Computation and Design of Moving Boundary Systems

Wei Shyy
Hong Kong University of Science and Technology
weishyy@ust.hk
Dr. C.-K. Kang, Dr. C.-K. Kuan
University of Michigan
Multiphase Flows - Topologically Varying Moving Boundaries

Multiphase flow in Engineering problems
- Cryogenic liquid in rocket fuel tanks
- Droplet collision/atomization in combustor
- Micro-fluidic devices

Challenges of interfacial flow computation
- Multiscale behavior
- Surface tension/phase change modeling
- Discontinuous fluid properties
- Interface break-up/merge
- Complex solid geometries

Combustion and atomization in a generic combustor

GE Genx Engine B747-7, B787
**Atomization Dynamics**

Global dimensionless parameters

- **Weber number**  \( \text{We} = \frac{\rho l U_0 D_0}{\sigma} \)
  - Inertia vs. surface tension vs. viscosity
  - Free surface flow at varying \( \text{We} \), including topological changes due to break-up/merger
    - small Weber number: maintaining spherical shape
    - large Weber number: unstable → break up as ligaments and droplets
  - Injector Weber number in combustor chamber – \( O(10^5) \) or higher
    - unstable impingement sheet → fragment to ligament
    - Secondary droplet-droplet collision at \( \text{We}_d \sim O(1) \) to \( O(10^3) \)

- **Reynolds number**  \( \text{Re} = \frac{\rho D_0 U_0}{\mu_l} \)

- **Ohnesorge number**  \( Oh = \frac{\mu_l}{(\sigma \rho D_0)^{1/2}} \)

Direct simulation very challenging even without turbulence
Atomization versus Weber Number

- High Weber number regime critical to propulsion and power devices
- Challenging to measure/compute due to time/length scale disparity
- Little information on detailed droplet collision dynamics

Chen & Yang ‘14 JCP
Combined Eulerian-Lagrangian Method with Local Adaptive Mesh Refinement

Cut-plane view of dynamic adaptive mesh and interface profile

Cell-based unstructured adaptive mesh refinement
  - Highly flexible cell-by-cell adaptation

- Unstructured data:
  - Performance of field equations solver is independent of refinement level.
    - No tree-like hierarchy (constant data-fetching cost)
    - No level-level interpolation (communication)
Low Weber Number regime

**Computational Parameters**

\[ \text{Density ratio} = 666.1; \text{Viscosity ratio} = 179.3 \]

\[ \text{Re} = \frac{\rho_{\text{drop}} U_{\text{impact}} d}{\mu_{\text{drop}}} \]

\[ \text{We} = \frac{\rho_{\text{drop}} U_{\text{impact}}^2 d}{\sigma} \]

Impact Parameter, \( B = h / d \)

**Schematic of collision regime**

**Table:**

<table>
<thead>
<tr>
<th>Cas</th>
<th>We</th>
<th>B</th>
<th>Re</th>
<th>Outcome (Qian &amp; Law, 97)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>60.1</td>
<td>0.55</td>
<td>302.8</td>
<td>Separation with satellite</td>
</tr>
<tr>
<td>2</td>
<td>32.8</td>
<td>0.08</td>
<td>210.8</td>
<td>Coalescence</td>
</tr>
<tr>
<td>3</td>
<td>37.2</td>
<td>0.01</td>
<td>228.0</td>
<td>Separation</td>
</tr>
<tr>
<td>4</td>
<td>61.4</td>
<td>0.06</td>
<td>296.5</td>
<td>Separation with satellite</td>
</tr>
</tbody>
</table>
CASE (1): Off-Center Separation (We = 60.1, B = 0.55) higher Weber # & impact factor

Present computations

Qian & Law JFM 1997

Singh & Shyy, JCP 2007
Kuan, Pan & Shyy, JFM 2014
**Binary droplet collision at high Weber number**

**Motivation**

- To improve the understanding of morphologies and instabilities of free surface
- Collisions at the low Weber number regimes were explored by experiments [1] and numerical computations.
- Analysis at high Weber number regimes (from 200 to thousands) is much challenging
  - Experiment at Weber number 200-5000 (Pan et al.[2])
    - Fingering, fingering and separation, breakup, prompt splattering
  - Instabilities
  - Gas-liquid interfaces appear in multiple length scale → costly computation

Pan, Chou & Tseng, Physical Review E, 80, 2009
Kuan, Pan & Shyy, J Fluid Mech, 2014.
Droplet collision at high Weber number - validation

We 210

Exp

0 ms  0.390 ms  0.585 ms  0.780 ms  0.975 ms  1.754 ms  2.144 ms  2.729 ms

Comp

0.159 ms  0.403 ms  0.583 ms  0.775 ms  0.981 ms  1.753 ms  2.151 ms  2.729 ms

We 277

Exp

0 ms  0.195 ms  0.390 ms  0.585 ms  0.780 ms  0.975 ms  1.754 ms  1.949 ms  2.534 ms  2.924 ms  3.509 ms

Comp

0.0 ms  0.207 ms  0.387 ms  0.581 ms  0.775 ms  0.981 ms  1.756 ms  1.949 ms  2.530 ms  2.931 ms  3.499 ms
Droplet collision at high Weber number - validation

We 442

Droplet collision at high Weber number - validation

We 878

Droplet collision at high Weber number - validation

We 1520
Interface evolution: General observation

The merged droplet extrudes a circular sheet

A toroid rim attached to the sheet

Longitudinal instability on the rim

Rim Inertia >> surface tension force

Rim grows with bead-like structures

The rim retract or slowdown

The rim retracts and aggregate

Fingering at retracting phase; disintegration

Fingering/necking and then disintegration of the rim

Necking and thinning of the detached rim \(\rightarrow\) breakup

Increasing Weber number
Droplet collision at high Weber number - animation

- We = 1520 (Eulerian grid size 23 million, Lagrangian marker size 2.5 million)

Drop Collision Dynamics Needs to be Incorporated into Atomization Model
Drones in the Sky (WSJ Nov 11, ‘14)

Google, DJI

Amazon
Finally, a Drone You Can Own

The DJI Phantom 2 Vision is a 2.5 pound flying camera

By Alex Fitzpatrick

Remote-controlled aircraft hobbyists have been trying to MacGyver small digital cameras to their airborne contraptions for years. Chinese manufacturer DJI has finally come along and rendered all that tinkering unnecessary, selling what’s essentially a flying camera ready-to-go out of the box: The DJI Phantom 2 Vision.

Setup

There’s an acronym in remote-controlled flying: “RTF,” or “Ready-to-Fly.” Many times, products advertise themselves as “RTF” when that’s really only half-true—but that’s not the case here. All you’ve got to do to get airborne with the Phantom 2 Vision is charge up the flight battery and Wi-Fi range extender, screw on its four propellers and pop four AA-size batteries into the controller. After you download and set up the mobile app and attach the range extender and phone clip to the controller, you’re clear for liftoff.

Flying

I’ve never met a remote-controlled aircraft quite this easy to fly. At two-and-a-half pounds, it has a heft that’s helpful for stability (though I wouldn’t risk it on a particularly windy day). The quadcopter’s four engines and propellers allow it to maneuver like a helicopter, making it pretty nimble once you get the hang of it—it flies on three axes and can hover with minimal pilot input thanks to its internal GPS system.

The Phantom’s control scheme was a little counter-intuitive at first—it would be nice if it was customizable, as is the case with more complex (and often very expensive) RC airplane transmitters. But with a little practice, flying the Phantom gets simple quick. Bringing it back to terra firma manually, however, can be a bit tricky—I damaged a propeller on what might be called a “hard landing.” But there’s a GPS-based auto-land feature that’s useful for the uninitiated, and attaching a new prop was less than a five-minute job with a tool DJI provides for the task. And there’s a saying in aviation: Any landing where you can use the plane again is a great landing.

I should probably note that somebody with less R/C flying experience might find the learning curve a bit more steep. Helpful tip: Try to keep the back of the aircraft facing you until you start learning how to “mirror” the controls when it’s facing a different direction than you are. It would be neat if DJI provided some kind of game inside its mobile app to get a feel for the controls in a virtual environment before advancing to the real deal.

DJI and Drone
Size vs Flexibility

As vehicle becomes smaller/slower, Re is reduced, wing movement more important, & fluid-structure interaction is more pronounced
Hummingbird (Wei Shyy ©)

**Reynolds number**

$$Re = \frac{\rho_f U_{\text{ref}} c}{\mu}$$

**Reduced frequency**

$$k = \frac{\pi f c}{U_{\text{ref}}}$$

**Thickness ratio**

$$h_s^* = \frac{h_s}{c}$$

**Density ratio**

$$\rho^* = \frac{\rho_s}{\rho_f}$$

**Frequency ratio**

$$f / f_1$$

**Effective stiffness**

$$\Pi_1 \sim \frac{E h_s^3}{\rho_f U_{\text{ref}}^2}$$

- **Re**: fluid inertia vs. viscosity
- **k**: unsteadiness
- **h_s^***: wing thickness vs. chord
- **\rho^***: wing density vs. fluid density
- **f / f_1**: motion vs. natural frequency
- **\Pi_1**: wing stiffness vs. dynamic pressure

**Wing Deformation, Lift Generation**

$$\Psi(Re, h_s^*, \rho^*, k, f / f_1)$$

Shyy et al., Intro Flapping Wing Aerodynamics, 2013
Unsteady Lift Mechanisms of Flapping Airfoil

- Delayed Stall Enhances lift
- Wake Capture Enhances lift
- Jet Interaction Reduces lift

Re = 100
2h_a /c = 3.0
\alpha_a = 45°
\phi = 90°
Unsteady Lift Mechanisms of Flapping Wing

Rotating Starting Vortex Enhances/reduce lift?

Tip Vortex reduces lift?

Re = 100
2h_a /c = 2.0
α_a = 45°
φ = 60°
Pareto Fronts: Lift vs. Power of Rigid Wing
Surrogate Model facilitates the investigation

△ High Lift: Advanced rotation; High AoA

△ Low Power: Delayed rotation; Low AoA
Optimal Lift for Flexible Wing - Symmetric Rotation

Just as for the rigid wings, all three rotational modes are observed:

For the flexible wing, the symmetric rotational mode → highest lift ( > 1.6) [1]

Kang & Shyy, J Royal Society Interface 2014;
Shyy et al, Intro Flapping Wing Aerodynamics, Cambridge Univ Press 2013
Fluid-Structural Interactions

Fluid

**mass:**

\[ \nabla \cdot u^* = 0 \]

**momentum:**

\[ \frac{k}{\pi} \frac{\partial u^*}{\partial t^*} + u^* \cdot \nabla u^* = -\nabla p^* + \frac{1}{Re} \nabla^2 u^* \]

Structure

\[ \Pi_0: \text{Effective inertia} \]

\[ \rho^* h^* \left( \frac{k}{\pi} \right)^2 \frac{\partial^2 w_3^*}{\partial t^*} + \Pi_1 \nabla^4 w_3^* = f^* \]

\[ \Pi_1: \text{Effective stiffness} \]

\[ \Pi_1 = \frac{E h_s^*}{12 \rho_f U_{\text{ref}}^2} \]

Beam: 1D

\[ h^* = St \frac{\pi}{k} \cos(2\pi t^*) \]

Kinematics

Structure

Non-dim. variables

\[ Re = \frac{\rho_f U_{\text{ref}} c_m}{\mu} \]

Reduced frequency

\[ k = \frac{\omega c_m}{2 U_{\text{ref}}} \]

Density ratio

\[ \rho^* = \frac{\rho_s}{\rho_f} \]

Thickness ratio

\[ h_s^* = \frac{h_s}{c_m} \]

Strouhal number

\[ St = \frac{k h_s}{\pi c_m} \]

Non-dimensionalized with Velocity: \( U_{\text{ref}} \) Length: \( c_m \) Time: \( 2\pi/\omega \)
Force Deforming a Flexible Wing

\[ C_F \sim \frac{h_a}{c} \left\{ \frac{1}{Re} O(1) + St O(1) \right\} + St k O(1) \]

- Vortex impulse
- Viscous \( \sim f^0 \)
- Inertia \( \sim f^1 \)
- Added mass \( \sim f^2 \)

- Global assessment for preliminary design: scaling parameters
- Stress distribution for aerodynamics/control design: Kinematics, shape, properties
Scaling for

**Force Generation**

\[
\log_{10} \left( \frac{C_F}{\Pi_1} \right) = 1.21 \log_{10} \gamma + 2.78
\]

\[ R^2 = 0.98216 \]

\[
\frac{\langle C_F \rangle}{\Pi_1} \sim \gamma
\]

Mean force / effective stiffness \sim max. relative wing tip deformation

---

**Power Input**

\[
\log_{10} \left( \frac{C_P}{\beta_2} \right) = 2.15 \log_{10} \gamma + 4.78
\]

\[ R^2 = 0.99 \]

\[
\beta_2 = \frac{\Pi_1^2}{k^2 + 4\pi\Pi_0}
\]

---

- Plunging chordwise flexible airfoils in forward flight, water, Re = 9.0 \times 10^3
- Plunging spanwise flexible wing in forward flight, water, Re = 3.0 \times 10^4
- Flapping isotropic Zimmerman wing in still air, Re = 1.5 \times 10^3
- Insect Flyers
Frequency Selection Optimal Propulsive Efficiency

\[ \eta = \frac{\langle C_T \rangle}{\langle C_P \rangle} \]

Propulsive efficiency scaling

\[ \eta \sim \left\{ 1 - \left( \frac{\omega}{\omega_1} \right)^2 \right\}^{0.82} \omega^{0.36} \]

<table>
<thead>
<tr>
<th>Literature</th>
<th>( \omega_{\text{opt}} )</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>Vallena et al. (2009)</td>
<td>0.3</td>
<td>Hover, 2D airfoil, torsion spring model</td>
</tr>
<tr>
<td>Yin &amp; Luo (2010)</td>
<td>0.4-0.5</td>
<td>Hover, 2D airfoil, membrane model</td>
</tr>
<tr>
<td>Ramananarivo et al. (2011)</td>
<td>0.5-0.6</td>
<td>Self-propelled flapper experiment</td>
</tr>
<tr>
<td><strong>Current study</strong></td>
<td><strong>0.4-0.5</strong></td>
<td><strong>Scaling analysis</strong></td>
</tr>
</tbody>
</table>

Example 1: 2% thick aluminum wing with 20 cm chord, 50 cm half span
→ motion frequency: 6.6 Hz for optimal propulsion

Example 2: 2% thick aluminum wing with 2 cm chord, 5 cm half span
→ motion frequency: 55 Hz for optimal propulsion