

**PROBLEM SET 5****Due Date: March 17th, 2005**

Problem 1. *Design of a Commercial Transport Winglet for Minimum Induced Drag.* The objective of this problem set is to modify the *baseline* wing-winglet configuration for a commercial transport provided in order to achieve optimum performance as measured by obtaining minimum induced drag at a lift coefficient, $C_L = 0.6$. For this purpose, you will follow the approach sketched out in the paper provided in class:

Smith, S. C., "Treffitz-Plane Drag Minimization at Transonic Speeds", SAE General, Corporate & Regional Aviation Meeting & Exposition, SAE Paper 971478. Wichita, KS, April, 1997.

which requires that you obtain sensitivity derivatives of the *induced drag*, D_i , of the configuration with respect to the design variables in the problem. For simplicity, we will only deal with two design variables which are the *toe-in* angles of the root and tip sections of the last portion of the winglet which are highlighted in Figure 1 below.

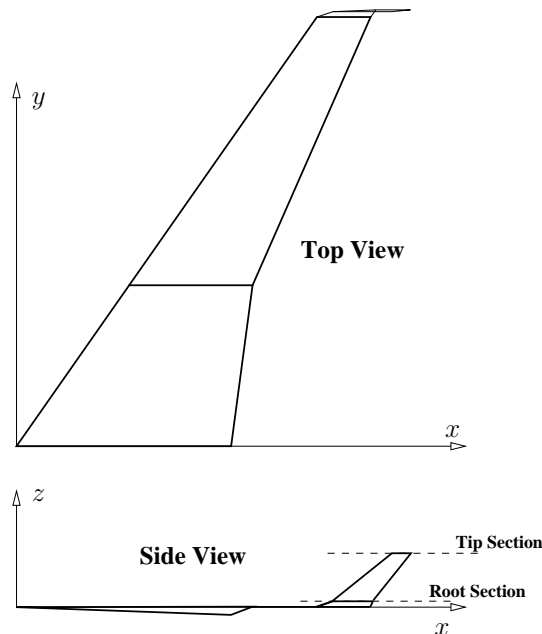


Figure 1: Top and Side View of Commercial Transport Baseline Configuration.

However, since we will be imposing a total lift constraint, we need to include a third design variable, the angle of attack of the complete configuration, α .

The aerodynamic analysis program that you will use to redesign this wing-winglet configuration is called LinAir4 and was written by Prof. Kroo. A modified version (for this problem set) of LinAir can be accessed from any web browser at

<http://adg.stanford.edu/aa208/linair4/1a.html>

The web page contains an explanation of the various parameters in the input and output files. A sample input file for the baseline configuration in Figure 1 can be found on the course web page. You can cut and paste this input file into the LinAir window.

Using the method discussed in class, derive the values of the optimum *toe-in* angles at the root and tip sections of the last portion of the winglet. Your solution should include a discussion of all the important steps that you followed to achieve the optimum.

A suggested plan of approach includes:

1. For a given wing-winglet configuration (whether it corresponds to the baseline or to a perturbed shape) you must be able to obtain the values of both the jump in potential across the wake, μ_i , and the velocity normal to the wake, ω_i , along the projected location of the wake on the Trefftz-plane. Assume that the wake travels in the direction of the free-stream from the trailing edge of the configuration. Also assume that the Trefftz-plane is far enough downstream and that the vortex filaments that intersect it are perpendicular to it, so that you can do all your downwash calculations as if the flow in the Trefftz-plane were two-dimensional.
2. Using the finite difference method, approximate the sensitivity derivatives required, $\frac{\partial \mu_i}{\partial x_j}$ and $\frac{\partial \omega_i}{\partial x_j}$, where x_j is the j -th design variable. In this case, $j = 1, 2, 3$, corresponding to α_{root} , α_{tip} , and α . Using the finite difference method, you can obtain a reasonable approximation to these sensitivities with

$$\frac{\partial \mu_i}{\partial x_j} = \frac{\mu_{ij} - \mu_{0i}}{\delta x_j}.$$

Remember that this is only a first-order approximation so you must take a small step in the design variables δx_j so that the approximation is not inaccurate (i.e., the truncation error of the approximation dominates), yet big enough so that subtractive cancellation does not take place.

3. Construct the matrix $[dic]$ and the right-hand-side vector $\{dic_0\}$ in Equations (7) and (8) of the paper by S. C. Smith.
4. Invert the resulting problem (including the lift constraint) to get the optimum values of the design variables.

Overplot the spanwise load distributions of the baseline and optimum designs and answer the following questions:

1. What is the magnitude in the improvement on D_i derived from the change in these design variables?
2. In light of your calculations of ω_i and μ_i , can you explain qualitatively what is the effect of the change in the winglet twist distribution?
3. How does the optimum design off-design behavior compare with that of the baseline design? Consider the clean-wing climb design point with $C_L = 1.0$.