Spectral Difference Method for Sliding-Mesh Simulations, and Stratified Solar Convection

Chunlei Liang

joint work with Bin Zhang and Daniel Wang
Department of Mechanical and Aerospace Engineering
The George Washington University
Washington, DC
also with Mark Miesch of NCAR for the work of solar convection

AJ80th @Stanford
Nov. 21, 2014
My academic career

1. postdoc @ Glasgow Scotland
2. hired by Antony in November 2007 @ Stanford
3. Since September 2010 Assistant Professor @ George Washington University
Section 1

Thanks to Professor Jameson’s Work on High-order Methods
Proof of the stability of the spectral difference method

Proof was done in early 2009 published in JSC 2010
Spectral Difference Method

1. Flux points: Legendre polynomial roots and two end points
2. Solution points: Chebyshev Gauss points
3. Tensor product
Other topics Prof. Jameson has worked on

- New Flux Reconstruction Method
- High-order Methods for Large Eddy Simulation
- High-order Methods for Complex Geometries
- High-order Methods for Moving Boundary Problems
- GPUs
Special Issue 2014

Special Issue of High-order Methods for Computational Fluid Dynamics

**in honor of Professor Jameson’s 80th Birthday**

published by Computers & Fluids in 2014

edited by Chunlei Liang, Krzysztof Fidkowski, Per-Olof Persson and Peter Vincent
ACL 2009
Section 2

Spectral Difference Method for Sliding-Mesh Simulations
Sliding-mesh interface
Dynamic construction of curved mortars
Euler vortex: problem setup

\[
\begin{align*}
    u &= U_\infty \left\{ \cos \theta - \frac{\epsilon y_r}{r_c} \exp \left( \frac{1 - x_r^2 - y_r^2}{2r_c^2} \right) \right\} \\
    v &= U_\infty \left\{ \sin \theta + \frac{\epsilon x_r}{r_c} \exp \left( \frac{1 - x_r^2 - y_r^2}{2r_c^2} \right) \right\} \\
    \rho &= \rho_\infty \left\{ 1 - \frac{(\gamma - 1)(\epsilon M_\infty)^2}{2} \exp \left( \frac{1 - x_r^2 - y_r^2}{r_c^2} \right) \right\}^{\frac{1}{\gamma - 1}} \\
    p &= p_\infty \left\{ 1 - \frac{(\gamma - 1)(\epsilon M_\infty)^2}{2} \exp \left( \frac{1 - x_r^2 - y_r^2}{r_c^2} \right) \right\}^{\frac{\gamma}{\gamma - 1}}
\end{align*}
\]

In this test we set \((U_\infty, \rho_\infty, p_\infty) = (1, 1, 1), \ M_\infty = 0.3, \ \theta = \arctan(1/2), \ \epsilon = 1, \ r_c = 1.\)

Domain size is \(0 \leq x, y \leq 10,\) initial location of vortex is \((x_0, y_0) = (5, 5).\) Periodic boundary condition is used.
Euler vortex: animations

Results from 4th order scheme:
### Euler vortex: order of accuracy

<table>
<thead>
<tr>
<th>cells</th>
<th>L1 error</th>
<th>order</th>
<th>L2 error</th>
<th>order</th>
</tr>
</thead>
<tbody>
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<td>180</td>
<td>3.16E-4</td>
<td>-</td>
<td>8.13E-4</td>
<td>-</td>
</tr>
<tr>
<td>700</td>
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**Table:** Error and order of accuracy of the 3\(^{rd}\) order scheme

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<tbody>
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<td>180</td>
<td>5.43E-5</td>
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<td>4.00</td>
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</tbody>
</table>

**Table:** Error and order of accuracy of the 4\(^{th}\) order scheme
Taylor-Couette flow: problem setup

The exact solution of the velocity is

\[ v_{\theta} = \omega r_1 \frac{r_2/r - r/r_2}{r_2/r_1 - r_1/r_2} \]  

(1)

In this test we choose \( \omega = 1, r_1 = 1, r_2 = 2 \).

Mach number on the inner wall is \( Ma = 0.1 \).

Reynolds number based on inner cylinder speed and radius is \( Re = 10 \).

Isothermal wall boundary condition is used for both cylinders.
Taylor-Couette flow: meshing and flow
Taylor-Couette flow: Order of accuracy

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</thead>
<tbody>
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Table: Error and order of accuracy of the 3\textsuperscript{rd} order scheme

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</tbody>
</table>

Table: Error and order of accuracy of the 4\textsuperscript{th} order scheme
What’s good with this method?

1. Simple
2. High-order accurate
3. Efficient
4. Easy to parallelize
2D stirred vessel: setup

\[ \omega = 1 \]
\[ Re = \omega D^2 / \nu = 100, \]  
\( D \) is inner cylinder diameter
\[ Ma = 0.1 \) on inner wall
adiabatic wall bc on blades
isothermal wall elsewhere
mesh has 14990 cells
2D stirred vessel: pressure contours

initial period (3rd order scheme):
2D stirred vessel: animations

converged solution (3rd order scheme):
Heaving and pitching airfoil: setup

NACA 0012 airfoil

\[ h = A \cos(2\pi \omega t) \], where \( A = 0.25, \, \omega = 0.4 \)

\[ \theta = \alpha \sin(2\pi \omega t) \], where \( \alpha = 60^\circ \)

\[ Re = U_\infty c / \nu = 1000 \]
Heaving and pitching airfoil: Meshing

Mesh with 9315 cells:
Heaving and pitching: animations
Heaving and pitching airfoil: force coefficients

Results from 3rd order scheme:
Heaving and pitching airfoil: animations

(a) Pressure
(b) Vorticity
Section 3

Spectral Difference Method for Solar Convection
Differential Rotation (Helioseismology)

Figure: Differential rotation
Solar Interior

**Figure:** Gough and McIntyre, *Nature*, 1998
Variable-resolution meshing technique

Figure: Conforming mesh (a) and Nonconforming mesh with variable resolution (b)
Fully compressible flow Model in a rotating reference frame

Rotates with reference to z axis at a rate of $\Omega_0$.
The hydrodynamic equations are:

$$\frac{\partial \rho}{\partial t} = -\nabla \cdot (\rho \mathbf{u})$$  \hspace{1cm} (2)

$$\frac{\partial (\rho \mathbf{u})}{\partial t} = -\nabla \cdot \rho \mathbf{uu} - \nabla \rho + \nabla \cdot \boldsymbol{\tau} + \rho \mathbf{g} - 2\rho \Omega_0 \times \mathbf{u}$$  \hspace{1cm} (3)

$$\frac{\partial (E)}{\partial t} = -\nabla \cdot ((E + p)\mathbf{u}) + \nabla \cdot (\boldsymbol{\tau} \cdot \mathbf{u} - \mathbf{f}) + \rho \mathbf{u} \cdot \mathbf{g}$$  \hspace{1cm} (4)

where $E$ is the total energy per unit volume, $\boldsymbol{\tau}$ is viscous stress tensor, $\mathbf{f}$ is entropy/radiative/conductive flux and $\mathbf{g}$ is gravitational acceleration.
What make the implementation hard?

1. Hydrostatic balance:

\[ \frac{dp_e}{dr} = -\rho_e g \tag{5} \]

2. Thermal equilibrium:

\[ \frac{d}{dr} \left( r^2 \rho_e T_e \kappa \frac{ds_e}{dr} \right) = 0 \tag{6} \]

3. Radiation plays an important role for energy transport


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Verify fully compressible codes

In need of creating new benchmark cases for stratified convection in spherical shells!

1. reduce Reynolds number
2. reduce Rayleigh number
3. spin faster
4. increase luminosity of the star
5. neglect the near surface shear layer
Unstructured Grid

Total cells: 294,912; Total DOFs: 18,874,368.

Figure: Unstructured Hexahedral Grid
Scalability of CHORUS
Initial equilibrium conditions

- Hydrostatic balance:

\[ \frac{dp_e}{dr} = -\rho_e g \]  \hspace{1cm} (7)

- Thermal equilibrium:

\[ \frac{d}{dr} \left( r^2 \rho_e T_e \kappa \frac{ds_e}{dr} \right) = 0 \]  \hspace{1cm} (8)

- Ideal gas law:

\[ p = \rho RT \]  \hspace{1cm} (9)
Boundary Conditions

- **Impenetrable**

  \[ u_r = 0 \] \hspace{1cm} (10)

- **Stress-free**

  \[ \frac{\partial}{\partial r} \left( \frac{u_\theta}{r} \right) = \frac{\partial}{\partial r} \left( \frac{u_\phi}{r} \right) = 0 \] \hspace{1cm} (11)

  Angular momentum conservation is re-enforced during every rotational cycle.

- A constant heat flux \( f_b \) at the bottom

- The temperature at the top is fixed.
Time history of the total kinetic energy: Solar model
Mollweide view of radial velocity for the Sun simulation

(a) Compressible code

(b) NCAR ASH code
Mollweide view of radial velocity at $0.95R_\odot$

This is the first time that an unstructured grid code successfully predicts the differential rotation of the sun!
Ongoing work

1. 3D simulation of a marine propeller
2. Oblate stars
Submitted manuscripts


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Happy Birthday to Antony!
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