AERO-STRUCTURAL WING PLANFORM OPTIMIZATION

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☞ Aircraft Weight Estimates

$$\text{Wing}_{bend} \approx 45\ \%\ \text{Wing weight}$$

$$\text{Wing weight} \approx 50\ \%\ \text{Weight}_{wing+tail+fuslage}$$

$$\text{Wing weight} \approx 30\ \%\ \text{Empty weight (no engine)}$$

We use Wing_{bend} to predict the total wing weight.

This simplified model does not analyse buckling.

☞ Mathematical Formulation for Euler Equations

Consider the Euler equations of steady flow

$$\frac{\partial}{\partial x_i} f_i(w) = 0.$$

In a fixed computational domain, with coordinates ξ_i, the Euler equations become,

$$R(w, S) = \frac{\partial}{\partial \xi_i} F_i(w) = \frac{\partial}{\partial \xi_i} S_{ij} f_j(w) = 0.$$

Suppose one wishes to minimize the cost function of a boundary integral

$$I = \int_{\mathcal{B}} \mathcal{M}(w, S)\, d\mathcal{B}_\xi + \int_{\mathcal{B}} \mathcal{N}(w, S)\, d\mathcal{B}_\xi$$

where

$\int_{\mathcal{B}} \mathcal{M}(w, S)\, d\mathcal{B}_\xi$ could be an **aerodynamic cost function**, e.g. C_D,

$\int_{\mathcal{B}} \mathcal{N}(w, S)\, d\mathcal{B}_\xi$ could be a **structural cost function**, e.g. **wing weight**.

☞ Mathematical Formulation for Euler Equations (cont.)

Then one can augment the cost function through Lagrange multiplier ψ as

$$I = \int_{\mathcal{B}} \mathcal{M}(w, S)\, d\mathcal{B}_\xi + \int_{\mathcal{D}} \psi^T \mathcal{R}(w, S)\, d\mathcal{D}_\xi + \int_{\mathcal{B}} \mathcal{N}(w, S)\, d\mathcal{B}_\xi.$$

A shape variation δS causes a variation

$$\delta I = \int_{\mathcal{B}} \delta\mathcal{M}\, d\mathcal{B}_\xi + \int_{\mathcal{D}} \psi^T \frac{\partial}{\partial \xi_i} \delta F_i\, d\mathcal{D}_\xi + \int_{\mathcal{B}} \delta\mathcal{N}\, d\mathcal{B}_\xi$$

The second term can be integrated by parts to give

$$\int_{\mathcal{B}} n_i \psi^T \delta F_i d\mathcal{B}_\xi - \int_{\mathcal{D}} \frac{\partial \psi^T}{\partial \xi_i} \delta F_i\, d\mathcal{D}_\xi.$$

☞ Mathematical Formulation for Euler Equations (cont.)

Now, choosing ψ to satisfy the adjoint equation

$$(S_{ij}\frac{\partial f_j}{\partial w})^T\frac{\partial \psi}{\partial \xi_i} = 0$$

with appropriate boundary conditions, explicit dependence on δw is eliminated, allowing the cost variations to be expressed in terms of δS and the adjoint solution.

In the case of the inverse design problem with cost function

$$I = \frac{1}{2}\int_{\mathcal{B}} (p - p_d)^2 \, dS,$$

where p_d is the desired surface pressure, the adjoint boundary condition is

$$\psi_j n_j = p - p_d$$

☞ Mathematical Formulation for Euler Equations (cont.)

Thus one obtains

$$\delta I = \int \mathcal{G} \delta \mathcal{F} \, d\xi = \langle \mathcal{G}, \delta \mathcal{F} \rangle$$

where $\mathcal{G}$ is the infinite dimensional gradient (Frechet derivative) and $\delta \mathcal{F}$ is the shape variations. Then one can make an improvement by setting

$$\delta \mathcal{F} = -\lambda \mathcal{G}$$

In fact the gradient $\mathcal{G}$ is generally of a lower smoothness class than the shape $\mathcal{F}$. Hence it is important to restore the smoothness. This may be affected by passing to a weighted Sobolev inner product of the form

$$\langle u, v \rangle = \int (uv + \epsilon \frac{\partial u}{\partial \xi} \frac{\partial v}{\partial \xi}) \, d\xi$$

This is equivalent to replacing $\mathcal{G}$ by $\bar{\mathcal{G}}$, where in one dimension

$$\bar{\mathcal{G}} - \frac{\partial}{\partial \xi} \bar{\mathcal{G}} \frac{\partial \bar{\mathcal{G}}}{\partial \xi} = \mathcal{G}, \quad \bar{\mathcal{G}} = \text{zero at end points}$$

and making a shape change $\delta \mathcal{F} = -\lambda \bar{\mathcal{G}}$.

☞ Adjoint development of the structural cost function

Suppose

$$I = C_W,$$

where

$$C_W = \frac{-\beta}{cos(\Lambda)^2} \int_0^{\frac{b}{2}} \int_{z^*}^{\frac{b}{2}} \oint_{wing} \frac{p(x,z)(z-z^*)}{t(z^*)} dx dz dz^* \tag{1}$$

To form the corresponding adjoint boundary condition, C_W must be expressed as $\int_{\mathcal{B}} d\mathcal{B}_\xi$ in the computational domain, or $\int\int dxdz$ in a physical domain to match the pressure boundary term.

To switch the order of integral of (1), introduce a Heaviside function

$$H(z-z^*) = \begin{cases} 0, & z < z^* \\ 1, & z > z^* \end{cases}$$

Then (1) can be rewritten as

$$\begin{aligned} C_W &= \frac{-\beta}{cos(\Lambda)^2} \int_0^{\frac{b}{2}} \int_0^{\frac{b}{2}} \oint_{wing} \frac{p(x,z)H(z-z^*)(z-z^*)}{t(z^*)} dx dz dz^* \\ &= \frac{-\beta}{cos(\Lambda)^2} \int_0^{\frac{b}{2}} \oint_{wing} p(x,z)K(z) dx dz, \end{aligned} \tag{2}$$

where

$$K(z) = \int_0^{\frac{b}{2}} \frac{H(z-z^*)(z-z^*)}{t(z^*)} dz^*$$
$$= \int_0^{z} \frac{z-z^*}{t(z^*)} dz^*$$

In the computational domain,

$$C_W = \frac{-\beta}{cos(\Lambda)^2} \oint_{\mathcal{B}} p(\xi_1, \xi_3) K(\xi_3) S_{22} d\xi_1 d\xi_3, \tag{3}$$

and $K(\xi_3)$ is a one-to-one mapping of $K(z)$.

☞ Adjoint development of the structural cost function

For simplicity, it is assumed that the portion of the boundary that undergoes shape modifications is restricted to the coordinate $\xi_2 = 0$. Then the adjoint boundary condition may be simplified by incorporatng the conditions

$$n_1 = n_3 = 0, \quad n_2 = 1 \quad and \quad d\mathcal{B}_\xi = d\xi_1 d\xi_3,$$

so that the only variation δF_2 needs to be considered at the wall boundary. Moreover, the condition that there is no flow through the wall boundary at $\xi_2 = 0$ is equivalent to

$$U_2 = 0,$$

and

$$\delta U_2 = 0$$

when the boundary shape is modified. Consequently,

$$\delta F_2 = \delta p \begin{Bmatrix} 0 \\ S_{21} \\ S22 \\ S23 \\ 0 \end{Bmatrix} + p \begin{Bmatrix} 0 \\ \delta S_{21} \\ \delta S22 \\ \delta S23 \\ 0 \end{Bmatrix}. \tag{4}$$

The variation of C_W is

$$\delta C_W = \frac{-\beta}{cos(\Lambda)^2} \oint_{\mathcal{B}} \delta p K S_{22} + p\delta \left(K S_{22}\right) d\xi_1 d\xi_3, \tag{5}$$

Since δF_2 and δC_W depend only on the pressure, it allows a complete cancellation of dependency of the boundary integral on δp, and the adjoint boundary condition reduces to

$$\psi_2 S_{21} + \psi_3 S_{22} + \psi_4 S_{23} = \frac{-\beta}{cos(\Lambda)^2} K S_{22} \tag{6}$$