Airfoil Design Methods

The process of airfoil design proceeds from a knowledge of the boundary layer properties and the relation between geometry and pressure distribution. The goal of an airfoil design varies. Some airfoils are designed to produce low drag (and may not be required to generate lift at all.) Some sections may need to produce low drag while producing a given amount of lift. In some cases, the drag doesn't really matter - it is maximum lift that is important. The section may be required to achieve this performance with a constraint on thickness, or pitching moment, or off-design performance, or other unusual constraints. Some of these are discussed further in the section on historical examples.

One approach to airfoil design is to use an airfoil that was already designed by someone who knew what he or she was doing. This "design by authority" works well when the goals of a particular design problem happen to coincide with the goals of the original airfoil design. This is rarely the case, although sometimes existing airfoils are good enough. In these cases, airfoils may be chosen from catalogs such as Abbott and von Doenhoff's Theory of Wing Sections, Althaus' and Wortmann's Stuttgarter Profilkatalog, Althaus' Low Reynolds Number Airfoil catalog, or Selig's "Airfoils at Low Speeds".

The advantage to this approach is that there is test data available. No surprises, such as a unexpected early stall, are likely. On the other hand, available tools are now sufficiently refined that one can be reasonably sure that the predicted performance can be achieved. The use of "designer airfoils" specifically tailored to the needs of a given project is now very common. This section of the notes deals with the process of custom airfoil design.

Methods for airfoil design can be classified into two categories: direct and inverse design.

Direct Methods for Airfoil Design

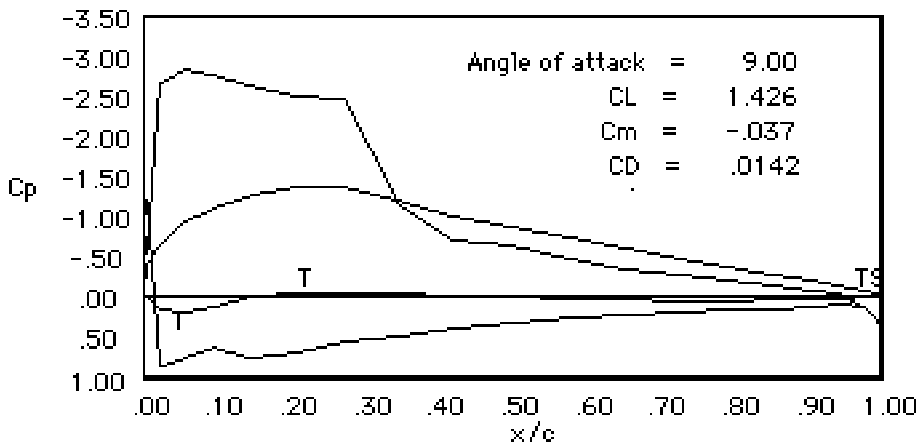
The direct airfoil design methods involve the specification of section geometry and the calculation of pressures and performance. One evaluates the given shape and then modifies the shape to improve the performance.

The two main subproblems in this type of method are

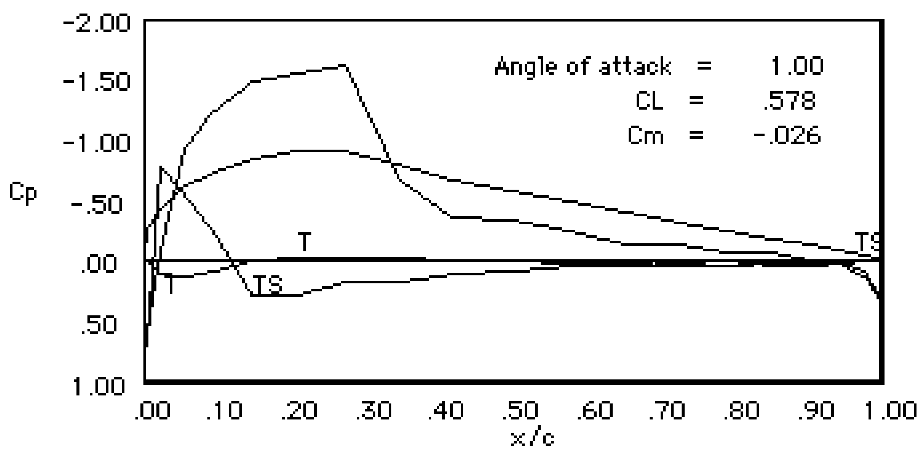
1. the identification of the measure of performance
2. the approach to changing the shape so that the performance is improved

The simplest form of direct airfoil design involves starting with an assumed airfoil shape (such as a NACA airfoil), determining the characteristic of this section that is most problemsome, and fixing this problem. This process of fixing the most obvious problems with a given airfoil is repeated until there is no major problem with the section. The

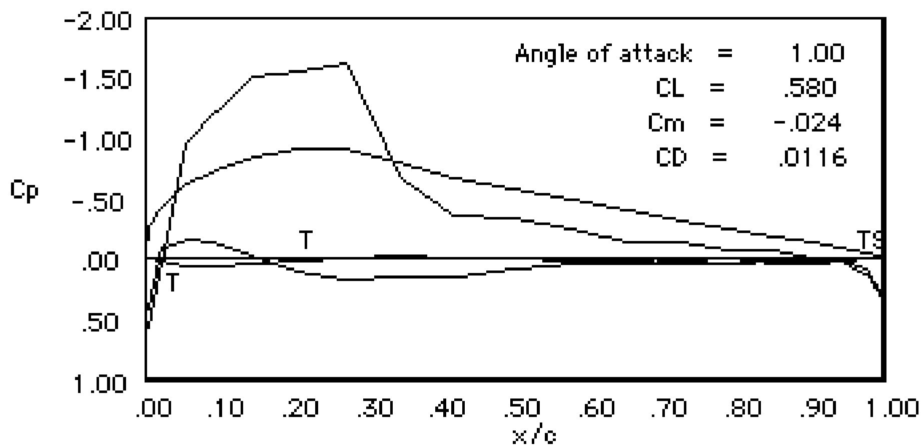
design of such airfoils, does not require a specific definition of a scalar objective function, but it does require some expertise to identify the potential problems and often considerable expertise to fix them. Let's look at a simple (but real life!) example.
A company is in the business of building rigid wing hang gliders and because of the low speed requirements, they decide to use a version of one of Bob Liebeck's very high lift airfoils. Here is the pressure distribution at a lift coefficient of 1.4. Note that only a small amount of trailing edge separation is predicted. Actually, the airfoil works quite well, achieving a C_{lmax} of almost 1.9 at a Reynolds number of one million.



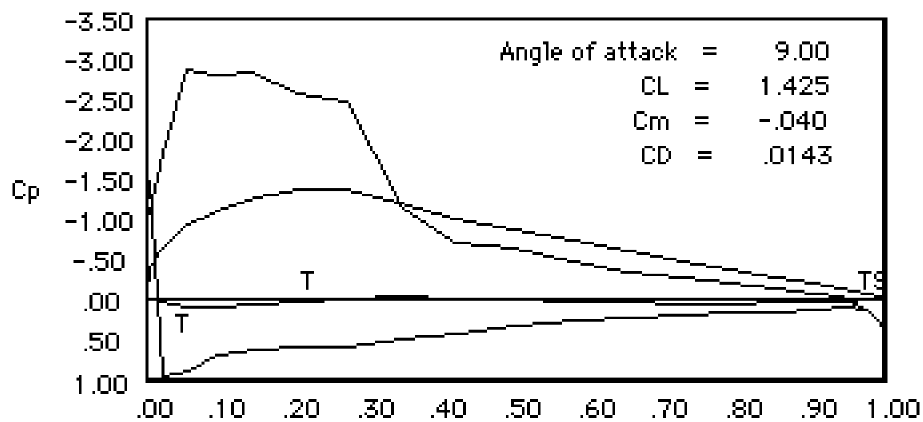
This glider was actually built and flown. It, in fact, won the 1989 U.S. National Championships. But it had terrible high speed performance. At lower lift coefficients the wing seemed to fall out of the sky. The plot below shows the pressure distribution at a C_l of 0.6. The pressure peak on the lower surface causes separation and severely limits the maximum speed. This is not too hard to fix.



By reducing the lower surface "bump" near the leading edge and increasing the lower surface thickness aft of the bump, the pressure peak at low C_l is easily removed. The lower surface flow is now attached, and remains attached down to a C_l of about 0.2. We must check to see that we have not hurt the C_{lmax} too much.



Here is the new section at the original design condition (still less than C_{lmax}). The modification of the lower surface has not done much to the upper surface pressure peak here and the C_{lmax} turns out to be changed very little. This section is a much better match for the application and demonstrates how effective small modifications to existing sections can be. The new version of the glider did not use this section, but one that was designed from scratch with lower drag.



Sometimes the objective of airfoil design can be stated more positively than, "fix the worst things". We might try to reduce the drag at high speeds while trying to keep the maximum CL greater than a certain value. This could involve slowly increasing the amount of laminar flow at low C_l's and checking to see the effect on the maximum lift. The objective may be defined numerically. We could actually minimize C_d with a constraint on C_{lmax}. We could maximize L/D or $C_l^{1.5}/C_d$ or C_{lmax} / C_d @$C_{ldesign}$. The selection of the figure of merit for airfoil sections is quite important and generally cannot be done without considering the rest of the airplane. For example, if we wish to build an airplane with maximum L/D we do not build a section with maximum L/D because the section C_l for best Cl/C_d is different from the airplane C_L for best C_L/C_D.

Inverse Design

Another type of objective function is the target pressure distribution. It is sometimes possible to specify a desired C_p distribution and use the least squares difference between the actual and target C_p's as the objective. This is the basic idea behind a variety of methods for inverse design. As an example, thin airfoil theory can be used to solve for the shape of the camberline that produces a specified pressure difference on an airfoil in potential flow.

The second part of the design problem starts when one has somehow defined an objective for the airfoil design. This stage of the design involves changing the airfoil shape to improve the performance. This may be done in several ways:

1. By hand, using knowledge of the effects of geometry changes on C_p and C_p changes on performance.
2. By numerical optimization, using shape functions to represent the airfoil geometry and letting the computer decide on the sequence of modifications needed to improve the design.

Typical Airfoil Design Problems

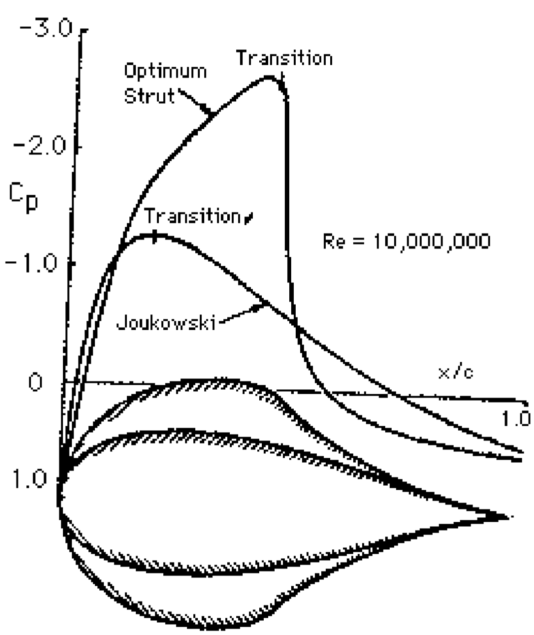
Regardless of the design goals and constraints, one is faced with some common problems that make airfoil design difficult. This section deals with the common issues that arise in the following design problems:

- Design for maximum thickness
- Design for maximum lift
- Laminar boundary layer airfoil design
- High lift or thickness transonic design
- Low Reynolds number airfoil design
- Low or positive pitching moment designs
- Multiple design points

Thick Airfoil Design

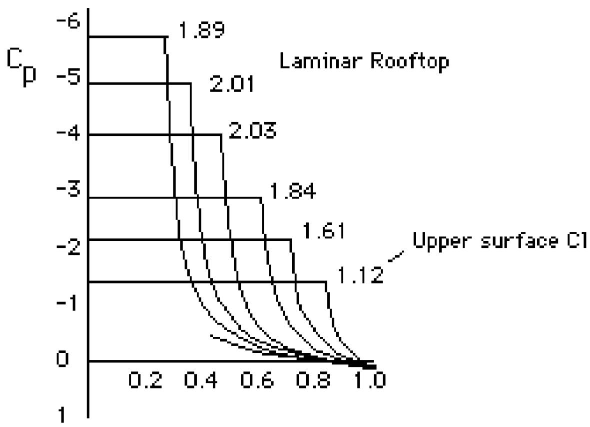
The difficulty with thick airfoils is that the minimum pressure is decreased due to thickness. This results in a more severe adverse pressure gradient and the need to start recovery sooner. If the maximum thickness point is specified, the section with maximum thickness must recover from a given point with the steepest possible gradient. This is just the sort of problem addressed by Liebeck in connection with maximum lift. The thickest possible section has a boundary layer just on the verge of separation throughout the recovery.

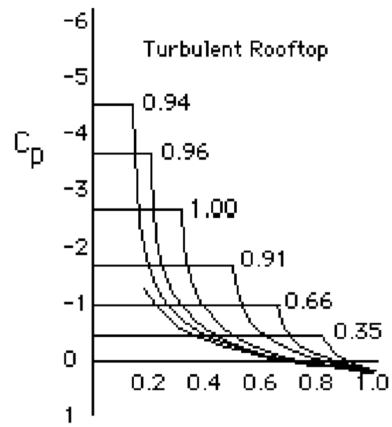
The thickest section at Re = 10 million is 57% thick, but of course, it will separate suddenly with any angle of attack.



High Lift Airfoil Design

To produce high lift coefficients, we require very negative pressures on the upper surface of the airfoil. The limit to this suction may be associated with compressibility effects, or may be imposed by the requirement that the boundary layer be capable of negotiating the resulting adverse pressure recovery. It may be shown that to maximize lift starting from a specified recovery height and location, it is best to keep the boundary layer on the verge of separation*. Such distributions are shown below for a Re of 5 million. Note the difference between laminar and turbulent results. The thickest section at Re = 10 million is 57% thick, but of course, it will separate suddenly with any angle of attack.

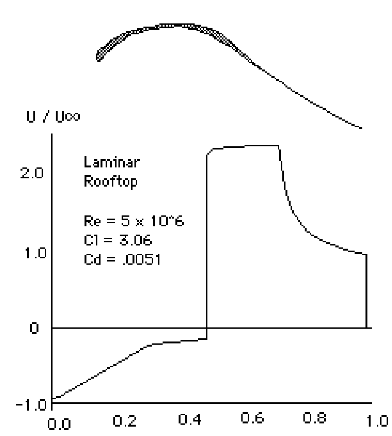




For maximum airfoil lift, the best recovery location is chosen and the airfoil is made very thin so that the lower surface produces maximum lift as well. (Since the upper surface Cp is specified, increasing thickness only reduces the lower surface pressures.)

Well, almost. If the upper surface Cp is more negative than -3.0, the perturbation velocity is greater than freestream, which means, for a thin section, the lower surface flow is upstream. This would cause separation and the maximum lift is achieved with an upper surface velocity just over 2U and a bit of thickness to keep the lower surface near stagnation pressure. A more detailed discussion of this topic may be found in the section on [high lift systems.]

Liebeck's Famous Maximum Lift Airfoil



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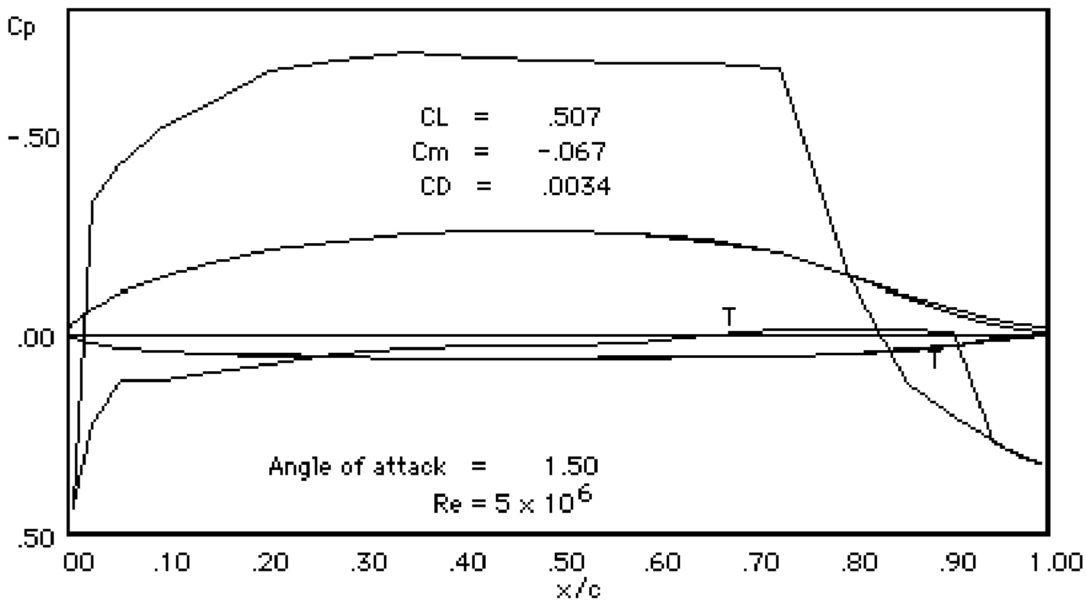
*This conclusion, described by Liebeck, is easily derived if Stratford's criterion or the laminar boundary layer method of Thwaites is used. For other turbulent boundary layer criteria, the conclusion is not at all obvious and indeed some have suggested (Kroo and Morris) that this is not the case.

Laminar Airfoil Design

Laminar flow may be useful for reducing skin friction drag, increasing maximum lift, or reducing heat transfer. It may be achieved without too much work at low Reynolds numbers by maintaining a smooth surface and using an airfoil with a favorable pressure gradient. The section below shows how the pressures may be tailored to achieve long runs of laminar flow on upper and/or lower surfaces.

Again, the Stratford-like pressure recovery is helpful in achieving the maximum run of favorable gradient on either upper or lower surfaces.



Transonic Airfoil Design

The transonic airfoil design problem arises because we wish to limit shock drag losses at a given transonic speed. This effectively limits the minimum pressure coefficient that can be tolerated. Since both lift and thickness reduce (increase in magnitude) the minimum Cp, the transonic design problem is to create an airfoil section with high lift and/or thickness without causing strong shock waves. One can generally tolerate some supersonic flow without drag increase, so that most sections can operate efficiently as "supercritical airfoils". A rule of thumb is that the maximum local Mach numbers should not exceed about 1.2 to 1.3 on a well-designed supercritical airfoil. This produces a considerable increase in available C_l compared with entirely subcritical designs.

Supercritical sections usually refer to a special type of airfoil that is designed to operate efficiently with substantial regions of supersonic flow. Such sections often take advantage of many of the following design ideas to maximize lift or thickness at a given Mach number:

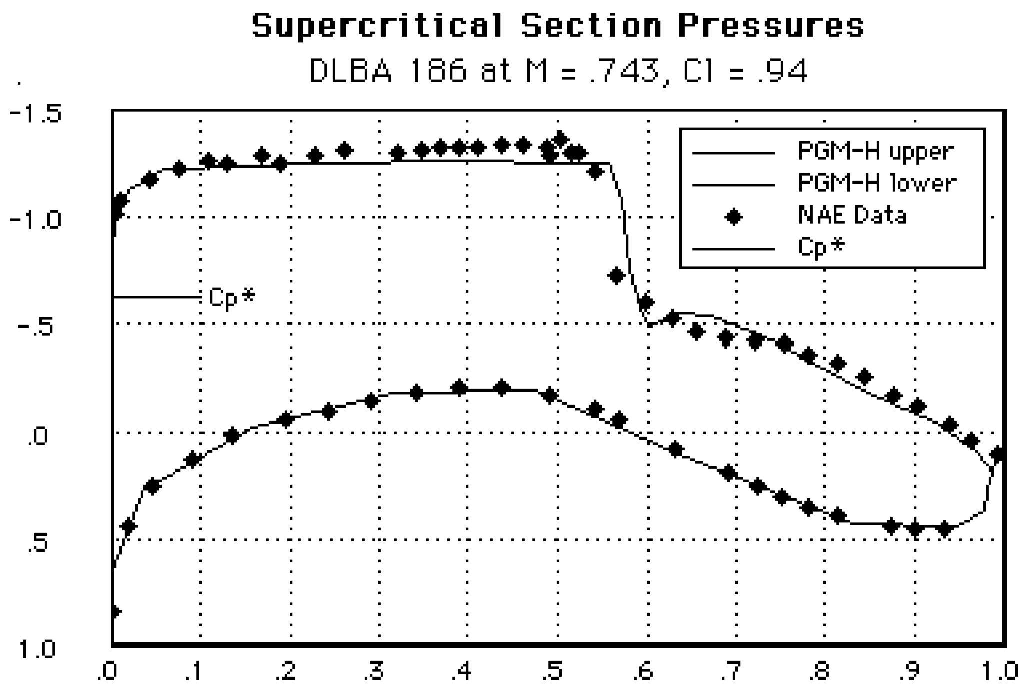
- Carry as much lift as is practical on the aft potion of the section where the flow is subsonic. The aft lower surface is an obvious candidate for increased loading (more positive Cp), although several considerations discussed below limit the extent to which this approach can be used.
- Make sure that sufficient lift is carried on the forward portion of the upper surface. As the Mach number increases, the pressure peak near the nose is diminished and without additional blunting of the nose, possible extra lift will be lost in this region.
- The lower surface near the nose can also be loaded by reducing the lower surface thickness near the leading edge. This provides both lift and positive pitching moment.
- Shocks on the upper surface near the leading edge produce much less wave drag than shocks aft of the airfoil crest and it is feasible, although not always best, to design sections with forward shocks. Such sections are known as "peaky" airfoils and were used on many transport aircraft.
- The idea of carefully tailoring the section to obtain locally supersonic flow without shockwaves (shock-free sections) has been pursued for many years, and such sections have been designed and tested. For most practical cases with a range of design CL and Mach number, sections with weak shocks are favored.

One must be cautious with supercritical airfoil design. Several of these sections have looked promising initially, but led to problems when actually incorporated into an aircraft design. Typical difficulties include the following.

- Too much aft loading can produce large negative pitching moments with trim drag and structural weight penalties.

- The adverse pressure gradient on the aft lower surface can produce separation in extreme cases.
- The thin trailing edge may be difficult to manufacture.
- Supercritical, and especially shock-free designs often are very sensitive to Mach and CL and may perform poorly at off-design conditions. The appearance of "drag creep" is quite common, a situation in which substantial section drag increase with Mach number occurs even at speeds below the design value.

The section with pressures shown below is typical of a modern supercritical section with a weak shock at its design condition. Note the rooftop C_p design with the minimum C_p considerably greater above C_p^*.



Low Reynolds Number Airfoil Design

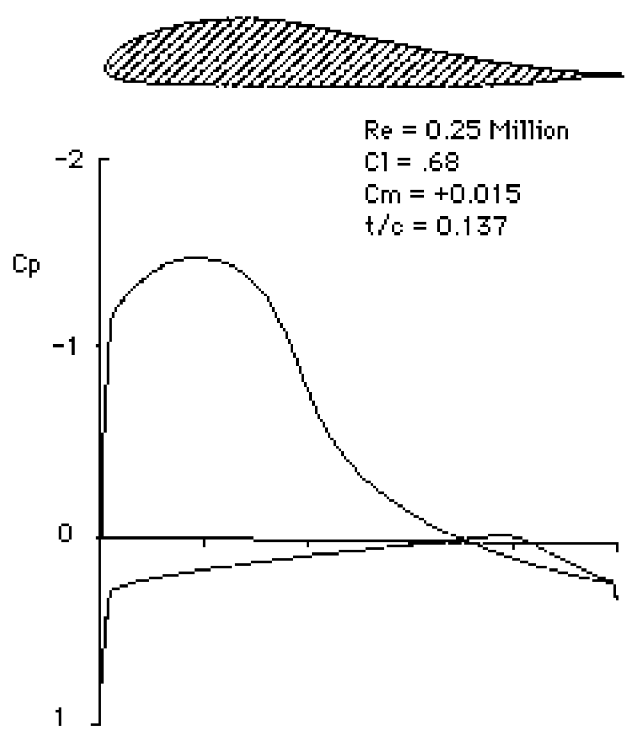
Low Reynolds numbers make the problem of airfoil design difficult because the boundary layer is much less capable of handling an adverse pressure gradient without separation. Thus, very low Reynolds number designs do not have severe pressure gradients and the maximum lift capability is restricted.

Low Reynolds number airfoil designs are cursed with the problem of too much laminar flow. It is sometimes difficult to assure that the boundary layer is turbulent over the steepest pressure recovery regions. Laminar separation bubbles are common and unless properly stabilized can lead to excessive drag and low maximum lift.

At very low Reynolds numbers, most or all of the boundary layer is laminar. Under such conditions the boundary layer can handle only gradual pressure recovery. Based on the expressions for laminar separation, one finds that an all-laminar section can generate a CL of about 0.4 or achieve a thickness of about 7.5%, (Try this with PANDA.)

Low Moment Airfoil Design

When the airfoil pitching moment is constrained, it is not always possible to carry lift as far back on the airfoil as desired. Such situations arise in the design of sections for tailless aircraft, helicopter rotor blades, and even sails, kites, and giant pterosaurs. The airfoil shown here is a Liebeck section designed to perform well at low Reynolds numbers with a positive C_{mo}. Its performance is not bad, but it is clearly inferior in C_{lmax} when compared to other sections without a Cm constraint. (C_{lmax} =1.35 vs. 1.60 for conventional sections at Re = 500,000.)



Multiple Design Point Airfoils

One of the difficulties in designing a good airfoil is the requirement for acceptable off-design performance. While a very low drag section is not too hard to design, it may separate at angles of attack slightly away from its design point. Airfoils with high lift capability may perform very poorly at lower angles of attack.

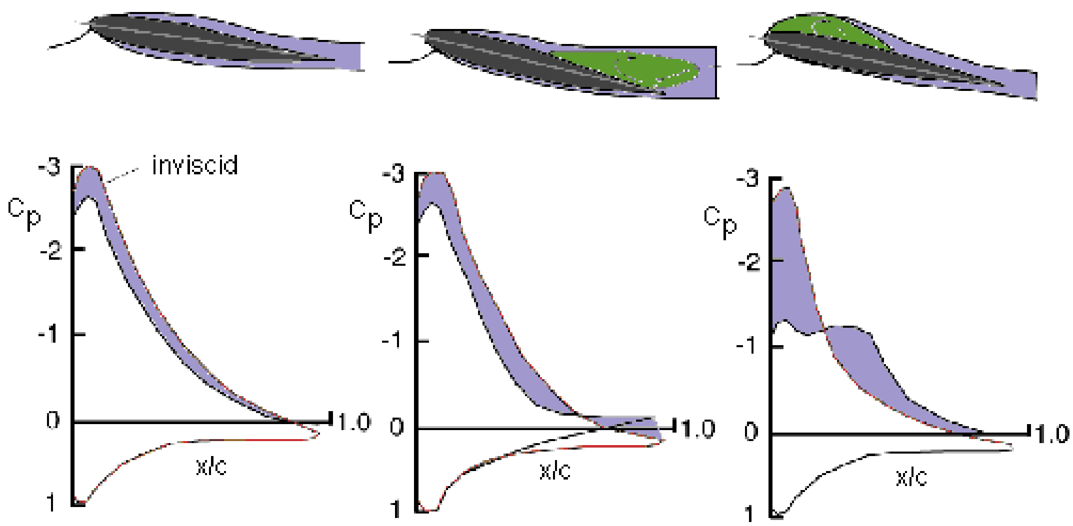
One can approach the design of airfoil sections with multiple design points in a well-defined way. Often it is clear that the upper surface will be critical at one of the points and we can design the upper surface at this condition. The lower surface can then be designed to make the section behave properly at the second point. Similarly, constraints such as C_{mo} are most affected by airfoil trailing edge geometry.

When such a compromise is not possible, variable geometry can be employed (at some expense) as in the case of high lift systems.

7.3. Separation

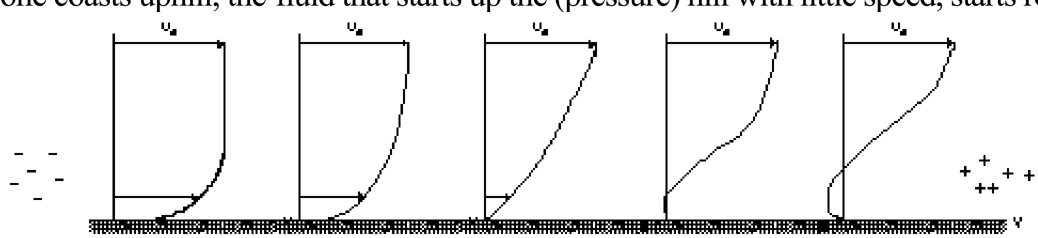
When the flow near the surface reverses its direction and flows upstream, there must be a place, generally a bit farther upstream, where streamlines meet and then leave the surface. This is separation and it is caused by the presence of an adverse pressure gradient. When this occurs, the assumptions that the u component of velocity is larger than the v component and that certain derivatives in the x direction may be ignored, no longer are valid. Thus, coupling an inviscid analysis with a simple boundary layer calculation does not work. One must resort to experiment or Navier-Stokes solutions.

The changes in the flow pattern, and associated forces and moments are large. Drag usually increases substantially and airfoil lift usually drops. The effect is generally Reynolds number dependent.



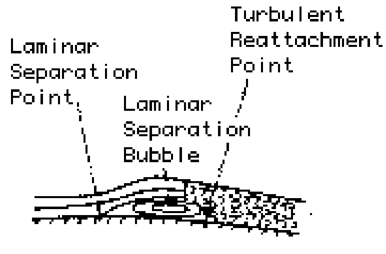
The figure above shows how the flow pattern and pressure distribution is affected by separation. On the left, the pressures are modified slightly by the boundary layer; in the center image, separation near the trailing edge has reduced the Cp and lift; leading edge separation dramatically reduces the suction peaks and reduces lift.

The presence of an adverse pressure gradient (increasing pressure) causes a deceleration of the fluid. Just as when one coasts uphill, the fluid that starts up the (pressure) hill with little speed, starts rolling backward after a while.



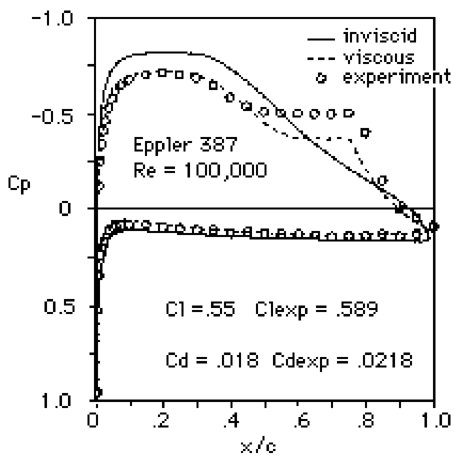
This picture explains why flow does not separate as readily at higher Reynolds numbers. In that case, the velocity profile is "fuller" with the high external velocities extending down closer to the surface. Turbulent boundary layers also have greater velocity near the surface and are therefore better able to handle adverse pressure gradients.

Since the velocity near the surface in a laminar boundary layer has lower velocity than its turbulent counterpart, the laminar boundary layer is more likely to separate. When this occurs, the laminar boundary layer leaves the surface and usually undergoes transition to turbulent flow away from the surface. This process takes place over a certain distance that is inversely related to the Reynolds number, but if it happens quickly enough, the flow may reattach as a turbulent boundary layer and continue along the surface.



Laminar separation with Turbulent Reattachment

This phenomenon has significant effects on airfoil pressure distributions at low Reynolds numbers.



To compute when separation will occur, we can solve the Navier-Stokes equations or apply one of several separation criteria to solutions of the boundary layer equations.

[Laminar Separation Criteria]
[Turbulent Separation Criteria]

7.3.1. Laminar Separation Criteria

Since in Thwaites method we essentially assume a shape for the profile, we can tell when the flow in the boundary layer reverses. This happens when:

$$\lambda = -0.09 = \frac{\theta^2}{\nu} \frac{dU_e}{dx}$$

The exact value is not very important since λ changes quickly near the area of separation.

Another criterion that does not require numerical integration of the boundary layer equations is one due to Stratford. This criterion asserts that laminar separation occurs when:

$$C_p' \left(x' \frac{dC_p'}{dx} \right)^2 = 0.0104$$

Cp' and x' are the canonical pressure coefficient and effective boundary layer length.

Stratford's laminar separation criteria appropriately reflects the deleterious effect of the adverse gradient's severity and length.

Because the laminar boundary layer is prone to transition in an adverse gradient, it is difficult to predict whether the flow will transition or separate first. Sometimes the flow separates, transitions, and then reattaches in what is called a laminar separation bubble. The length of the bubble is a function of the pressure gradient and Reynolds number, growing longer as the Reynolds number is reduced. In any case, laminar separation is to be avoided in airfoil design. This is done in several ways including forcing transition with surface roughness elements (grit) or building in a special transition region in the pressure distribution.

Once the flow is turbulent, we must apply an entirely different set of separation criteria, described in following sections.

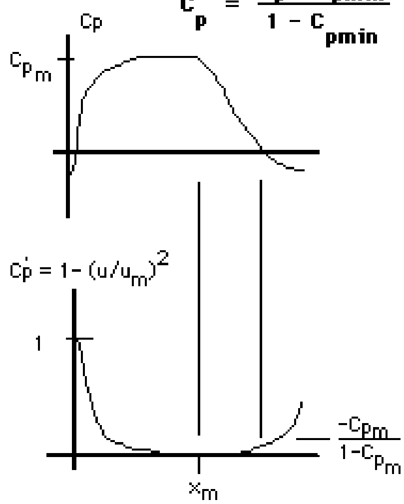
7.3.1.1. Canonical Pressure Distribution

Separation criteria are often stated in terms of the so-called canonical pressure coefficient, Cp'.

This is defined as: $C_p' = 1 - \frac{U^2}{U^2_{max}}$

Thus, the canonical pressure coefficient varies from 0 at the start of the pressure recovery to 1 at stagnation.

$$C_p' = \frac{C_p - C_{pmin}}{1 - C_{pmin}}$$



7.3.2. Turbulent Separation Criteria

Turbulent separation criteria are the most useful since most pressure recovery is done using turbulent boundary layers. There are many criteria that are used.

Minimum C_p
The simplest criterion is that used to estimate when the flow will separate from the leading edge of an airfoil. This rule of thumb states that there is a minimum value of C_p that can be tolerated. Numbers such as -10 to -13 are sometimes used, but this is a very crude rule and applies only to cases of leading edge separation.

Loftin's criterion
A related, but somewhat more sophisticated method is attributed to Loftin and states that the maximum value of C_p', the canonical pressure coefficient, after the start of recovery is +0.88. This is not a conservative estimate, however, and cannot be relied on for a wide range of airfoils.

Shape Factor
Perhaps the most reliable criterion is that based on the computed boundary layer quantities. It has been shown that separation is very likely when the value of the shape factor, H exceeds 2.2 to 2.4.

Stratford's Criterion
In 1959 Stratford devised a rather simple criterion for the separation of turbulent boundary layers. Similar to his laminar separation criterion, this rule states that separation will occur when:

$$C_p' \sqrt{x' \frac{dC_p'}{dx}} = \left(\frac{Re}{10^6}\right)^{0.1} S$$

Where the constant S is 0.35 when $d^2p/dx^2 < 0$ (concave recovery)
and 0.39 when $d^2p/dx^2 > 0$ (convex recovery)

The Reynold's number in the Stratford formula is based on the local effective length of the boundary layer, x', and the maximum velocity, U_m.

The formula is based on a great deal of empirical data and is only valid for $C_p' < 4/7$, but it is very useful in the design of airfoil sections. Stratford's method usually is conservative, predicting separation just a bit before the methods based on explicit computation of the shape factor.

Comparison with the corresponding laminar flow formula shows how turbulent boundary layers are very much more resistant to separation. Note also that the expression for x is different for turbulent boundary layers. More detail on the definition of effective boundary layer length is presented at the end of this section.

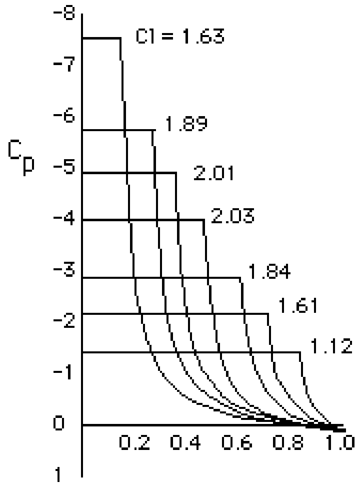
Stratford's criterion may be used to compute the shape of the pressure distribution that is everywhere on the edge of separation. This is a useful distribution for many reasons. Most importantly, it permits the most rapid possible recovery from a given minimum pressure. This, or something approaching it, would be used in the design of sections with maximum extent of laminar flow or sections with maximum lift or maximum thickness. This will be discussed in a subsequent section, but here we show how the particular C_p distribution is derived.

We start by taking Stratford's criterion as a differential equation describing the C_p variation and integrate the expression for the resulting C_p. This is not as straightforward as it appears since the formula is only valid for $C_p' < 4/$

Stratford effectively assumed a constant value of the boundary layer shape factor (e.g. H = 2.0) over this section and derived:

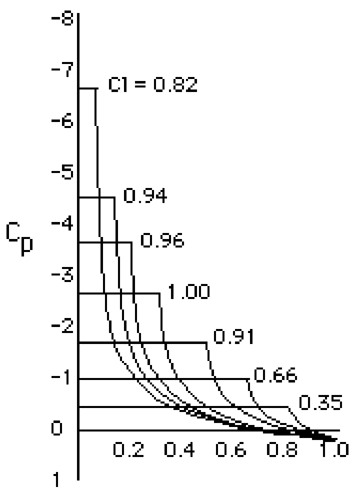
$$C_p' = 1 - \frac{a}{\sqrt{x + b}} \quad \text{for } C_p' > \frac{4}{7}$$

The values of a and b are chosen so that the slope and value of C_p' match at $C_p' = 4/7$.



Upper surface pressures with Stratford recovery to $C_p = 0.20$ at trailing edge.

Laminar Rooftop, Re = 5 x 10^6



Upper surface pressures with Stratford recovery to $C_p = 0.20$ at trailing edge.

Turbulent Rooftop, Re = 5 x 10^6

Liebeck airfoils with Stratford pressure recoveries designed for maximum lift.