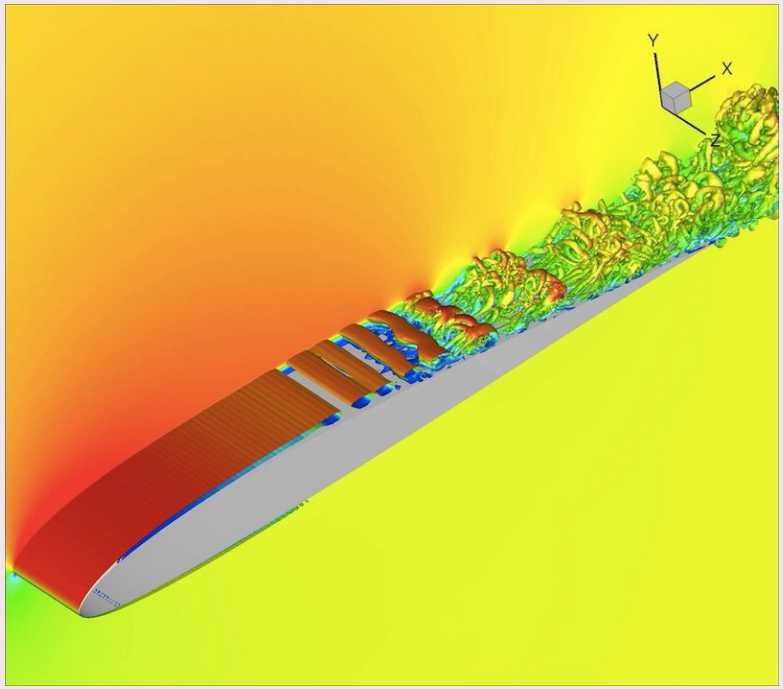
1. Castonguay, P., D. Williams, P. Vincent, M. Lopez, and A. Jameson (2011). On the development of a high-order, multi-GPU enabled, compressible viscous flow solver for mixed grids. AIAA P., vol. 2011-3229
2. Jameson, A. (2010). A proof of the stability of the spectral difference method for all orders of accuracy. J. Sci. Comput., vol. 45(1)
3. Jameson, A. (2011). Advances in bringing high-order methods to practical applications in computational fluid dynamics. AIAA P., vol. 2011-3226
4. Ou, K. and A. Jameson (2011). Unsteady adjoint method for the optimal control of advection and Burgers equations using high-order spectral difference method. AIAA P., vol. 2011- 24
5. Vincent, P. and A. Jameson (2011). Facilitating the adoption of unstructured high-order methods amongst a wider community of fluid dynamicists. Math. Model. Nat. Phenom., vol. 6(3)
6. Vincent, P., P. Castonguay, and A. Jameson (2010). A new class of high-order energy stable flux reconstruction schemes. J. Sci. Comput., vol. 47(1)
7. Vincent, P., P. Castonguay, and A. Jameson (2011). Insights from von Neumann analysis of high-order flux reconstruction schemes. J. Comput. Phys., vol. 230(22)
8. Williams, D., P. Castonguay, P. Vincent, and A. Jameson (2011). An extension of energy stable flux reconstruction to unsteady, non-linear, viscous problems on mixed grids. AIAA P., vol. 2011-3405
9. Lodato, G., P. Castonguay, and A. Jameson, Structural LES modeling with high-order spectral difference schemes. In Annual Research Briefs (Center for Turbulence Research, Stanford University, 2011)
10. Jameson, A., P. Vincent, and P. Castonguay (2012). On the non-linear stability of flux reconstruction schemes. J. Sci. Comput., vol. 50(2)

Transitional Flow over SD7003 Airfoil

	Freestream Turbulence	Separation x_{sep}/c	Transition x_{tr}/c	Reattach. x_r/c
Radespiel et al.	0.08%	0.30	0.53	0.64
Ol et al.	0.10%	0.18	0.47	0.58
Galbraith Visbal	0%	0.23	0.55	0.65
Uranga et al.	0%	0.23	0.51	0.60
Present ILES*	**0%**	**0.23**	**0.53**	**0.64**

Experiments in green

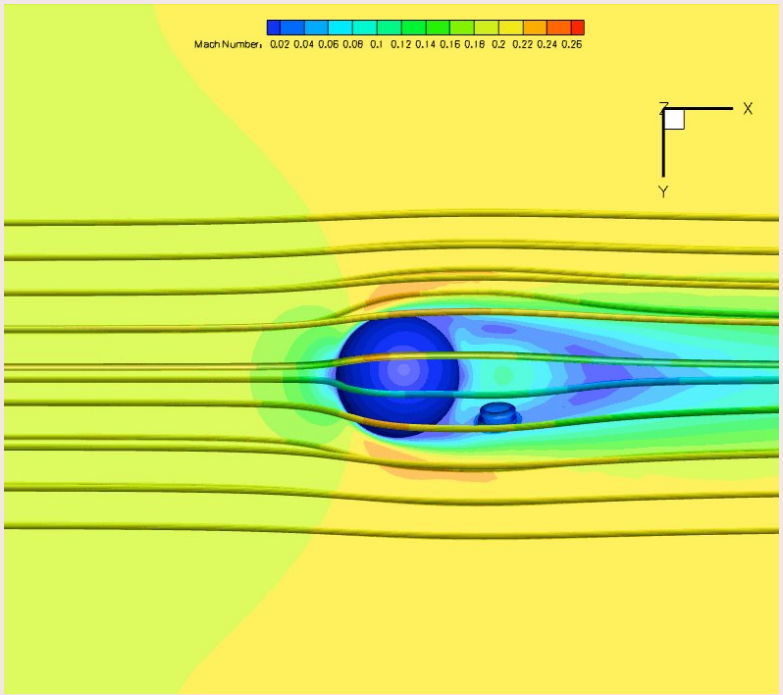
*1.7×10⁶ DoF



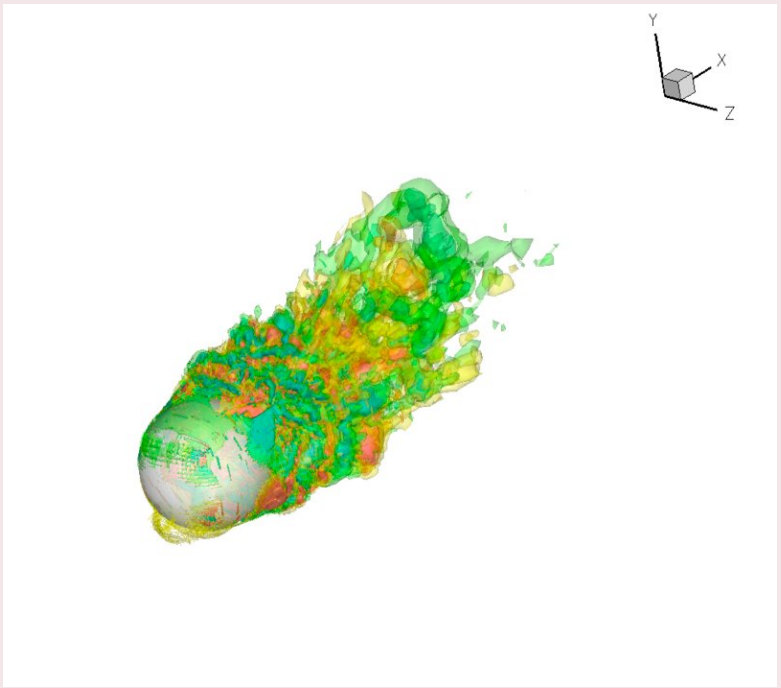
Iso-Entropy colored by Ma
Wing-Body
$Re = 5000$, SD, $N = 4$, 2.1×10^7 DoF

Castonguay, et al. (2010), AIAA Paper 2010-4626; Radespiel, et al. (2007), AIAA J., 45(6); Ol, et al. (2005), AIAA Paper 2005-5149; Galbraith, Visbal (2008), AIAA Paper 2008-225; Uranga, et al. (2009), AIAA Paper 2009-4131;

Flow Over Spheres



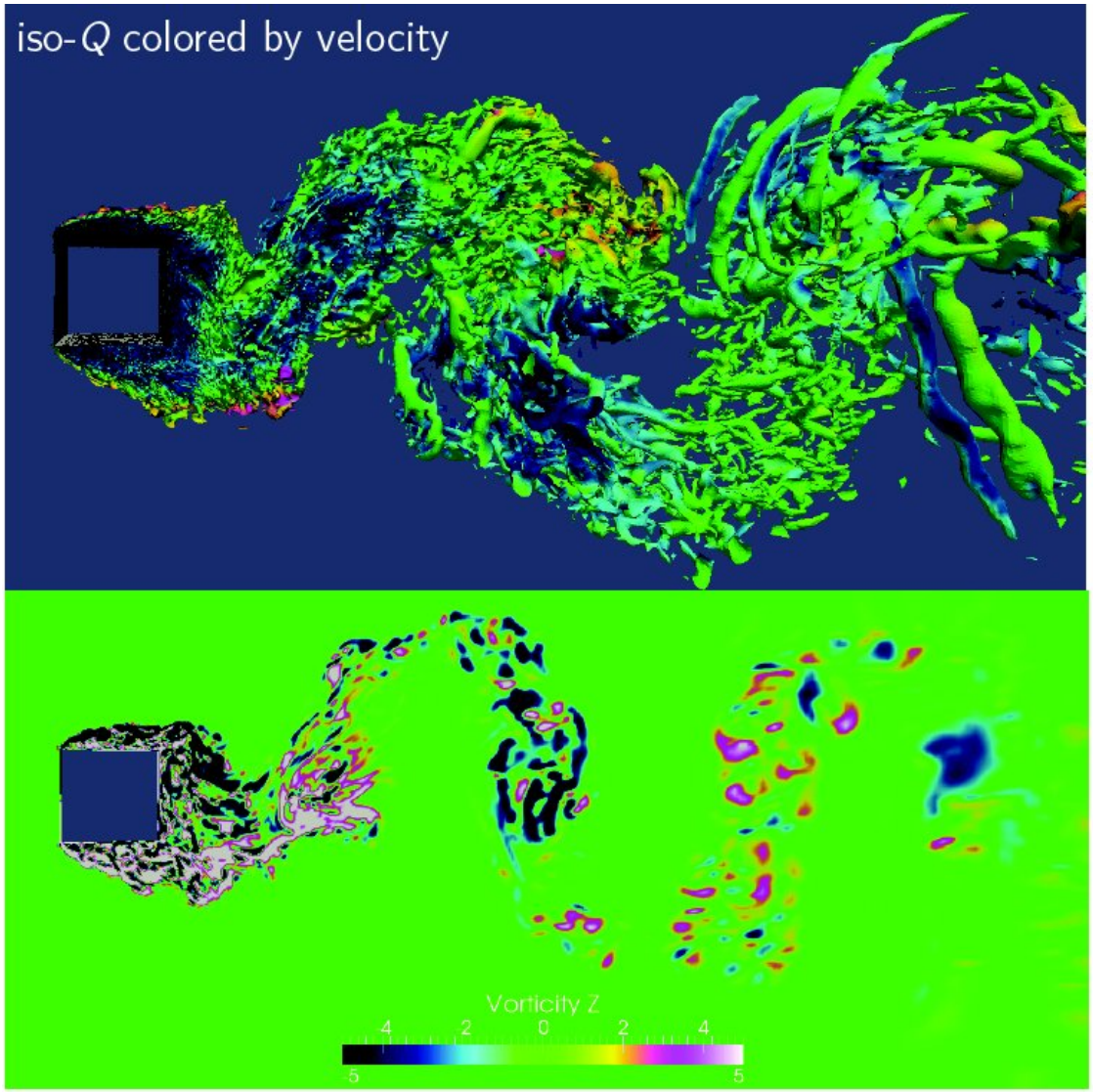
Mach Contours + Streamlines
Flow over a spinning sphere,
$Re = 300$, $Ma = 0.2$

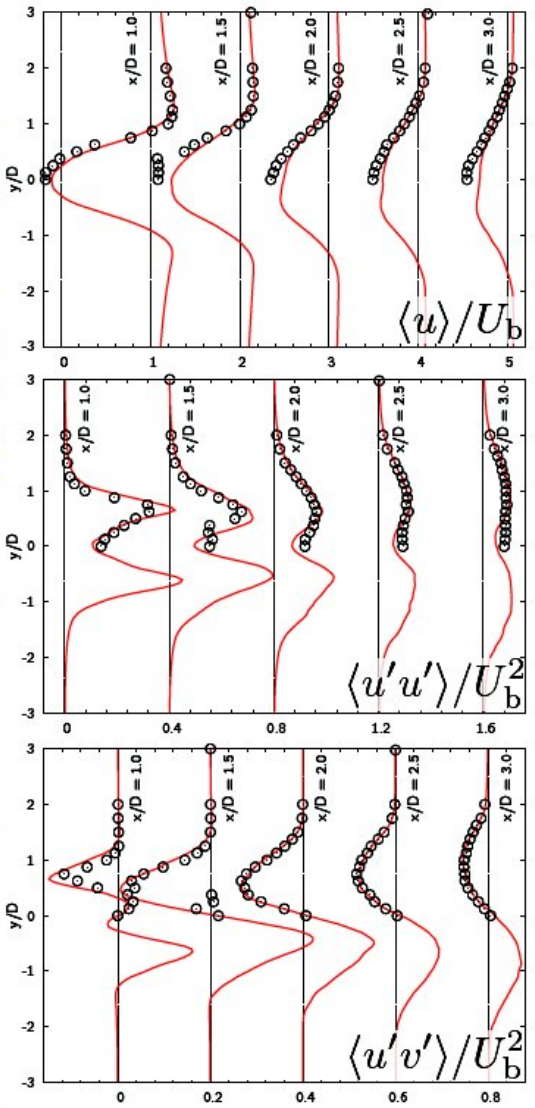


Iso-Entropy colored by Ma
Flow over a sphere
$Re = 10000$, $Ma = 0.2$

Ou, et al. (2011), AIAA Paper 2011-3668

Large Eddy Simulation of Flow Past Square Cylinder: $Re_D = 21400$





Preliminary results (work in progress)

Summary and Conclusions

Predicting the future is generally ill advised.
However, the following are the author's opinions:

- The early development of CFD in the Aerospace Industry was primarily driven by the need to calculate steady transonic flows: **this problem is quite well solved**.
- CFD has been on a plateau for the last 15 years with 2nd-order accurate FV methods for the RANS equations almost universally used in both commercial and government codes which can treat complex configurations.
- These methods cannot reliably predict complex separated, unsteady and vortex dominated flows.
- Ongoing advances in both numerical algorithms and computer hardware and software should enable an advance to LES for industrial applications within the foreseeable future.
- Research should focus on high-order methods with minimal numerical dissipation for unstructured meshes to enable the treatment of complex configurations.
- Eventually DNS may become feasible for high Reynolds number flows...

hopefully with a smaller power requirement than a wind tunnel.