AA200b - Applied Aerodynamics II

Winter Quarter 2004-05

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Sample Final Exam Problems **Due Date: Not due**

Problem 1. *Lumped Vortex Method.* Find the lift and moment on the slatted, flapped symmetric airfoil of the Figure below by representing the vortex system with three concentrated vortices, at the quarter chords of the three sections of the airfoil, and determining their strengths so as to satisfy the approximate flow tangency boundary condition from Equation (4-10) at the three-quarter-chord points on the three sections of the airfoil. Take $\delta_s = 5°$ and $\delta_f = 5°$.

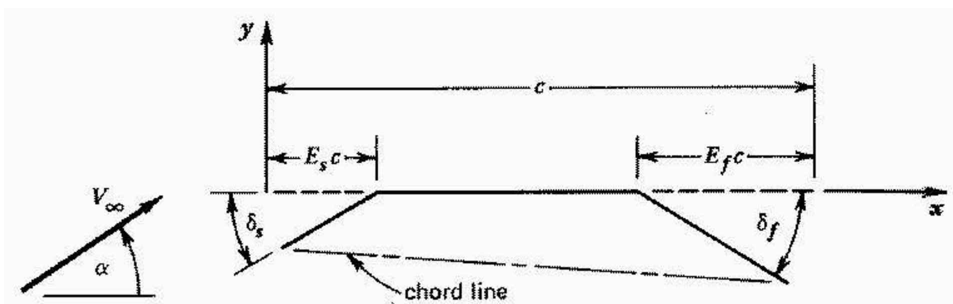

Figure 1: Multi-element Airfoil Geometry

Problem 2. *Boundary Layer Momentum Thickness, θ, using Thwaites' method* Thwaites' formula for the boundary layer momentum thickness is

$$\theta^2 = \frac{0.45\nu}{V_e^6}\int_0^x V_e^5 dx$$

1. Use this result to *estimate* (say within 20%) the momentum thickness θ for the nozzle shown above.
2. Why do you expect the ratio of the displacement thickness to the momentum thickness at $x = 0$ to be greater or less then that for the flat plate boundary layer?
3. Assuming the flow were laminar and *steady*, sketch the downstream flow and give a *crude* estimate of how far downstream from the nozzle you would expect the boundary layers from the upper and lower surfaces of the nozzle to merge on the centerline.
4. Why will the actual distance be very much less?

Problem 3. Answer the following multiple choice questions. These questions may have multiple answers. Circle ***ALL*** that apply.

1.1 For a thin airfoil with a parabolic camber distribution $\bar{Y}(x) = 4\epsilon\frac{x}{c}(c-x)$:

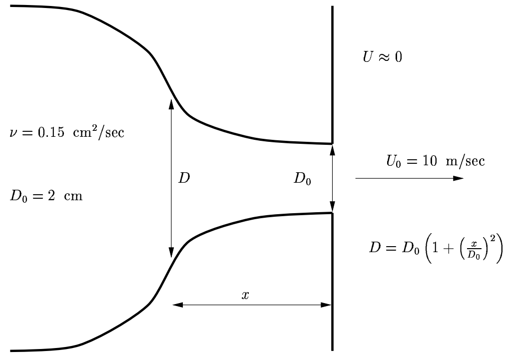

Figure 2: Nozzle Geometry for Problem 2

a. The aerodynamic center is located at the quarter chord point only for $\alpha = 0$.
b. The aerodynamic center is located at the quarter chord point for all angles of attack.
c. The center of pressure is located at the quarter chord point.
d. The center of pressure is located at the half chord point for $\alpha = 0$.
e. The aerodynamic center and the center of pressure *always* coincide, regardless of the values of ϵ and α.

1.2 If you were to discretize the x-momentum equation according to the boundary layer approximation, to obtain a relationship between the pressure gradient and the normal curvature of the tangential velocity profile *at the wall*, you would find that, looking from above the plate towards the plate itself:

a. A favorable pressure gradient induces a *concave* velocity profile making the boundary layer more stable.
b. A favorable pressure gradient induces a *convex* velocity profile making the boundary layer less stable.
c. An adverse pressure gradient induces a *convex* velocity profile making the boundary layer more stable.
d. An adverse pressure gradient induces a *concave* velocity profile making the boundary layer less stable.
e. An adverse pressure gradient induces a *convex* velocity profile making the boundary layer less stable.

1.3 Panel methods can be used, among other things, for the following types of flow calculations

a. Inviscid, incompressible, non-lifting flows around two-dimensional airfoils.
b. Inviscid, incompressible, lifting flows around two-dimensional airfoils.
c. Inviscid, incompressible, lifting flows around three-dimensional airfoils.
d. Inviscid, rotational, incompressible, lifting flows around two-dimensional airfoils.
e. Viscous, rotational, incompressible, one-dimensional flows.

1.4 Given the definition of the boundary layer *displacement thickness*, δ^*, the continuity equation can be integrated to yield the following expression, where u_e and v_e are the tangential and normal components of the velocity at the edge of the boundary layer.

a. $\frac{1}{u_e}\frac{d}{dx}\left[u_e(\delta - \delta^*)\right] = -\frac{v_e}{u_e}$

b. $\frac{1}{u_e}\frac{d}{dx}\left[u_e(\delta - \delta^*)\right] = \frac{v_e}{u_e}$

c. $\frac{1}{u_e}\frac{d}{dx}\left[u_e(\delta - \delta^*)\right] = \frac{d\delta}{dx} - \frac{v_e}{u_e}$

Problem 4. *Thin-Airfoil Theory.* For the NACA "1-series" of airfoils, the camberline was designed on the basis of this airfoil theory to yield a constant chordwise loading:

$$C_p(x, 0^+) - C_p(x, 0^-) = \text{constant}$$

Find $\bar{Y}(x)$ such that this is theoretically the case.

Problem 5. *Boundary Layer Momentum Thickness at a Stagnation Point.* The objective of this problem is to obtain an analytic formula for the momentum thickness at the stangnation point of a flow so that the estimate of this value can be used to impose initial conditions for Thwaites' method. Show that

$$\theta(0) = \sqrt{\frac{0.075\mu}{\rho\frac{dV_e(0)}{dx}}}$$

by using a first order Taylor series expansion for the boundary layer edge velocity, $V_e(x)$, at the stagnation point, $x = 0$ and by using this estimate in conjunction with Thwaites' method:

$$\frac{\rho}{\mu}\frac{d}{dx}(\theta^2 V_e^6) = 0.45 V_e^5$$

Notice that $\theta(0) \neq 0$. Do you find this surprising in any way?

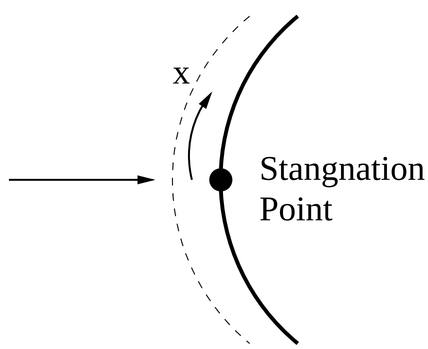

Figure 3: Stangnation Point Problem Coordinates

Problem 6. *Lifting-Line Theory.* For an untwisted constant-section wing of elliptical planform and aspect ratio 5, find the sectional and total lift and drag coefficients, according to lifting-line theory. Take the lift-curve slope m_0 to be 5.7, $V_\infty = 45$ m/sec, the wing loading $W/S = 1000$ N/m^2

(consider the level flight case, for which $L = W$), the altitude to be sea-level, and the span $b = 10$ m. Sea-level density can be taken to be $\rho_\infty = 1.2250$ Kg/m^3.

Also, plot the absolute and induced angles of attack as functions of spanwise position y, and calculate the power required to overcome the induced drag.