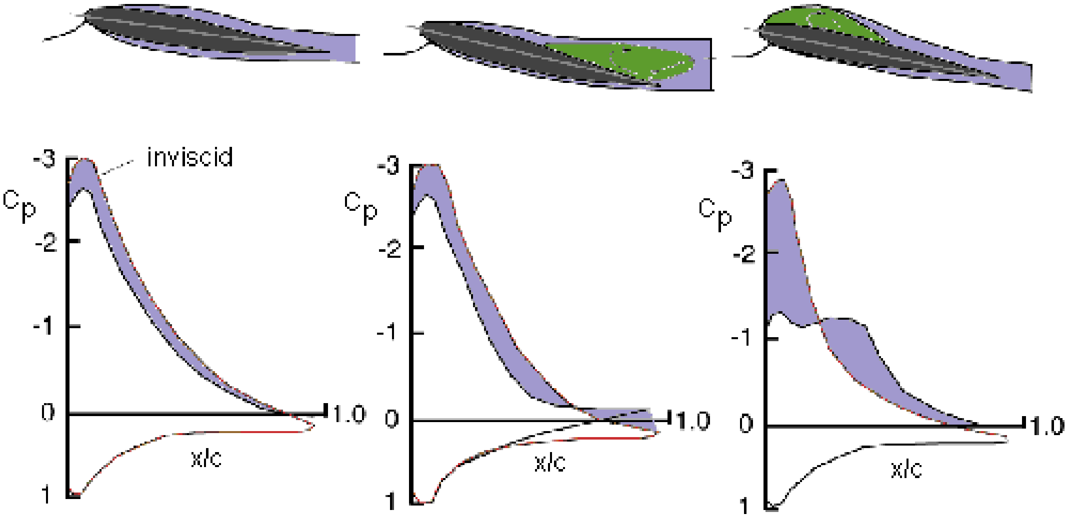
7.3. Separation

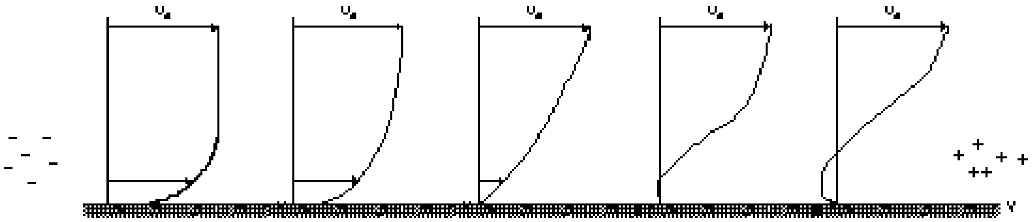
When the flow near the surface reverses its direction and flows upstream, there must be a place, generally a bit farther upstream, where streamlines meet and then leave the surface. This is separation and it is caused by the presence of an adverse pressure gradient. When this occurs, the assumptions that the u component of velocity is larger than the v component and that certain derivatives in the x direction may be ignored, no longer are valid. Thus, coupling an inviscid analysis with a simple boundary layer calculation does not work. One must resort to experiment or Navier-Stokes solutions.

The changes in the flow pattern, and associated forces and moments are large. Drag usually increases substantially and airfoil lift usually drops. The effect is generally Reynolds number dependent.



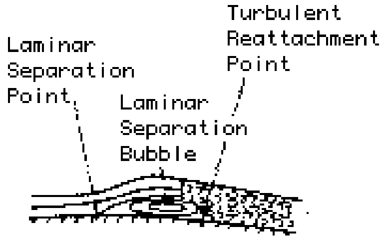
The figure above shows how the flow pattern and pressure distribution is affected by separation. On the left, the pressures are modified slightly by the boundary layer; in the center image, separation near the trailing edge has reduced the Cp and lift; leading edge separation dramatically reduces the suction peaks and reduces lift.

The presence of an adverse pressure gradient (increasing pressure) causes a deceleration of the fluid. Just as when one coasts uphill, the fluid that starts up the (pressure) hill with little speed, starts rolling backward after a while.



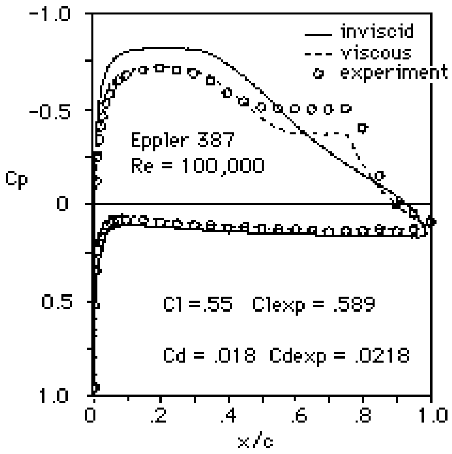
This picture explains why flow does not separate as readily at higher Reynolds numbers. In that case, the velocity profile is "fuller" with the high external velocities extending down closer to the surface. Turbulent boundary layers also have greater velocity near the surface and are therefore better able to handle adverse pressure gradients.

Since the velocity near the surface in a laminar boundary layer has lower velocity than its turbulent counterpart, the laminar boundary layer is more likely to separate. When this occurs, the laminar boundary layer leaves the surface and usually undergoes transition to turbulent flow away from the surface. This process takes place over a certain distance that is inversely related to the Reynolds number, but if it happens quickly enough, the flow may reattach as a turbulent boundary layer and continue along the surface.



Laminar separation with Turbulent Reattachment

This phenomenon has significant effects on airfoil pressure distributions at low Reynolds numbers.



To compute when separation will occur, we can solve the Navier-Stokes equations or apply one of several separation criteria to solutions of the boundary layer equations.

Laminar Separation Criteria

Turbulent Separation Criteria

7.3.1. Laminar Separation Criteria

Since in Thwaites method we essentially assume a shape for the profile, we can tell when the flow in the boundary layer reverses. This happens when:

$$\lambda = -0.09 = \frac{\theta^2}{\nu} \frac{dU_e}{dx}$$

The exact value is not very important since □ changes quickly near the area of separation.

Another criterion that does not require numerical integration of the boundary layer equations is one due to Stratford. This criterion asserts that laminar separation occurs when:

$$C_p' \left(x' \frac{dC_p'}{dx} \right)^2 = 0.0104$$

Cp' and x' are the canonical pressure coefficient and effective boundary layer length.

Stratford's laminar separation criteria appropriately reflects the deleterious effect of the adverse gradient's severity and length.

Because the laminar boundary layer is prone to transition in an adverse gradient, it is difficult to predict whether the flow will transition or separate first. Sometimes the flow separates, transitions, and then reattaches in what is called a laminar separation bubble. The length of the bubble is a function of the pressure gradient and Reynolds number, growing longer as the Reynolds number is reduced. In any case, laminar separation is to be avoided in airfoil design. This is done in several ways including forcing transition with surface roughness elements (grit) or building in a special transition region in the pressure distribution.

Once the flow is turbulent, we must apply an entirely different set of separation criteria, described in following sections.

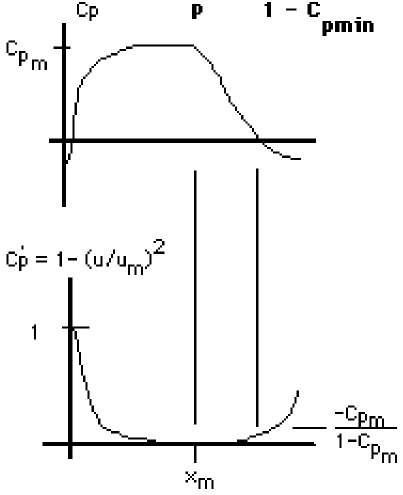
7.3.1.1. Canonical Pressure Distribution

Separation criteria are often stated in terms of the so-called canonical pressure coefficient, Cp'.

This is defined as: $C_p' = 1 - \frac{U^2}{U_{max}^2}$

Thus, the canonical pressure coefficient varies from 0 at the start of the pressure recovery to 1 at stagnation.

$$C_p' = \frac{C_p - C_{pmin}}{1 - C_{pmin}}$$



7.3.2. Turbulent Separation Criteria

Turbulent separation criteria are the most useful since most pressure recovery is done using turbulent boundary layers. There are many criteria that are used.

Minimum C_p
The simplest criterion is that used to estimate when the flow will separate from the leading edge of an airfoil. This rule of thumb states that there is a minimum value of C_p that can be tolerated. Numbers such as -10 to -13 are sometimes used, but this is a very crude rule and applies only to cases of leading edge separation.

Loftin's criterion
A related, but somewhat more sophisticated method is attributed to Loftin and states that the maximum value of C_p', the canonical pressure coefficient, after the start of recovery is +0.88. This is not a conservative estimate, however, and cannot be relied on for a wide range of airfoils.

Shape Factor
Perhaps the most reliable criterion is that based on the computed boundary layer quantities. It has been shown that separation is very likely when the value of the shape factor, H exceeds 2.2 to 2.4.

Stratford's Criterion
In 1959 Stratford devised a rather simple criterion for the separation of turbulent boundary layers. Similar to his laminar separation criterion, this rule states that separation will occur when:

$$C_p' \sqrt{x' \frac{dC_p'}{dx}} = \left(\frac{Re}{10^6}\right)^{0.1} S$$

Where the constant S is 0.35 when $d^2p/dx^2 < 0$ (concave recovery)
and 0.39 when $d^2p/dx^2 > 0$ (convex recovery)

The Reynold's number in the Stratford formula is based on the local effective length of the boundary layer, x', and the maximum velocity, U_m.

The formula is based on a great deal of empirical data and is only valid for $C_p' < 4/7$, but it is very useful in the design of airfoil sections. Stratford's method usually is conservative, predicting separation just a bit before the methods based on explicit computation of the shape factor.

Comparison with the corresponding laminar flow formula shows how turbulent boundary layers are very much more resistant to separation. Note also that the expression for x is different for turbulent boundary layers. More detail on the definition of effective boundary layer length is presented at the end of this section.

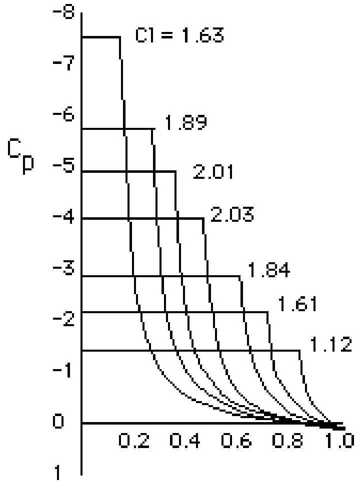
Stratford's criterion may be used to compute the shape of the pressure distribution that is everywhere on the edge of separation. This is a useful distribution for many reasons. Most importantly, it permits the most rapid possible recovery from a given minimum pressure. This, or something approaching it, would be used in the design of sections with maximum extent of laminar flow or sections with maximum lift or maximum thickness. This will be discussed in a subsequent section, but here we show how the particular C_p distribution is derived.

We start by taking Stratford's criterion as a differential equation describing the C_p variation and integrate the expression for the resulting C_p. This is not as straightforward as it appears since the formula is only valid for $C_p' < 4/$

Stratford effectively assumed a constant value of the boundary layer shape factor (e.g. H = 2.0) over this section and derived:

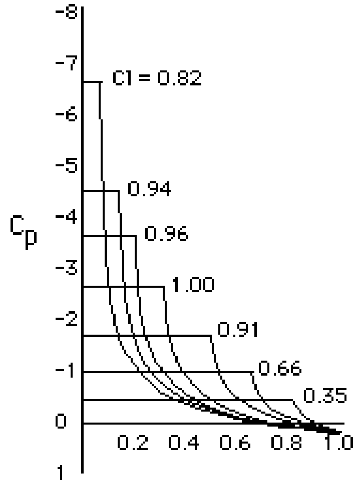
$$C_p' = 1 - \frac{a}{\sqrt{x + b}} \quad \text{for } C_p' > \frac{4}{7}$$

The values of a and b are chosen so that the slope and value of C_p' match at $C_p' = 4/7$.



Upper surface pressures with Stratford recovery to $C_p = 0.20$ at trailing edge.

Laminar Rooftop, $Re = 5 \times 10^6$



Upper surface pressures with Stratford recovery to $C_p = 0.20$ at trailing edge.

Turbulent Rooftop, $Re = 5 \times 10^6$

Liebeck airfoils with Stratford pressure recoveries designed for maximum lift.