COMPLETE CONFIGURATION AERO-STRUCTURAL OPTIMIZATION USING A COUPLED SENSITIVITY ANALYSIS METHOD

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Outline

• Introduction
  – High-fidelity aircraft design optimization
  – The need for aero-structural sensitivities
  – Sensitivity analysis methods

• Theory
  – Adjoint sensitivity equations
  – Lagged aero-structural adjoint equations

• Results
  – Optimization problem statement
  – Aero-structural sensitivity validation
  – Optimization results

• Conclusions
High-Fidelity Aircraft Design Optimization

- Start from a baseline geometry provided by a conceptual design tool.

- Required for transonic configurations where shocks are present.

- Necessary for supersonic, complex geometry design.

- High-fidelity analysis needs high-fidelity parameterization, e.g. to smooth shocks, favorable interference.

- Large number of design variables and complex models incur a large cost.
Aero-Structural Aircraft Design Optimization

- Aerodynamics and structures are core disciplines in aircraft design and are very tightly coupled.

- For traditional designs, aerodynamicists know the spanload distributions that lead to the true optimum from experience and accumulated data. What about unusual designs?

- Want to simultaneously optimize the aerodynamic shape and structure, since there is a trade-off between aerodynamic performance and structural weight, e.g.,

\[
\text{Range} \propto \frac{L}{D} \ln \left( \frac{W_i}{W_f} \right)
\]
The Need for Aero-Structural Sensitivities

- Sequential optimization does not lead to the true optimum.
- Aero-structural optimization requires coupled sensitivities.
- Add structural element sizes to the design variables.
- Including structures in the high-fidelity wing optimization will allow larger changes in the design.
Methods for Sensitivity Analysis

- **Finite-Difference:** very popular; easy, but lacks robustness and accuracy; run solver \( N_x \) times.

\[
\frac{df}{dx_n} \approx \frac{f(x_n + h) - f(x)}{h} + O(h)
\]

- **Complex-Step Method:** relatively new; accurate and robust; easy to implement and maintain; run solver \( N_x \) times.

\[
\frac{df}{dx_n} \approx \frac{\text{Im} [f(x_n + ih)]}{h} + O(h^2)
\]

- **(Semi)-Analytic Methods:** efficient and accurate; long development time; cost can be independent of \( N_x \).
Objective Function and Governing Equations

Want to minimize scalar objective function,

\[ I = I(x_n, y_i), \]

which depends on:

- \( x_n \): vector of design variables, e.g. structural plate thickness.
- \( y_i \): state vector, e.g. flow variables.

Physical system is modeled by a set of governing equations:

\[ R_k (x_n, y_i (x_n)) = 0, \]

where:

- Same number of state and governing equations, \( i, k = 1, \ldots, N_R \)
- \( N_x \) design variables.
Total sensitivity of the objective function:

\[
\frac{dI}{dx_n} = \frac{\partial I}{\partial x_n} + \frac{\partial I}{\partial y_i} \frac{dy_i}{dx_n}.
\]

Total sensitivity of the governing equations:

\[
\frac{d\mathcal{R}_k}{dx_n} = \frac{\partial \mathcal{R}_k}{\partial x_n} + \frac{\partial \mathcal{R}_k}{\partial y_i} \frac{dy_i}{dx_n} = 0.
\]
Solving the Sensitivity Equations

Solve the total sensitivity of the governing equations

$$\frac{\partial R_k}{\partial y_i} \frac{dy_i}{dx_n} = -\frac{\partial R_k}{\partial x_n}.$$

Substitute this result into the total sensitivity equation

$$\frac{dI}{dx_n} = \frac{\partial I}{\partial x_n} - \frac{\partial I}{\partial y_i} \left[ \frac{\partial R_k}{\partial y_i} \right]^{-1} \frac{\partial R_k}{\partial x_n},$$

where $\Psi_k$ is the adjoint vector.
Adjoint Sensitivity Equations

Solve the adjoint equations

$$\frac{\partial R_k}{\partial y_i} \Psi_k = -\frac{\partial I}{\partial y_i}. $$

Adjoint vector is valid for all design variables.

Now the total sensitivity of the function of interest $I$ is:

$$\frac{dI}{dx_n} = \frac{\partial I}{\partial x_n} + \Psi_k \frac{\partial R_k}{\partial x_n}.$$

The partial derivatives are inexpensive, since they don’t require the solution of the governing equations.
Two coupled disciplines: Aerodynamics ($A_k$) and Structures ($S_l$).

$$\mathcal{R}_{k'} = \begin{bmatrix} A_k \\ S_l \end{bmatrix}, \quad y_{i'} = \begin{bmatrix} w_i \\ u_j \end{bmatrix}, \quad \Psi_{k'} = \begin{bmatrix} \psi_k \\ \phi_l \end{bmatrix}.$$  

Flow variables, $w_i$, five for each grid point.

Structural displacements, $u_j$, three for each structural node.
Aero-Structural Adjoint Equations

\[
\begin{bmatrix}
\frac{\partial A_k}{\partial w_i} & \frac{\partial A_k}{\partial u_j} \\
\frac{\partial S_l}{\partial w_i} & \frac{\partial S_l}{\partial u_j}
\end{bmatrix}^T \begin{bmatrix} \psi_k \\ \phi_l \end{bmatrix} = - \begin{bmatrix} \frac{\partial I}{\partial w_i} \\ \frac{\partial I}{\partial u_j} \end{bmatrix}.
\]

- \( \frac{\partial A_k}{\partial w_i} \): a change in one of the flow variables affects only the residuals of its cell and the neighboring ones.
- \( \frac{\partial A_k}{\partial u_j} \): wing deflections cause the mesh to warp, affecting the residuals.
- \( \frac{\partial S_l}{\partial w_i} \): since \( S_l = K_{lj} u_j - f_l \), this is equal to \( -\frac{\partial f_l}{\partial w_i} \).
- \( \frac{\partial S_l}{\partial u_j} \): equal to the stiffness matrix, \( K_{lj} \).
- \( \frac{\partial I}{\partial w_i} \): for \( C_D \), obtained from the integration of pressures; for stresses, its zero.
- \( \frac{\partial I}{\partial u_j} \): for \( C_D \), wing displacement changes the surface boundary over which drag is integrated; for stresses, related to \( \sigma_m = S_{mj} u_j \).
Lagged Aero-Structural Adjoint Equations

Since the factorization of the complete residual sensitivity matrix is impractical, decouple the system and lag the adjoint variables,

\[
\frac{\partial A_k}{\partial w_i} \psi_k = - \frac{\partial I}{\partial w_i} - \frac{\partial S_l}{\partial u_j} \tilde{\phi}_l, \\
\text{Aerodynamic adjoint}
\]

\[
\frac{\partial S_l}{\partial u_j} \phi_l = - \frac{\partial I}{\partial u_j} - \frac{\partial A_k}{\partial u_j} \tilde{\psi}_k, \\
\text{Structural adjoint}
\]

Lagged adjoint equations are the single discipline ones with an added forcing term that takes the coupling into account.

System is solved iteratively, much like the aero-structural analysis.
Total Sensitivity

The aero-structural sensitivities of the drag coefficient with respect to wing shape perturbations are,

\[
\frac{dI}{dx_n} = \frac{\partial I}{\partial x_n} + \psi_k \frac{\partial A_k}{\partial x_n} + \phi_l \frac{\partial S_l}{\partial x_n}.
\]

- \(\partial I/\partial x_n\): \(C_D\) changes when the boundary over which the pressures are integrated is perturbed; stresses change when nodes are moved.

- \(\partial A_k/\partial x_n\): the shape perturbations affect the grid, which in turn changes the residuals; structural variables have no effect on this term.

- \(S_l/\partial x_n\): shape perturbations affect the structural equations, so this term is equal to \(\partial K_{lj}/\partial x_n u_j - \partial f_l/\partial x_n\).
3D Aero-Structural Design Optimization Framework

- Aerodynamics: FLO107-MB, a parallel, multiblock Navier-Stokes flow solver.
- Structures: detailed finite element model with plates and trusses.
- Coupling: high-fidelity, consistent and conservative.
- Geometry: centralized database for exchanges (jig shape, pressure distributions, displacements.)
- Coupled-adjoint sensitivity analysis: aerodynamic and structural design variables.
Sensitivity of $C_D$ wrt Shape

![Graph showing sensitivity of $C_D$ with respect to design variables $n$. The graph compares coupled adjoint and complex step methods with and without fixed displacements.]
Sensitivity of $C_D$ wrt Structural Thickness

![Graph showing sensitivity of $C_D$ with respect to structural thickness.](image-url)
Structural Stress Constraint Lumping

To perform structural optimization, we need the sensitivities of all the stresses in the finite-element model with respect to many design variables. There is no method to calculate this matrix of sensitivities efficiently.

Therefore, lump stress constraints

\[ g_m = 1 - \frac{\sigma_m}{\sigma_{\text{yield}}} \geq 0, \]

using the Kreisselmeier–Steinhauser function

\[ \text{KS} (g_m) = -\frac{1}{\rho} \ln \left( \sum_m e^{-\rho g_m} \right), \]

where \( \rho \) controls how close the function is to the minimum of the stress constraints.
Sensitivity of KS wrt Shape

![Graph showing sensitivity of KS with respect to shape design variable, n. The graph compares different methods: Coupled adjoint, Complex step, Coupled adjoint, fixed loads, and Complex, fixed loads.](image-url)
Sensitivity of KS wrt Structural Thickness

- Coupled adjoint
- Complex step
- Coupled adjoint, fixed loads
- Complex, fixed loads

Design variable, $n$

$dKS / dx_n$

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Computational Cost vs. Number of Variables

![Graph showing computational cost vs. number of variables](image)

- **Complex step**: 2.1 + 0.92 \( N_x \)
- **Finite difference**: 1.0 + 0.38 \( N_x \)
- **Coupled adjoint**: 3.4 + 0.01 \( N_x \)
Supersonic Business Jet Optimization Problem

Minimize:

\[ I = \alpha C_D + \beta W \]

where \( C_D \) is that of the cruise condition.

Subject to:

\[ KS(\sigma_m) \geq 0 \]

where KS is taken from a maneuver condition.

With respect to: external shape and internal structural sizes.

Natural laminar flow
supersonic business jet
Mach = 1.5, Range = 5,300nm
1 count of drag = 310 lbs of weight
Baseline Design
Aero-Structural Optimization Convergence History

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Aero-Structural Optimization Results
Comparison with Sequential Optimization

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<th>( C_D ) (counts)</th>
<th>( \sigma_{\text{max}} ) ( \sigma_{\text{yield}} )</th>
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Conclusions

- Present a framework for high-fidelity aero-structural design.

- The computation of sensitivities using the aero-structural adjoint is extremely accurate and efficient.

- Demonstrated the usefulness of the coupled adjoint by optimizing a supersonic business jet configuration with external shape and internal structural variables.