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Aerodynamic Shape Optimization for the World's Fastest P-51

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In this paper, an Adjoint-based automatic design methodology has been applied to redesign the wing of the P51 "Dago Red", an aircraft competing in the Reno Air Races. The aircraft reaches speeds above 500 MPH and encounters compressibility drag due to the appearance of shock waves. The objective is to delay drag rise without altering the wing structure. Hence the shape modifications are restricted to adding a bump on the wing surface, allowing only outward movement. Moreover, the changes are limited to part of the chord-wise range. Results indicate that the perturbations created by this bump propagate along the characteristics and are reflected back from the sonic line to weaken the shock. With this bump on the wing, it is expected that Dago Red can reach the speed of 550 MPH, which will create a new World's speed record of the propeller driven airplane.

I. Introduction

The North American P-51 Mustang was one of the greatest WWII fighters. It was the product of two highly advanced technologies: the American advanced structural-and-aerodynamic plane body and the British prestigious Rolls-Royce Merlin engine. The performance and maneuverability of the P-51 outfitted other WWII fighters, resulting in extremely high survivability. A total of 15,686 Mustangs were built since 1944 and about 280 P-51s still exist today,[1] with more than half still airworthy. Among those is the Dago Red, a modified version of the P51-D to race the Reno Air Race. With the engine supercharged to gain horsepower and the wing tip cropped to match low attitude flight of the air race, the Dago Red reaches speeds above 500 MPH and suffers from high compressibility drag due to shock formation.

In this paper we propose a wing modification to delay the drag rise without changing the wing structure. Thus we only allow adding material to the wing at specific locations. We implement an adjoint-based optimization to identify the shape of the added material. We will discuss the design methodology in section II, along with results in section III. Results indicate that by adding a small bump near the leading edge of the wing, shock waves can be weaken and the drag rise can be delayed. With this bump, the aircraft may reach the speed of 550 MPH and create a new world speed record.

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II. Design methodology

A. Aerodynamic design without altering wing structure

During the last decade, the adjoint-based optimization for wing design has been extensively studied by Jameson et al[5,4,9,7] The method effectively eliminates shock waves but also allows structural changes of the wing. With slight modification such as fixing the planform, designing only specific chord-wise range to avoid affecting the control surface, and implementing the gradients that point only outward from the wing, the method can be directly applied to this application without affecting the wing structure. This should result in a bump added on the wing surface, as shown in figure 1 for one wing section.

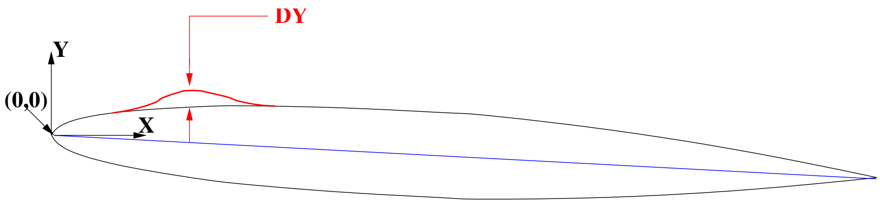


Figure 1. Added bump to achieve shock-free wing.

To design the shape of this bump, we use surface mesh points as the design parameters. There are 4096 surface mesh points on the wing surface and we only choose those that lie between the leading edge and 65 percents chord as the design parameters. We implement the adjoint method to calculate the gradient of these design parameters. Once the gradients are calculated, we implement an additional condition to allow only outward section movement;

$$\text{if} < 0 \quad \text{then} \quad g = 0, \tag{1}$$

assuming a positive g indicates outward movement. Finally, we smooth the gradients the smoothness of a modified wing.

B. The control theory approach to wing design problems

The control theory approach has been proposed for shape design since 1974[8] but it did not have much impact on aerodynamic design until its application to transonic flow.[3] The major impact arose from its capability to effectively handle a design problem that involves a large number of design variables and is governed by a complex mathematical model, such as fluid flow. The control theory approach is often called the adjoint method since the necessary gradients are obtained through the solution of the adjoint equations of the governing equations.

In the context of control theory, a wing design problem can be considered as:

$$\begin{aligned} Minimizing &\quad I(w,S) \\ w.r.t &\quad S \\ subjected\ to &\quad R(w,S)=0 \end{aligned}$$

where w is the flow variable, S is the vector of wing design parameters, and $R(w,S)=0$ is the flow equation.

For instance, for a drag minimization problem we can take $I = C_D$ which is an integral of flow w (pressure and shear force) over the wing S (represented by parameters such as airfoils). We modify S (the airfoils) to reduce the drag. The pressure and shear force are obtained from the flow equation $R = 0$ using CFD.

A change in S results in a change

$$\delta I = \left[\frac{\partial I}{\partial w}\right]^T \delta w + \left[\frac{\partial I}{\partial S}\right]^T \delta S, \tag{2}$$

and δw is determined from the equation

$$\delta R = \left[\frac{\partial R}{\partial w}\right] \delta w + \left[\frac{\partial R}{\partial S}\right] \delta S = 0. \tag{3}$$

The finite difference approach attempts to solve δw from equation (3) and substitute it into equation (2) to calculate δI. For a design problem of n design parameters e.g. $\mathcal{O}(S) = n$, this procedure requires a well-converged solution of $n+1$ flow analysis problems to obtain the design sensitivities. Thus it becomes impractical when n becomes large.

For an adjoint approach, we try to avoid solving for δw. This is done by introducing a Lagrange multiplier ψ, and subtracting the variation δR from the variation δI without changing the result. Thus, equation (2) can be replaced by

$$\begin{aligned}
\delta I &= \left[\frac{\partial I}{\partial w}\right]^T \delta w + \left[\frac{\partial I}{\partial S}\right]^T \delta S - \psi^T \left(\left[\frac{\partial R}{\partial w}\right] \delta w + \left[\frac{\partial R}{\partial S}\right] \delta S \right) \\
&= \left\{ \left[\frac{\partial I}{\partial w}\right]^T - \psi^T \left[\frac{\partial R}{\partial w}\right] \right\} \delta w + \left\{ \left[\frac{\partial I}{\partial S}\right]^T - \psi^T \left[\frac{\partial R}{\partial S}\right] \right\} \delta S \qquad (4)
\end{aligned}$$

Choosing ψ to satisfy the adjoint equation,

$$\left[\frac{\partial R}{\partial w}\right]^T \psi = \left[\frac{\partial I}{\partial w}\right], \qquad (5)$$

the first term is eliminated, and we find that

$$\delta I = \mathcal{G}^T \delta S, \qquad (6)$$

where

$$\mathcal{G}^T = \left[\frac{\partial I}{\partial S}\right]^T - \psi^T \left[\frac{\partial R}{\partial S}\right].$$

The advantage is that equation (6) is independent of δw, with the result that the gradient of I with respect to an arbitrary number of design variables can be determined without the need for additional flow-field evaluations.

Once the gradient vector $\mathcal{G}$ has been established, it may now be used to determine a direction of improvement. The simplest procedure is to make a step in the negative gradient direction (steepest descent method) by setting

$$\delta S = -\lambda \mathcal{G}$$

where λ is positive and small enough that the first variation is an accurate estimate of δI. The variation of the cost function then becomes

$$\begin{aligned}
\delta I &= -\lambda \mathcal{G}^T \mathcal{G} \\
&\leq 0
\end{aligned}$$

More sophisticated search procedures may be used such as quasi-Newton methods, which attempt to estimate the second derivative $\frac{\partial^2 I}{\partial S_i \partial S_j}$ of the cost function from changes in the gradient $\frac{\partial I}{\partial S}$ in successive optimization steps. These methods also generally introduce line searches to find the minimum in the search direction which is defined at each step. Reference[2] provides a good description for those techniques. However, not all the techniques are practical for our wing design problem. Line searches, for example, would require extra flow calculations, which we try to avoid. The complete detail and derivation using the Navier-Stokes can be found in reference.[6]

III. Results

We present a geometry of the bump and the benefit of this bump on the drag rise. The bump optimization was performed on the cropped P51-D wing, using the Reynolds-Averaged Navier Stokes Optimizer *Syn-107* on about 1 million mesh cells. The optimization was done at fixed $C_L = .1$ and Mach 0.78, corresponding to a flight condition to make new world speed record at 550 MPH. The original airfoil sections are shown in figure 2. Figure 3 shows the optimum shape of the bump. The size of this bump is about three quarters of an inch, which is big enough to be manufactured. The perturbations created by this bump propagate along the characteristics and are reflected back from the sonic line to weaken the shock. The improvement is shown in figure 4. Although we pay a penalty of higher drag below Mach .73, the drag rise can be delayed. This provide an opportunity to increase the maximum velocity and can be applied for world speed record making.

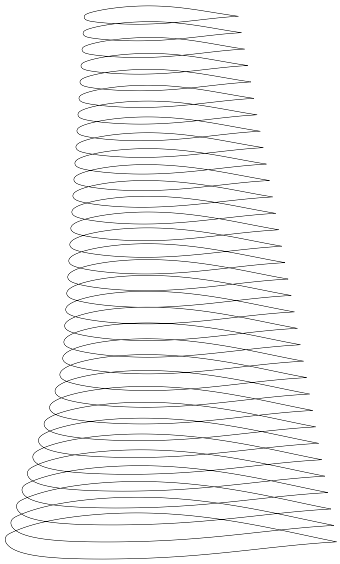


Figure 2. Original wing sections of the P51-D, cropped at 189 inches span station.

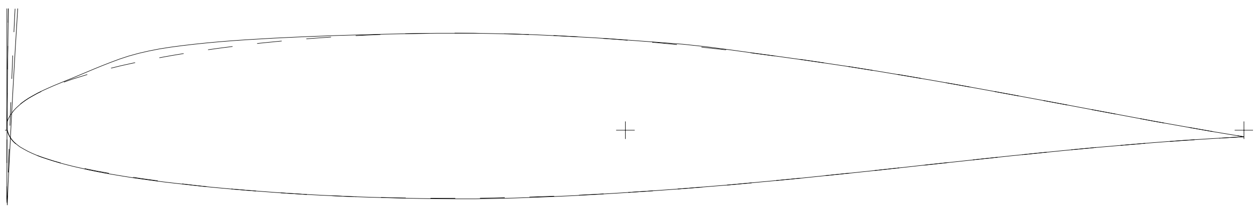
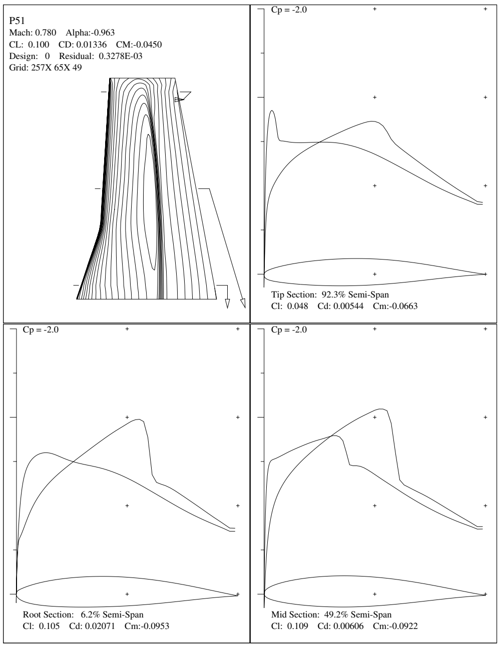
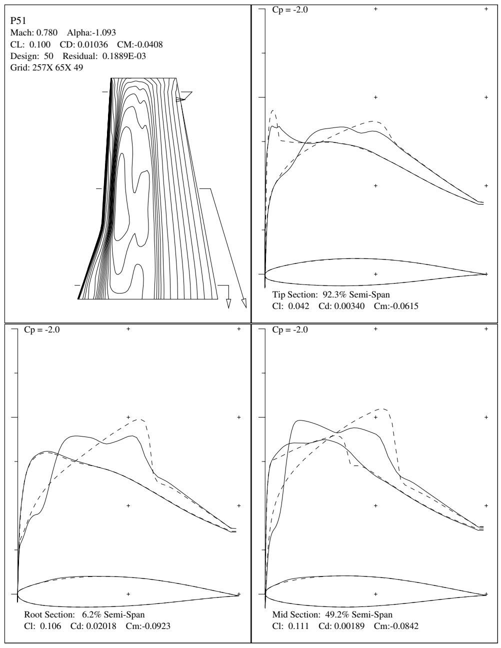


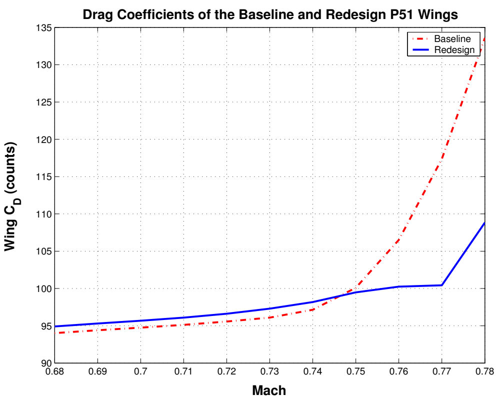
Figure 3. Shape of an optimum bump at a mid-span location. Dashed line represent an original wing section.



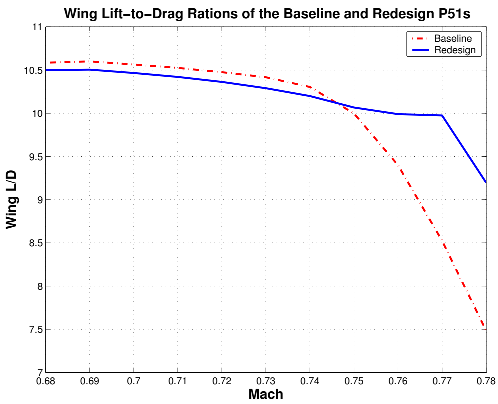
(a) Before the redesign



(b) After the redesign



(c) C_D



(d) L/D

Figure 4. Improvements earned by adding a bump on the original P51 wing. (a) C_p distributions of the original wing at Mach .78, (b) C_p distributions of the redesigned wing. The bump eliminates the shock, (c) C_D Vs. Mach number at fixed C_L 0.1, (d) L/D Vs. Mach at fixed C_L 0.1.

IV. Conclusion

By adding a bump near the wing leading edge, it is possible to delay a drag rise of the P51-D without altering the wing structure. The size of this bump is comparatively small, but large enough to be manufactured. The delay of drag rise benefit velocity gained by the aircraft using the same engine. The new maximum speed is expected to reach 550 MPH and can make a new world speed record. Because this bump can be installed quickly, it suggests a quick way to improve aerodynamic characteristic.

V. Acknowledgment

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