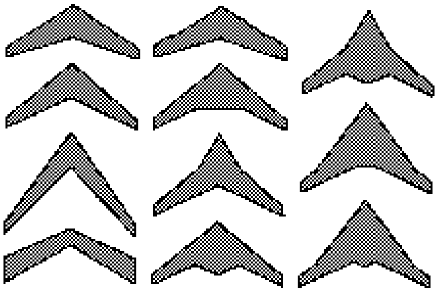
# 12. Wing Design



There are essentially two approaches to wing design. In the direct approach, one finds the planform and twist that minimize some combination of structural weight, drag, and $C_{L_{max}}$ constraints. The other approach involves selecting a desirable lift distribution and then computing the twist, taper, and thickness distributions that are required to achieve this distribution. The latter approach is generally used to obtain analytic solutions and insight into the important aspects of the design problem, but is is difficult to incorporate certain constraints and off-design considerations in this approach. The direct method, often combined with numerical optimization is often used in the latter stages of wing design, with the starting point established from simple (even analytic) results.

This chapter deals with some of the considerations involved in wing design, including the selection of basic sizing parameters and more detailed design. The chapter begins with a general discussion of the goals and trade-offs associated with wing design and the initial sizing problem, illustrating the complexities associated with the selection of several basic parameters. Each parameter affects drag and structural weight as well as stalling characteristics, fuel volume, off-design performance, and many other important characteristics.

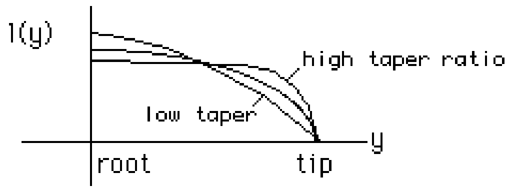
Wing lift distributions play a key role in wing design. The lift distribution is directly related to the wing geometry and determines such wing performance characteristics as induced drag, structural weight, and stalling characteristics. The determination of a reasonable lift and $C_l$ distribution, combined with a way of relating the wing twist to this distribution provides a good starting point for a wing design. Subsequent analysis of this baseline design will quickly show what might be changed in the original design to avoid problems such as high induced drag or large variations in $C_l$ at off-design conditions.

A description of more detailed methods for modern wing design with examples is followed by a brief discussion of nonplanar wings and winglets.

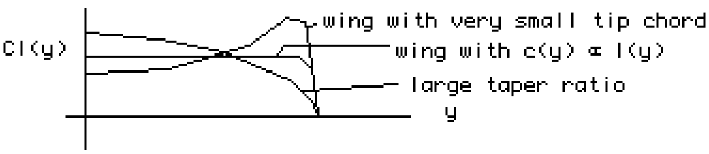
1. Wing Design Parameters
2. Lift Distributions
3. Wing Design in More Detail
4. Nonplanar Wings and Winglets
5. References

# 12.2.2. Wing Geometry and Lift Distribution

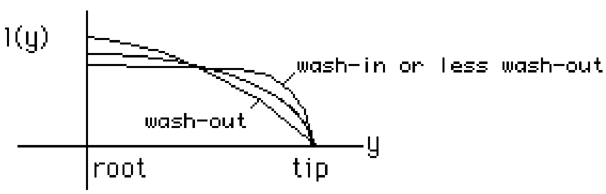
The wing geometry affects the wing lift and $C_l$ distributions in mostly intuitive ways. Increasing the taper ratio (making the tip chords larger) produces more lift at the tips, just as one might expect:




But because the section $C_l$ is the lift divided by the local chord, taper has a very different effect on the $C_l$ distribution.

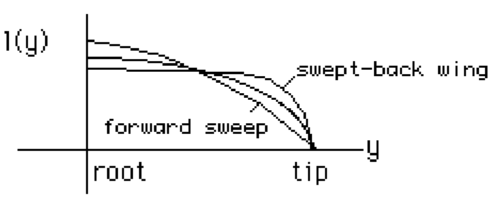



Changing the wing twist changes the lift and $C_l$ distributions as well. Increasing the tip incidence with respect to the root is called wash-in. Wings often have less incidence at the tip than the root (wash-out) to reduce structural weight and improve stalling characteristics.




Since changing the wing twist does not affect the chord distribution, the effect on lift and $C_l$ is similar.

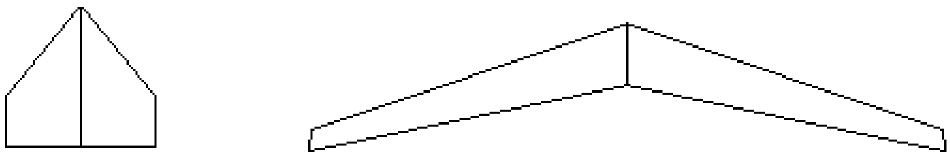
Wing sweep produces a less intuitive change in the lift distribution of a wing. Because the downwash velocity induced by the wing wake depends on the sweep, the lift distribution is affected. The result is an increase in the lift near the tip of a swept-back wing and a decrease near the root (as compared with an unswept wing.




This effect can be quite large and causes problems for swept-back wings.
The greater tip lift increases structural loads and can lead to stalling problems.

The effect of increasing wing aspect ratio is to increase the lift at a given angle of attack as we saw from the discussion of lifting line theory. But it also changes the shape of the wing lift distribution by magnifying the effects of all other parameters.

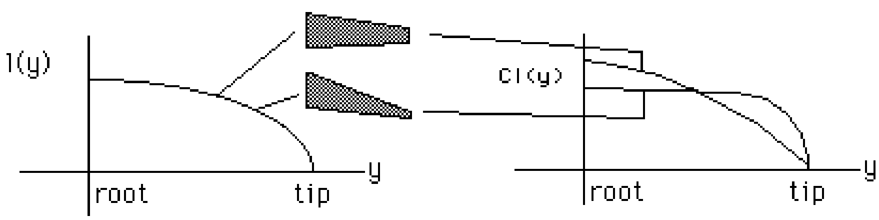
Low aspect ratio wings have nearly elliptic distributions of lift for a wide range of taper ratios and sweep angles. It takes a great deal of twist to change the distribution. Very high aspect ratio wings are quite sensitive, however and it is quite easy to depart from elliptic loading by picking a twist or taper ratio that is not quite right.



Note that many of these effects are similar and by combining the right twist and taper and sweep, we can achieve desirable distributions of lift and lift coefficient.

For example: Although a swept back wing tends to have extra lift at the wing tips, wash-out tends to lower the tip lift. Thus, a swept back wing with washout can have the same lift distribution as an unswept wing without twist.

Lowering the taper ratio can also cancel the influence of sweep on the lift distribution. However, then the $C_l$ distribution is different.




Today, we can relate the wing geometry to the lift and $C_l$ distributions very quickly by means of rapid computational methods. Yet, this more intuitive understanding of the impact of wing parameters on the distributions remains an important skill.

# 12.2.3. Lift Distributions and Performance

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Wing design has several goals related to the wing performance and lift distribution. One would like to have a distribution of $C_l(y)$ that is relatively flat so that the airfoil sections in one area are not "working too hard" while others are at low $C_l$. In such a case, the airfoils with $C_l$ much higher than the average will likely develop shocks sooner or will start stalling prematurely.

The induced drag depends solely on the lift distribution, so one would like to achieve a nearly elliptical distribution of section lift. On the other hand structural weight is affected by the lift distribution also so that the ideal shape depends on the relative importance of induced drag and wing weight.

With taper, sweep, and twist to "play with", these goals can be easily achieved at a given design point. The difficulty appears when the wing must perform well over a range of conditions.

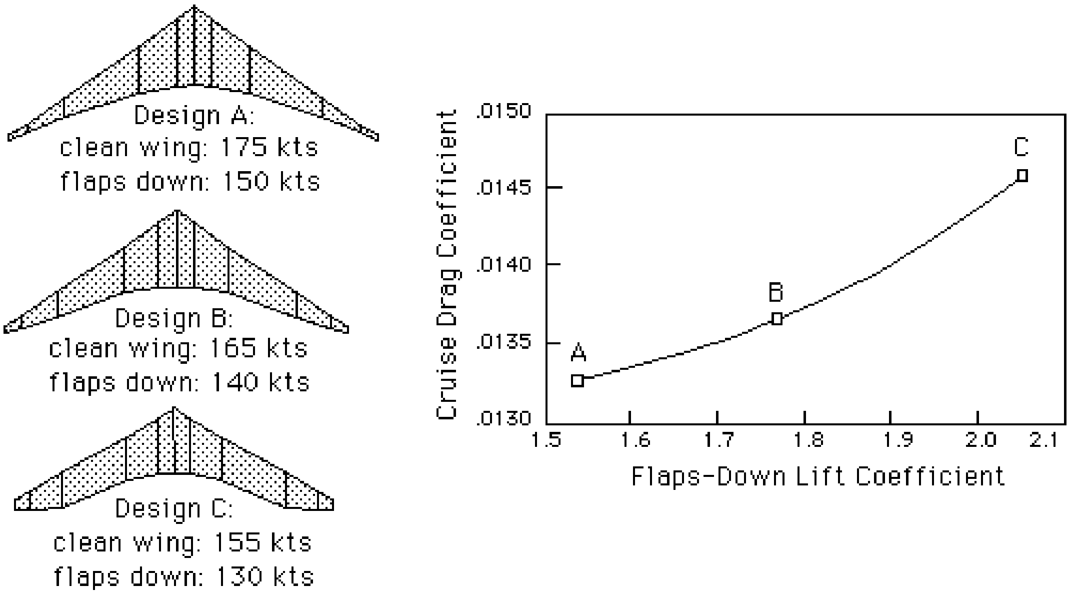
One of the more interesting tradeoffs that is often required in the design of a wing is that between drag and structural weight. This may be done in several ways. Some problems that have been solved include:

- Minimum induced drag with given span -- Prandtl
- Minimum induced drag with given root bending moment -- Jones, Lamar, and others
- Minimum induced drag with fixed wing weight and constant thickness -- Prandtl, Jones
- Minimum induced drag with given wing weight and specified thickness-to-chord ratio -- Ward, McGeer, Kroo
- Minimum total drag with given wing span and planform -- Kuhlman

... there are many problems of this sort left to solve and many approaches to the solution of such problems. These include some closed-form analytic results, analytic results together with iteration, and finally numerical optimization.

The best wing design will depend on the construction materials, the arrangement of the high-lift devices, the flight conditions ($C_L$, Re, M) and the relative importance of drag and weight. All of this is just to say that it is difficult to design just a wing without designing the entire airplane. If we were just given the job of minimizing cruise drag the wing would have a very high aspect ratio. If we add a constraint on the wing's structural weight based on a trade-off between cost and fuel savings then the problem is somewhat better posed but we would still select a wing with very small taper ratio. High t/c and high sweep are often suggested by studies that include only weight and drag.

The high lift characteristics of the design force the taper ratio and sweep to more usual values and therefore must be a fundamental consideration at the early stages of wing design. Unfortunately the estimation of $C_{L_{max}}$ is one of the more difficult parts of the preliminary design process. An example of this sensitivity is shown in the figure below.




The effect of a high lift constraint on optimal wing designs. Wing sweep, area, span, and twist, chord, and t/c distributions were optimized for minimum drag with a structural weight constraint. (Results from work of Sean Wakayama.

# 9.2.4. Computational Models

## Panel Methods

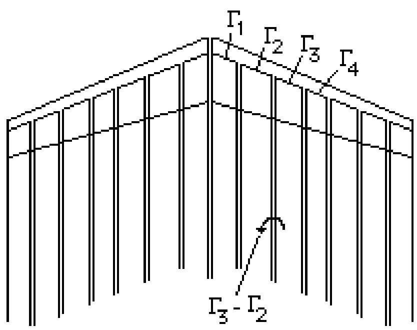
Many computational models and analysis methods are based on linear three-dimensional potential flow theory. These are discussed in the overview of panel methods in an earlier chapter.

In this section we take a look at the simplest panel method in more detail.

## Weissinger Method

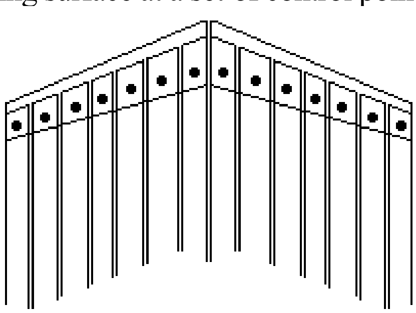
Weissinger theory or extended lifting line theory differs from lifting line theory in several respects. It is really a simple panel method (a vortex lattice method with only one chordwise panel), not a corrected strip theory method as is lifting line theory. This model works for wings with sweep and converges to the correct solution in both the high and low aspect ratio limits.

The version of this model used in the Wing Design program is actually a variant of Weissinger's method: it uses discrete skewed horseshoe vortices as shown.




Each horseshoe vortex consists of a bound vortex leg and two trailing vortices. This arrangement automatically satisfies the Helmholtz requirement that no vortex line ends in the flow. (The trailing vortices extend to infinity behind the wing.)

The basic concept is to compute the strengths of each of the "bound" vortices required to keep the flow tangent to the wing surface at a set of control points.



If the vortex of unit strength at station j produces a downwash velocity of AIC at station i, then the linear system of ij

equations representing the boundary conditions may be written:

$$[AIC]\{\Gamma_i\} = U_\infty \{\alpha\}$$

where {alpha} represents the angle of incidence of the sections along the span (assumed a flat plates). If the section has camber, the local angle of attack is taken as the angle from the zero lift line of the section.

The linear system of equations to be solved may also be written in terms of the angle of attack at the wing root and the twist amplitude. For wings with a linear distribution of twist (washout):

$$\{\alpha\} = \{\alpha_r\} - \frac{2\theta}{b}\{y\}$$

where:

{$\alpha_r$} is a vector containing the root angle of attack as each element

{y} is the spanwise coordinate, varying from 0 at the root to b/2.

{$\theta$} is the total twist (washout) in the wing from root to tip

Thus, the wing circulation distribution can be written as the sum of two distributions:

$$\{\Gamma\} = [AIC]^{-1}\,\alpha_r\,\{1\} - 2\theta/b\,[AIC]^{-1}\,\{y\}$$

Since the section lift (lift per unit length along the span) is related to the circulation by:

$$\{l\} = \rho\,U_\infty\{\Gamma\}$$

The lift distribution can be expressed as:

$$\{l\} = \alpha_r\,\{l_1\} + \theta\,\{l_2\}$$

where l1 and l2 are independent of the incidence angles and depend only on the planform shape of the wing.

Since the lift coefficient of the wing, $C_L$, is linearly related to the angle of attack we can also write the lift distribution in the following form:

$$\{l\} = C_L\,\{l_3\} + \theta\,\{l_4\}$$

The first term is known as the additional lift distribution and the second term is called the basic lift distribution. They scale linearly with the wing lift coefficient and the twist angle respectively. Additional information on basic and additional lift distributions is available in the section on wing design.

An interactive computation based on this idea is available on the internet. Use it to investigate the effect of wing shape on lift distributions or to design wings as discussed in the following sections. The source code is available as well.