AA200b - Applied Aerodynamics II

Winter Quarter 2004-05

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PROBLEM SET 2 **Due Date: February 4th, 2005**

Problem 1. *Boundary Layer Momentum Thickness, θ.* Given the definition of the momentum thickness

$$\theta \equiv \int_0^\infty \frac{u}{V_e}\left(1 - \frac{u}{V_e}\right) dy$$

come up with an explanation of its physical meaning in terms of momentum fluxes through the boundary layer. Show that this explanation holds mathematically.

Problem 2. *Quadratic Velocity Profile Fitting.* In the spirit of Pohlhausen's Method, assume that you can describe a laminar boundary layer profile with the quadratic formula

$$\begin{aligned} \frac{u(x,y)}{V_e(x)} &= A + B\frac{y}{\delta} + C\left(\frac{y}{\delta}\right)^2 \quad && \text{for } y < \delta(x) \\ &= 1 && \text{for } y > \delta(x) \end{aligned}$$

where $\delta(x)$ is the boundary layer thickness. Choose the coefficients so that the no-slip boundary condition is satisfied and so that $\frac{\partial u}{\partial y} = 0$ at the edge of the boundary layer.

From this velocity profile determine, for $V_e = \text{const}$, the values of $\theta(x)$, $c_f(x)$, $\delta(x)$, $\delta^*(x)$, and $H(x)$. What is the relative error in C_f incurred by using this simple velocity fit?

Problem 3. *Thwaites' method.* If the variables in the boundary layer problem are non-dimensionalized according to

$$\begin{aligned} \tilde{V_e} &\equiv \frac{V_e}{V_{ref}} \\ \tilde{x} &\equiv \frac{x}{L} \\ \tilde{\theta} &\equiv \frac{\theta}{L} \end{aligned}$$

the governing equation for Thwaites' method

$$\frac{\rho}{\mu}\frac{d}{dx}\left(\theta^2 V_e^6\right) = 0.45 V_e^5$$

becomes

$$\text{Re}\frac{d}{d\tilde{x}}\left(\tilde{\theta}^2 \tilde{V}_e^6\right) = 0.45 \tilde{V}_e^5$$

where

$$\text{Re} = \frac{\rho V_{ref} L}{\mu}.$$

Integrate Thwaites' differential equation analytically for the case

$$\tilde{V}_e = \tilde{x}^m$$

and plot the quantities $\frac{\mathrm{Re}_\theta}{\sqrt{\mathrm{Re}_x}}$, H, and $\sqrt{\mathrm{Re}_x c_f}$ as functions of m for $-0.09 < m < 1.0$. How do these results compare with the Falkner-Skan solution? Hint: you can use the formula

$$\tilde{\theta}(0) = \sqrt{\frac{0.075}{\mathrm{Re}\dot{\tilde{V}}_0}}$$

where

$$\dot{\tilde{V}}_0 = \frac{d\tilde{V}_e}{d\tilde{x}}(0)$$

to estimate the initial value of the boundary layer momentum thickness.