43rd AIAA Aerospace Sciences Meeting and Exhibit, January 10-13, 2005, Reno, NV

Enhancement of Adjoint Design Methods via Optimization of Adjoint Parameters

Sangho Kim* Kaveh Hosseini† Kasidit Leoviriyakit‡ and Antony Jameson§

Stanford University, Stanford, CA 94305, U.S.A

Three approaches of enhancing the Euler adjoint design methods using a non-linear gradient-based optimization package, SNOPT, have been investigated. First, the convergence of Euler and adjoint solutions was accelerated by optimizing the coefficients of the residual smoothing scheme and the CFL number. Second, the input parameters of Euler adjoint design methods were optimized such that the best aerodynamic shape can be achieved in a given number of design iterations. Finally the SNOPT software has also been used to provide line searches of the shape optimization parameters at each step and improved the robustness of the design methods. The numerical results showed the feasibility of integration of the SNOPT package and the adjoint software in order to speed-up, improve, and stabilize the performance of the design methods.

I. Introduction

Automatic design procedures that use Computational Fluid Dynamics(CFD) combined with gradient-based optimization techniques have been made possible by the availability of high performance computing platforms and by the development of new and efficient analysis and design algorithms. In particular the focus of CFD applications has shifted to aerodynamic design since the introduction of control theory to Aerodynamic Shape Optimization(ASO) in transonic flow by Jameson[1,2] in 1989. This control theory approach is often called the adjoint method, since the necessary gradients are obtained via the solution of the adjoint equations of the governing equations of interest. The adjoint method is extremely efficient since the computational expense incurred in the calculation of the complete gradient is effectively independent of the number of design variables. In fact this method has become a popular choice for design problems involving fluid flow and has been successfully used for the aerodynamic design of complete aircraft configurations .[3,4,5,6]

In Jameson's first works on the use of control theory,[1,7,8] every surface mesh point was used as a design variable. Using this approach, the complete design space of all airfoil shapes that can be represented by a given number of surface points can be spanned. In theory, this approach would also produce results closer to a true optimum if the optimum shape were to have a high frequency component that could not be captured with other shape functions with less compact support.

On the other hand, a problem of this choice is that the smoothness of the aerodynamic shape may not be preserved. This contradicts the assumption of first derivative continuity of the solution in the development of the adjoint formulation. Another way of understanding this problem is that this choice of using each grid point as a design variable admits very high frequencies in the design space. This admittance of high frequencies causes a higher degree of nonlinearity, and, in practice, the higher the degree of nonlinearity, the more computationally difficult it is to find an optimum.

Jameson was aware of this problem and developed an implicit gradient smoothing method in order to overcome the difficulty of discontinuity and proved the success of this approach in his previous works.[1,7,8]

*Postdoctoral Fellow, Department of Aeronautics and Astronautics

†Doctoral Candidate, Department of Aeronautics and Astronautics

‡Doctoral Candidate, Department of Aeronautics and Astronautics

§Thomas V. Jones Professor of Engineering, Department of Aeronautics and Astronautics, AIAA Fellow

Similar to his implicit residual smoothing[9] , he smoothed the gradient implicitly. This smoothing procedure eliminates high frequency components from the design space.

Once the necessary gradient information is evaluated, the aerodynamic shape should be updated in order to obtain an optimum configuration by an appropriate optimization algorithm. Advanced methods such as Newton or quasi-Newton methods can be adopted for this purpose and procedures using quasi-Newton methods have also been used for the aerodynamic shape design.[10] However, in conjunction with adjoint methods, second-derivative methods can not be always considered as the optimum choice due to the high cost in Hessian matrix evaluations. The large numbers of design variables used with surface point parameterizations also preclude the use of advanced optimization algorithms such as Newton or quasi-Newton methods due to the high cost of the associated matrix operations, and the fact that the number of design iterations required is proportional to the number of design variables. For this reason, Jameson's earlier works with the use of mesh design variables used the simple steepest descent method with no line searches. Instead of line searching, repeated small steps are taken, simulating a time dependent process. These issues are explored in the paper on the Brachistochrone problem by Jameson and Vassberg.[11] The implicit gradient smoothing may also be regarded as a preconditioner which allows the use of much larger steps for the search procedure and leads to a large reduction in the number of design iterations needed for convergence. The efficiency of Jameson's steepest descent method results from the elimination of the need for exact gradients and the effectiveness of implicit smoothing as a preconditioner.

Determining the value of the smoothing coefficient and the size of the step for the shape change in the steepest descent method are critical for a successful design in practice. They have been determined by trial and error. Since CFD analysis and design codes are becoming more complex, there is an increasing number of parameters that a user needs to choose as input and it is very time consuming to find a proper combination of parameters.

In this work, we propose to enhance the existing adjoint-based design method by determining the control parameters of CFD analysis and design process by the use of the non-linear gradient-based optimizer package, SNOPT.[12,13] SNOPT is a gradient-based package that uses a sequential quadratic programming (SQP) algorithm that obtains search directions from a sequence of quadratic programming subproblems. Each QP subproblem minimizes a quadratic model of a certain Lagrangian function subject to a linearization of the constraints. The SNOPT package was successfully used by Hosseini and Alonso[14,15] to optimize various input parameters for explicit Euler and Navier-Stokes flow solvers, including the multistage coefficients of the modified Runge-Kutta schemes. The parameter optimization investigated previously by Hosseini and Alonso will be summarized in the next section.

II. Parameter optimization

CFD input parameters govern various aspects of a code's building blocks such as multistage scheme, multigrid algorithm, artificial dissipation terms, entropy fix, etc. An inappropriate choice of input parameters can result in slow convergence, oscillations, inaccurate solutions, or divergence. Often times, users are required to spend a lot of time experimenting with various parameters to obtain reasonably fast convergence to an accurate solution. Finding the magical combination of parameters that would make a code perform to its full potential has become almost an art that is mastered only by the most advanced users who know all the intricacies of a given code.

Many of the parameters that users choose as being optimal, are the result of trial and error. In some cases though, more systematic approaches have been used in finding optimal input parameters. Such approaches may become particularly valuable when code developers add new features to older codes that extend their capabilities. New features usually introduce additional input parameters and sometimes alter the algorithms for which the trial and error method had produced optimal parameters.

A. Trial and error

An example of input parameters optimized by trial and error is the case of multistage coefficients for modified Runge-Kutta schemes. Using trial and error, Jameson and Martinelli produced several sets of multistage coefficients for modified Runge-Kutta schemes.[16,9,17] In order to illustrate their results, they applied Fourier analysis to the case of the one-dimensional model wave equation with third order artificial dissipation.

$$w_t + a w_x + \mu_a \Delta x^3 w_{xxxx} = 0. \tag{1}$$

Fourier analysis used in conjunction with stability domain contour plots offers a simple yet powerful visual tool in order to understand some of the effects of various input parameters on scheme stability, or propagation and damping of error modes. The following set of coefficients that we will refer to as the 5-stage Martinelli-Jameson (MJ) coefficients produces a large Fourier stability domain and is popularly used in CFD, especially in schemes based on scalar artificial dissipation and central-differencing.

$$\begin{aligned}
\alpha_1 &= 1/4 & \beta_1 &= 1 \\
\alpha_2 &= 1/6 & \beta_2 &= 0 \\
\alpha_3 &= 3/8 & \beta_3 &= 14/25 \\
\alpha_4 &= 1/2 & \beta_4 &= 0 \\
\alpha_5 &= 1 & \beta_5 &= 11/25.
\end{aligned} \tag{2}$$

Fig. 2-a shows the stability domain of the MJ coefficients and the locus of the Fourier residual operator z for the discrete version of Eq. 1 using central differencing without residual smoothing.

B. Analytical approach

Some authors have tried to optimize multistage coefficients using more analytical approaches. Tai uses a geometric method that forces the locus of z to pass through the zeros of a multistage scheme in order to reduce the amplification factor.[18]

In an attempt to adapt Runge-Kutta multistage methods to codes modified by preconditioners, other authors have minimized the Fourier amplification factors of discrete Euler or Navier-Stokes equations for classical Runge-Kutta multistaging rather than the more advantageous modified Runge-Kutta approach.[19,20] Hosseini and Alonso have performed systematic optimizations of various objective functions for modified Runge-Kutta schemes.[14] They have shown that, at least in the case of the Euler flow solvers, objective functions based on the minimization of the amplification factor do not necessarily produce the fastest convergence rates in actual codes. Convergence seems to be a strong function of propagation and a weak function of damping. Fig. 2-b shows that the trial and error method used by Jameson and Martinelli had produced results that were very close to being optimal.

Hosseini and Alonso further showed that despite its simplicity, the one-dimensional model wave equation produces remarkably good indications for an actual flow solver's behavior. In reality, the locus of the one-dimensional scalar Fourier operator z, produces the envelope of the loci of the eigenvalues of the matrix two-dimensional Euler Fourier operator Z if at least a Block-Jacobi preconditioner is used as seen on Fig. 3.[15]

C. Direct numerical approach

Given the ever increasing performance of computers and the easier access to parallel machines, it has become possible to adopt a new optimization approach based on actual flow solvers rather than on simple analytical models. Hosseini and Alonso used the FORTRAN gradient-based optimization package SNOPT in conjunction with FLO103[21,22,17]– an actual two-dimensional flow solver– in order to find optimal input parameters including multistage coefficients, CFL number, artificial dissipation terms, entropy fix cutoff, etc. Not only was faster convergence achieved, but it was also shown that in certain cases, the input parameters found by SNOPT produced Fourier residual coefficients falling outside of the stability domain. Therefore analytical methods prove to have limitations despite giving good indications of the direction to take in choosing input parameters.[15] On the other hand, gradient-based optimizers have their own limitations in that they can only find local optima. Fig. 4 from Hosseini and Alonso's ten-variable optimizations for squared-preconditioned Euler cases using SNOPT and FLO103,[15] shows improved stability domains for $M = 0.1$ and $M = 0.4$ while reduced stability domains for $M = 0.01$ and $M = 0.8$. A practical approach to the input parameter optimization problem could therefore combine all three approaches (trial and error, analytical, direct numerical).

III. Residual Smoothing Parameter Optimization

Hosseini and Alonso have intentionally left out the usage of residual smoothing in their works[14,15] in order to isolate the effects of preconditioners, dissipation schemes and multistage coefficients on the convergence acceleration produced. It is obvious that the use of residual smoothing can have an additional beneficial effect on accelerating both the convergence of flow solution and, consequently, the design optimization convergence. The proper optimization of this feature has been investigated in the present work and will be addressed in this section, focusing on the effects on the efficiency of the design methods,

A. Description of Implicit Residual Smoothing

The general idea behind this technique is to increase the time step limit by replacing the residual at one cell in the flow field by a weighted average of the residuals at the neighboring cells. The average is calculated implicitly and can be expressed in the two-dimensional case:

$$(1 - \epsilon_i \delta_{xx})\,(1 - \epsilon_j \delta_{yy})\,\bar{R}_{i,j} = R_{i,j}, \tag{3}$$

where ϵ_i and ϵ_j control the level of smoothing, and $\bar{R}_{i,j}$ is the updated value of the residual that is obtained by solving the equation implicitly in each coordinate direction using a tridiagonal solver. A complete discussion of the stability character and overall benefit of this acceleration method is provided by Jameson and Baker.[23,24] A study of the limits of the smoothing parameter was done by Martinelli,[17] which leads to a conclusion that the smoothing parameters in each direction should satisfy the following inequalities.

$$\epsilon_i = max\left\{\frac{1}{4}\left[\left(\frac{CFL^*}{CFL}\lambda_i\right)^2 - 1\right], 0\right\}$$

$$\epsilon_j = max\left\{\frac{1}{4}\left[\left(\frac{CFL^*}{CFL}\lambda_j\right)^2 - 1\right], 0\right\},$$

where λ_i and λ_j are the scaled spectral radii of the flux Jacobian matrices in each direction, CFL^* is the new Courant number, and CFL is the maximum allowable Courant number for the scheme without implicit residual averaging. This residual averaging with locally varying coefficients has been proved to be effective especially on highly stretched meshes.

B. Optimization of Residual Smoothing

In the usual practice of residual averaging, the coefficients are modified by

$$\epsilon_i^* = \beta_i \epsilon_i$$

$$\epsilon_j^* = \beta_j \epsilon_j.$$

Here, the proper values of β_i, β_j, and CFL^* depend on the flight conditions, flow conditions, and aspect ratio of meshes, etc. Moreover they must be empirically tuned by trial and error. For example from our previous experiences, the recommended values to double the permissible Courant number for two-dimensional Euler calculations are found as $\beta_i = \beta_j = 0.6$.

In the adjoint design methods using the steepest descent optimization algorithm, the flow and adjoint evaluations are required in each design iteration. The computational cost for these required flow and adjoint calculations is the most time consuming portion of the design method. In the present work the values of β_i, β_j, β_k and CFL^* for the three-dimensional Euler calculations are optimized using the SNOPT package such that an accurate flow convergence level (10^{-4}) can be reached within the minimum number of multigrid cycles.

C. Results of Residual Smoothing Optimization

To demonstrate some advantage of our approach, we performed a shape optimization using Euler calculations on the Boeing 747 wing-body configuration at a fixed angle of attack, $\alpha = 2.3°$, and $M_\infty = 0.87$ using a C-H grid of size $192 \times 32 \times 32$. The average density residual, R, was selected as the cost function and the residual smoothing coefficients and CFL number were used as the design variables for the optimization using SNOPT. The optimization of the residual smoothing parameters has been performed in a way that SNOPT called the Euler solver and received the value of the cost function, R, while Euler solver received the updated coefficients and the CFL number from SNOPT for a new flow evaluation. The optimized residual smoothing coefficients and new CFL number are listed in table 1. Using these optimized parameters the average density residual after 200 multigrid iterations, R_{200}, was improved by two orders of magnitude. As shown in Fig. 5-a, the number of multigrid iterations required for an engineering accuracy was reduced from 100 to 67. A separate study to find the optimum adjoint solver parameters was not made in this work. However, as one would expect, a similar improvement in adjoint solution convergence has been monitored in Fig. 5-b and table 1 using the same optimized parameters.

Euler adjoint solutions typically converge faster than Euler solutions for given input parameters, since the adjoint equations are a set of linear partial differential equations with the same eigenvalues as Euler equations. Also only a minor level of convergence is needed to get a sufficiently accurate adjoint gradient. Thus, due to the convergence acceleration for both the Euler and the adjoint solutions with these optimized parameters, the overall adjoint design method could be improved by 30%.

Table 1. Results for residual smoothing optimization

Case : Euler	β_i	β_j	β_k	CFL	R_{200}
Reference or Initial	0.6	0.6	0.6	9.0	1.631E-06
Optimal	0.459	0.471	0.450	7.18	1.914E-08

Case : Adjoint	β_i	β_j	β_k	CFL	R_{200adj}
Reference or Initial	0.6	0.6	0.6	6.5	1.570E-09
Optimal	0.459	0.471	0.450	7.18	1.048E-10

IV. Enhancement of Shape Optimization Procedure

A. Description of Implicit Gradient Smoothing

In this section, we demonstrate the concept of implicit gradient smoothing which is combined with the steepest descent method using fixed step size.

Let $\mathcal{F}$ represent the design variable, and $\mathcal{G}$ the gradient. An improvement could then be made with a shape change

$$\delta\mathcal{F} = -\lambda\mathcal{G}. \tag{4}$$

In fact, however, the gradient $\mathcal{G}$ is generally of a lower smoothness class than the shape $\mathcal{F}$, with the result that this process may fail to converge or even become unstable. In order to preserve the smoothness we redefine the gradient to correspond to a weighted Sobolev inner product of the form

$$< u, v >= \int \left(uv + \epsilon \frac{\partial u}{\partial \xi}\frac{\partial v}{\partial \xi} \right) d\xi.$$

Thus we define a modified gradient $\bar{\mathcal{G}}$ such that

$$\delta I =< \bar{\mathcal{G}}, \delta\mathcal{F} > .$$

In the one dimensional case, taking $\bar{\mathcal{G}} = 0$ at the end points, integration by parts yields

$$\delta I = \int \left(\bar{\mathcal{G}} - \frac{\partial}{\partial \xi_1}\epsilon\frac{\partial \bar{\mathcal{G}}}{\partial \xi_1} \right) \delta\mathcal{F} d\xi.$$

Then $\bar{\mathcal{G}}$ is obtained by solving the smoothing equation

$$\bar{\mathcal{G}} - \frac{\partial}{\partial \xi_1} \epsilon \frac{\partial}{\partial \xi_1} \bar{\mathcal{G}} = \mathcal{G}. \tag{5}$$

In the multi-dimensional case the smoothing is applied in product form. Finally we set

$$\delta \mathcal{F} = -\lambda \bar{\mathcal{G}} \tag{6}$$

with the result that

$$\delta I = -\lambda < \bar{\mathcal{G}}, \bar{\mathcal{G}} > \quad < 0,$$

unless $\bar{\mathcal{G}} = 0$, and correspondingly $\mathcal{G} = 0$.

When second-order central differencing is applied to Eq. (5), the equation at a given node, i, can be expressed as

$$\bar{\mathcal{G}}_i - \epsilon \left(\bar{\mathcal{G}}_{i+1} - 2\bar{\mathcal{G}}_i + \bar{\mathcal{G}}_{i-1} \right) = \mathcal{G}_i, \qquad 1 \leq i \leq n,$$

where $\mathcal{G}_i$ and $\bar{\mathcal{G}}_i$ are the point gradients at node i before and after the smoothing respectively, and n is the number of design variables equal to the number of mesh points in this case. Then,

$$\bar{\mathcal{G}} = A\mathcal{G},$$

where A is the $n \times n$ tri-diagonal matrix such that

$$A^{-1} = \begin{bmatrix} 1+2\epsilon & -\epsilon & 0 & . & 0 \\ \epsilon & . & . & & \\ 0 & . & . & . & \\ . & & . & . & -\epsilon \\ 0 & & & \epsilon & 1+2\epsilon \end{bmatrix}.$$

Now using the steepest descent method in each design iteration, a step, $\delta \mathcal{F}$, is taken such that

$$\delta \mathcal{F} = -\lambda A \mathcal{G}. \tag{7}$$

As can be seen from the form of this expression, implicit smoothing may be regarded as a preconditioner which allows the use of much larger steps for the search procedure and leads to a large reduction in the number of design iterations needed for convergence.

B. Enhancement of the Design Optimization Procedure

In our optimization code SYN88, two main parameters that control the optimization process are the smoothing parameters ϵ and the step size parameter λ. An approach we have been using is to use constant values of ϵ and λ for the entire process.

While this approach can produce a shock free solution efficiently, finding two constants by trial and error is not necessarily easy due to the following issues: first, since only the abstract expression of $\mathcal{G}$ in Eq. 7 is available, the stability analysis and thus, a general rule for stability may not be established. Second there is no pre-existing knowledge of the design space where multiple local optimum points, flat contours or even discontinuities might exist. For the optimization of the more complex system of configurations where many different types of design variables and the corresponding design parameters will be introduced, more difficulties will be expected.

In this paper, we incorporate SNOPT into SYN88 so that the process of deciding the right values of ϵ and λ by the user is bypassed. We have proposed two enhancement approaches. First, the optimization of input parameters of Euler adjoint design methods has been made such that an optimum aerodynamic shape can be reached in a given number of design iterations. In this approach since a full shape optimization using adjoint methods will be carried out in every SNOPT iteration for parameter optimization, there will be a huge disadvantage in terms of computational savings. However once SNOPT finds an optimal set of design parameters, an even better optimum may be achieved by the adjoint methods using these optimized parameters. Alternatively, the SNOPT package can provide line searches to decide the values of design parameters at every adjoint shape design iteration. An improvement in the robustness of the shape design methods without any noticeable addition of computational cost will be a target in this approach. This approach may be challenging since the function evaluation for SNOPT should now includes the effect of the shape change as well.

C. Results of Gradient Smoothing Optimization

Parameter Optimization for Boeing 747 Wing-Body Configuration Wing Redesign

A drag minimization of the Boeing 747 wing-body configuration was performed by modifying the wing with a fixed fuselage shape. An Euler calculation was carried out for the wing-body configuration at a fixed coefficient of lift, $C_L = 0.45$, and $M_\infty = 0.85$ using a C-H grid of size $192 \times 32 \times 32$. In order to provide a reference for drag minimization, the original optimization method without using SNOPT has first performed a wing shape design. The values of ϵ and λ were set to 8.0 and 0.2 respectively after several trial and error corrections were made. Using these values of ϵ and λ the total wing drag was reduced from $C_D = 0.01092$ to $C_D = 0.01007$ while keeping C_L fixed close to 0.45. Next SNOPT optimizes ϵ and λ for the adjoint design code to produce the best possible shape within 10 shape design iterations. The optimal values of ϵ and λ were found to be 4.0 and 0.253, and using these values the shape design was repeated. Fig. 6 shows a comparison of the flow over the initial configuration with that over the improved configuration after 10 design cycles. The final pressure contours on the upper surface are displayed in the upper left quadrant, while the other three quadrants superpose the C_p distributions at root, mid, and tip sections. The total wing drag was reduced from $C_D = 0.01092$ to $C_D = 0.00986$ at the same fixed $C_L = 0.45$. The design results with and without SNOPT parameter optimization are listed in the table 2. It can be seen that a lower final C_D was obtained using parameters optimized by SNOPT.

Table 2. Results for residual smoothing optimization

Case	ϵ	λ	design iterations	$C_{D\,final}(\Delta C_D)$
Parameter by trial and error	8.0	0.2	17	0.01007(0.00085)
Parameter by SNOPT	4.0	0.253	10	0.00986(0.00106)

D. Results Using SNOPT to Optimize the Step Parameters at Each Step

1. Boeing 747 Wing-Body Configuration Wing Redesign

Our other approach is to use SNOPT to carry out line searches for the best step parameters at each design cycle. This has been tested for Boeing 747 wing redesign. Now starting with the initial $\epsilon = 8.0$ and $\lambda = 0.2$, they are updated by SNOPT at every shape design iteration. The values of ϵ and λ were 8.006 and 0.239 after the design. Similarly to the reference case, the total wing drag was successfully reduced from $C_D = 0.01092$ to $C_D = 0.01004$ in 17 design iterations. Though there was no substantial improvement over the reference design case, now tuning efforts for ϵ and λ have been avoided and with a proper boundary setting in design parameter, the design method becomes more stable. Interestingly, we also found that the resulting ϵ and λ values were very dependent on the initial values. This is due to the fact that the design space is very flat with respect to ϵ and λ and the SNOPT optimality tolerance should be set relatively higher in order to get an optimum in a small number of design iterations.

2. Supersonic Business Jet Design

Similarly to the previous test case, Euler calculations were carried out on a $192 \times 32 \times 32$ C-H grid for a supersonic business jet wing-body configuration at a fixed coefficient of lift, $C_L = 0.18$, and $M_\infty = 1.8$. In this design case, a polynomial function was introduced as a design variable and a direct camber line modification was forced, by putting the function on top of the base camber line in order to accelerate the design convergence. Now, not only ϵ and λ, but also the step size, λ_{camber}, for camber movement have to be decided by SNOPT. In 33 design iterations the final converged values were $\epsilon = 2.002$, $\lambda = 8.007$, and $\lambda_{camber} = 0.376$, while the wing section converged to the solid line as shown in Fig. 7. The total wing drag was reduced from $C_D = 0.01594$ to $C_D = 0.01450$.

3. Boeing 747 Wing Planform Optimization

Finally we extend our design methodology to wing planform optimization. This design case is quite complicated due to the increased complexity of the design space. In the previous sections, only two or three design parameters needed to be determined but in the present section the values of 8 total design parameters have been updated at every adjoint design iteration.

The shape changes in the wing section needed in order to improve the transonic wing design are quite small. However, in order to obtain a true optimum design, larger scale changes such as changes in the wing planform (sweepback, span, chord, section thickness, and taper) should be considered. Because these directly affect the structural weight, a meaningful result can only be obtained by considering a cost function that accounts for both the aerodynamic characteristics and the weight.

Figure 1. Simplified wing planform of a transport aircraft.

Following references[25,26] we redesign both wing section and planform to minimize a cost function including both drag and structural weight terms of the form:

$$I = \alpha_1 C_D + \alpha_2 C_W. \tag{8}$$

The wing section is modeled by surface mesh points, and the wing planform is modeled by the design variables shown in figure 1 as root chord (c_1), mid-span chord (c_2), tip chord (c_3), span (b), sweepback(Λ), and wing thickness ratio (t).

In fact, it has been shown that both drag and structural weight can be improved simultaneously with the use of the steepest descent method in the previous wing planform optimization work by Leoviriyakit.[26]

Using SNOPT in this type of complex problems to include a line search, has improved the robustness of the design process and has proved the concept that the time and efforts required to find a proper set of input design parameters can be removed by the use of a direct numerical software.

Fig. 8 shows the result after 58 design iterations using SNOPT for line searches for λ, ϵ and 6 wing planform parameters. The total cost function, $I = C_D + 0.07C_W$ has reduced from 0.01431 to 0.01202. The design convergence history is shown in Fig. 9.

V. Conclusion

The feasibility of improving the existing adjoint method by finding an optimal combination of flow analysis and design input parameters has been investigated in this work. The input parameter optimization problem could be performed by three approaches, trial and error, analytical, and direct numerical approaches. A non-linear gradient-based optimization package, SNOPT was chosen as a direct numerical tool for parameter optimization, and various numerical tests, in conjunction with the three-dimensional Euler adjoint design software, SYN88, were carried in order to investigate the benefits out of using this choice of parameter optimization. The numerical results show the adjoint design method can be improved in shape design speed, performance, and stability by integrating the method with a parameter optimization tool such as, but not limited to, SNOPT. The benefits may be greater for parameter optimization for a complex system where, in particular, neither analytical nor trial and error approach are practical.

VI. Acknowledgment

This work has benefited greatly from the support of the Air Force Office of Scientific Research under grant No. AF F49620-98-1-2005.

References

[1] A. Jameson. Aerodynamic design via control theory. *Journal of Scientific Computing*, 3:233–260, September 1989.

[2] A. Jameson. Optimum aerodynamic design using CFD and control theory. *AIAA paper 95-1729*, AIAA 12th Computational Fluid Dynamics Conference, San Diego, CA, June 1995.

[3]J. Reuther, J. J. Alonso, J. C. Vassberg, A. Jameson, and L. Martinelli. An efficient multiblock method for aerodynamic analysis and design on distributed memory systems. *AIAA paper 97-1893*, June 1997.

[4]J. Reuther, J.J. Alonso, M.J. Rimlinger, and A. Jameson. Aerodynamic shape optimization of supersonic aircraft configurations via an adjoint formulation on parallel computers. *AIAA paper 96-4045*, 6th AIAA/NASA/ISSMO Symposium on Multidisciplinary Analysis and Optimization, Bellevue, WA, September 1996.

[5]O. Baysal and M. E. Eleshaky. Aerodynamic design optimization using sensitivity analysis and computational fluid dynamics. *AIAA paper 91-0471*, 29th Aerospace Sciences Meeting, Reno, Nevada, January 1991.

[6]W. K. Anderson and V. Venkatakrishnan. Aerodynamic design optimization on unstructured grids with a continuous adjoint formulation. *AIAA paper 97-0643*, 35th Aerospace Sciences Meeting and Exhibit, Reno, Nevada, January 1997.

[7]A. Jameson. Automatic design of transonic airfoils to reduce the shock induced pressure drag. In *Proceedings of the 31st Israel Annual Conference on Aviation and Aeronautics, Tel Aviv*, pages 5–17, February 1990.

[8]A. Jameson. Optimum aerodynamic design via boundary control. In *AGARD-VKI Lecture Series, Optimum Design Methods in Aerodynamics*. von Karman Institute for Fluid Dynamics, 1994.

[9]A. Jameson. Transonic flow calculations. *Princeton University Report MAE 1651*, Princeton University, April 1984.

[10]J. Reuther. Aerodynamic shape optimization using control theory. *Ph. D. Dissertation*, University of California, Davis, Davis, CA, June 1996.

[11]A. Jameson and J.C. Vassberg. Studies of alternative numerical optimization methods applied to the brachistochrone problem. *Computational Fluid Dynamics*, 9:281–296, 2000.

[12]P. E. Gill, W. Murray, and M. H. Wright. *Practical Optimization*. Academic Press, Inc., New York, 1981.

[13]P. E. Gill, W. Murray, and M. A. Saunders. User's guide for snopt (version 6.0): A fortran package for large-scale nonlinear programming. Technical report, Department of Mathematics, University of California, San Diego, December 2002.

[14]K. Hosseini and J. J. Alonso. Optimum of multistage coefficients for explicit multigrid flow solvers. *AIAA paper 2003-3705*, AIAA 16th Computational Fluid Dynamics Conference, Orlando, FL, June 2003.

[15]K. Hosseini and J. J. Alonso. Practical implementatin and improvement of preconditioning methods for explicit multistage flow solvers. *AIAA paper 2004-0763*, 42nd AIAA Aerospace Sciences Meeting & Exhibit, Reno, NV, January 2004.

[16]A. Jameson. Multigrid algorithms for compressible flow calculations. In W. Hackbusch and U. Trottenberg, editors, *Lecture Notes in Mathematics, Vol. 1228*, pages 166–201. Proceedings of the 2nd European Conference on Multigrid Methods, Cologne, 1985, Springer-Verlag, 1986.

[17]L. Martinelli. Calculation of viscous flows with a multigrid method. *Ph. D. Dissertation*, Princeton University, 1987.

[18]C. H. Tai. *Acceleration Techniques for Explicit Euler Codes*. PhD thesis, University of Michigan, Ann Arbor, MI, 1990.

[19]J. F. Lynn. *Multigrid Solution of the Euler Equations with Local Preconditioning*. PhD thesis, University of Michigan, Ann Arbor, MI, 1995.

[20]W. L. Kleb. Efficient multi-stage time marching for viscous flows via local preconditioning. *AIAA Paper* 99-3267, 1999.

[21]A. Jameson. Solution of the Euler equations for two dimensional transonic flow by a multigrid method. *Applied Mathematics and Computations*, 13:327–356, 1983.

[22]L. Martinelli and A. Jameson. Validation of a multigrid method for the Reynolds averaged equations. *AIAA paper 88-0414*, 1988.

[23]A. Jameson. A vertex based multigrid algorithm for three dimensional compressible flow calculations. In *the ASME Symposium on Numerical Methods for Compressible Flow*, Anaheim, 1986.

[24]A. Jameson and T.J. Baker. Solution of the Euler equations for complex configurations. *AIAA paper 83-1929*, AIAA 6th Computational Fluid Dynamics Conference, Danvers, MA, July 1983.

[25]K. Leoviriyakit, S. Kim, and A. Jameson. Aero-structural wing planform optimization using the Navier-Stokes equations. *AIAA paper 2004-4479*, 10th AIAA/ISSMO Multidisciplinary Analysis and Optimization Conference, Albany, New York, August 30 - September 1 2004.

[26]K. Leoviriyakit. *Wing Optimization via an Adjoint Method. PhD Dissertation*, Stanford University, Stanford, CA, December 2004.

(a) Case of the MJ coefficients optimized through trial and error. The locus of the Fourier residual operator z corresponds to a maximum CFL number $\sigma = 3.93$ and an artificial dissipation coefficient $\mu_a = \frac{1}{32}$

(b) Case of the optimized multistage coefficients obtained using MATLAB's gradient-based optimization function `fmincon` with the one-dimensional model wave equation keeping CFL as high as possible while retaining minimum damping. The locus of the Fourier residual operator z corresponds to a maximum CFL number $\sigma = 3.99$ and an artificial dissipation coefficient $\mu_a = \frac{1}{32}$

Figure 2. Stability domain and locus of the Fourier residual operator z.

(a) One-dimensional model wave equation

(b) Un-preconditioned Euler equations for $M = 0.1$, $\text{Æ} = 1$, and $\varphi = 30^\circ$

(c) Block-Jacobi preconditioned Euler equations for $M = 0.1$, $\text{Æ} = 1$, and $\varphi = 30^\circ$

Figure 3. Stability domain and amplification factor for the MJ coefficients with $\sigma = 3.6$, $\mu_a = \frac{2.5}{32}$

(a) $M = 0.01$

(b) $M = 0.1$

(c) $M = 0.4$

(d) $M = 0.8$

Figure 4. Stability domains for optimized coefficients obtained through SNOPT for a squared-preconditioned FLO103 Euler flow solver.

(a) Flow convergence history

(b) Adjoint convergence history

Figure 5. Residual smoothing optimization

Figure 6. Solution after 10 design iterations, Euler drag minimization at fixed $C_L = 0.45$. using the SNOPT optimized $\epsilon = 8.0$ and $\lambda = 0.2$, Boeing 747 wing redesign. Initial C_p : $---$, Redesigned C_p : ———

Figure 7. Solution after 33 adjoint design iterations using SNOPT line searches, Euler drag minimization at fixed $C_L = 0.18$, Super Sonic Business Jet Wing Body. Initial C_p : $---$, Redesigned C_p : ———

Figure 8. Solution after 58 adjoint design iterations usng SNOPT line searches, Euler drag and wing weight minimization at fixed $C_L = 0.45$, Boeing 747 wing and planform redesign. Initial C_p : $---$, Redesigned C_p : ———

Figure 9. Adjoint shape design convergence history using SNOPT line searches