AERODYNAMIC SHAPE OPTIMIZATION OF WINGS INCLUDING PLANFORM VARIABLES

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☞ Outline

- ➢ Introduction and Motivation.
- ➢ Planform optimization including wing weight estimation.
- ➢ Adjoint Formulation.
- ➢ Results.
- ➢ Conclusion and Future work.

☞ Introduction

➢ While aerodynamic prediction methods based on CFD are now well established, accurate, and robust, the ultimate need in the design process is to find the optimum shape which maximizes the aerodynamic performance.

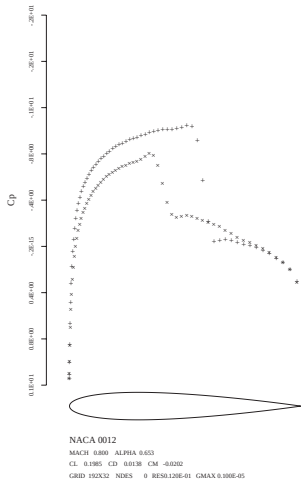


Figure 1: Baseline of NACA0012 airfoil (calculated in 3.42 sec.)

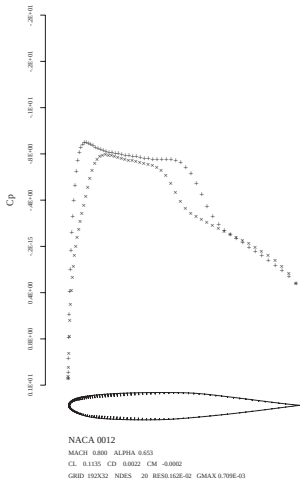


Figure 2: Redesign of NACA0012 airfoil (calculated in 19.75 sec.)

➢ Up to now, our work has focused on the use of high-dimensional parameterizations of aerodynamic surfaces.

➢ Assume: Rigid wing and Fixed planform.

☞

➢ Result: Shock free wing within 5 minutes (using 1 1.7 gHz processor).

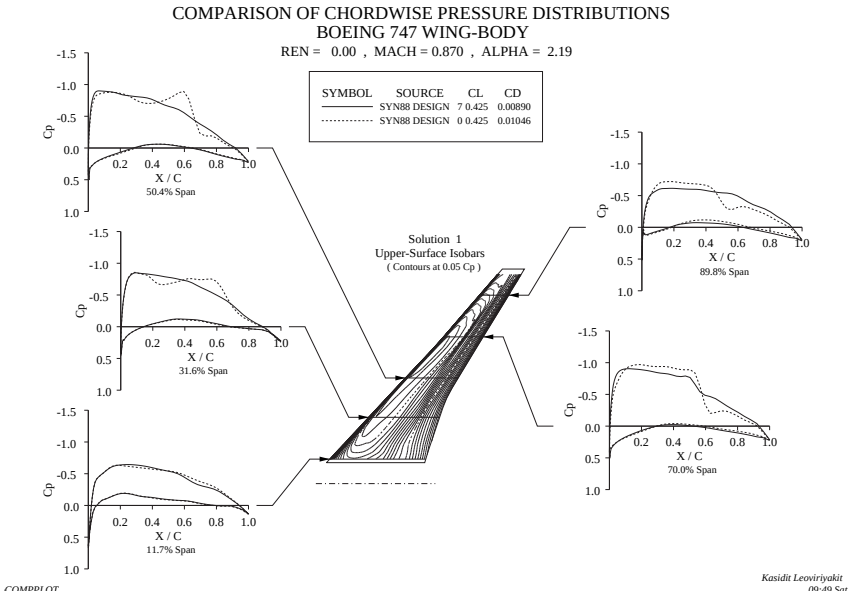


Figure 3: Redesign of B747-200

➢ Can we do more?

☞ Planform Optimization

- ➢ Wing planform modification can lead to large variations/improvements in wing performance but also effect:
  - Wing weight.
  - Stability & Control issues.
- ➢ The results of aerodynamic-only optimization that involves planform variables are highly suspect because the decrease in drag may come at the expense of an increase in wing weight.
- ➢ Hence it is essential to account for the effect of planform change on wing weight.
- ➢ The relative importance of drag and weight depends on the overall design, but may be estimated from the Breguet range equation.
- ➢ In this work we pursue "Aerodynamic Optimization with simple estimation of wing weight".
- ➢ In the future we plan to introduce a more complete structure model.

☞ Cost Function for Planform Design

A natural choice of the cost function would be

$$I = \alpha_1 C_D + \alpha_2 \frac{1}{2} \int_{\mathcal{B}} (p - p_d)^2 dS + \alpha_3 C_W \tag{1}$$

with

$$C_W = \frac{\mathcal{W}_{wing}}{q_\infty S_{ref}} \tag{2}$$

where α_1, α_2, α_3 are properly chosen weighting constants.

A simple way to model the weight of the wing is to use a Statistical Group Weight equation based on sophisticated regression analysis.

☞ Statistical Group Weight Model

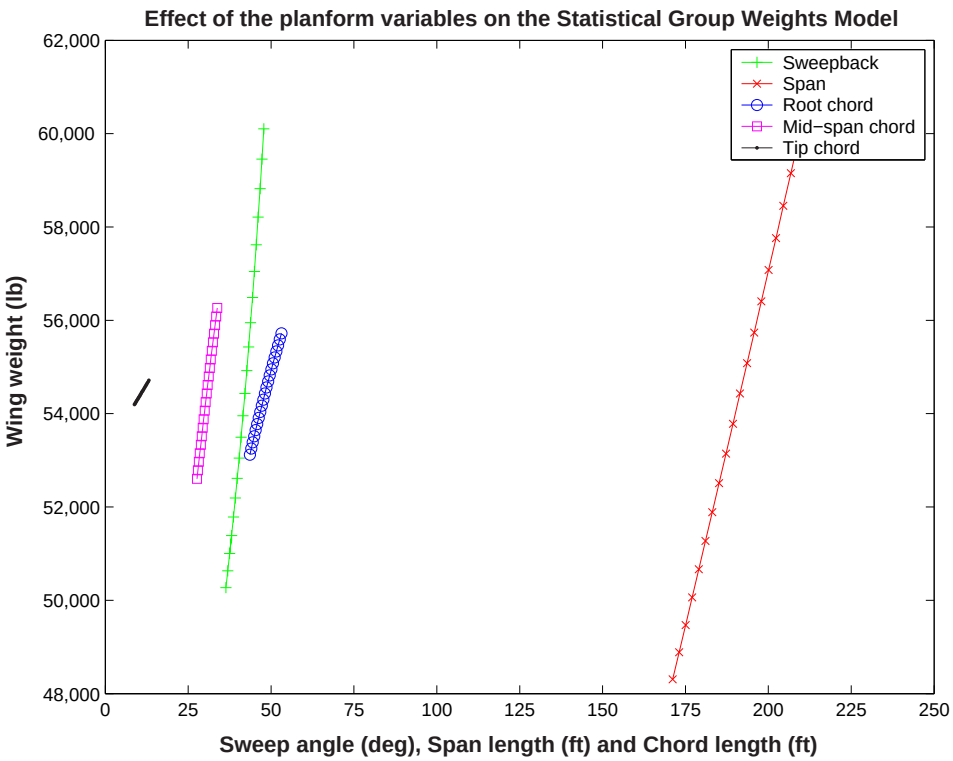
For cargo/transportation wing weight, one can use

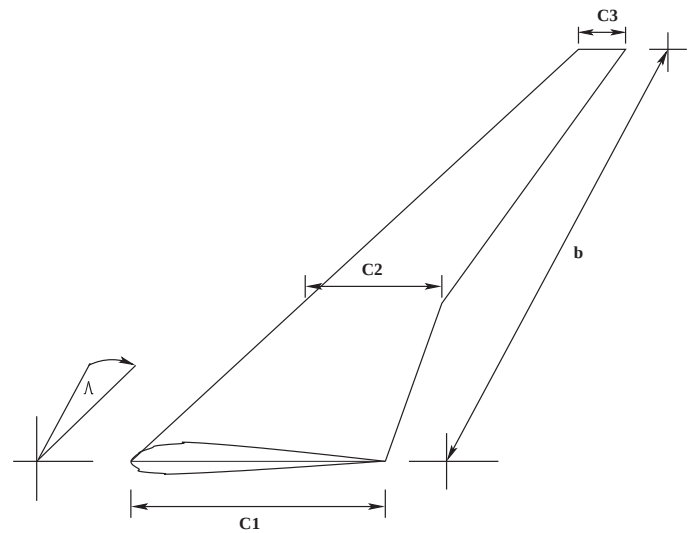
$$\mathcal{W}_{weight} = 0.0051(W_{dg}N_z)^{0.557}S_w^{0.649}A^{0.5}(t/c)_{root}^{-0.4}(1+\lambda)^{0.1}cos(\Lambda)^{-1.0}S_{csw}^{0.1} \quad (3)$$

where

A = aspect ratio,
N_z = ultimate load factor; = 1.5 x limit load factor,
S_{csw} = control surface area (wing-mounted),
S_w = trapezoidal wing area,
t/c = thickness to chord ratio,
W_{dg} = flight design gross weight,
Λ = wing sweep, and
λ = taper ratio at 25 % MAC.

☞ Characteristic of Weight Model





☞ Choice of weighting constant

- The choice of α_1 and α_3 greatly effects the optimum shape.
- Maximizing the range of an aircraft provides a guide to their values.
- The **simplified range equation** can be expressed as

$$R = \frac{V}{C}\frac{L}{D} log \frac{W_1}{W_2}$$

where W_2 is the empty weight of the aircraft. With fixed $\frac{V}{C}$, W_1, and L, the variation of R can be stated as

$$\begin{aligned}
\delta R &= \frac{V}{C}\left(\delta\left(\frac{L}{D}\right) log \frac{W_1}{W_2} + \frac{L}{D}\delta\left(log \frac{W_1}{W_2}\right)\right) \\
&= \frac{V}{C}\left(-\frac{\delta D}{D}\frac{L}{D} log \frac{W_1}{W_2} - \frac{L}{D}\frac{\delta W_2}{W_2}\right) \\
&= -\frac{V}{C}\frac{L}{D} log \frac{W_1}{W_2}\left(\frac{\delta D}{D} + \frac{1}{log \frac{W_1}{W_2}}\frac{\delta W_2}{W_2}\right)
\end{aligned}$$

☞ Choice of weighting constant (cont.)

➢ Then,

$$\frac{\delta R}{R} = -\left(\frac{\delta C_D}{C_D} + \frac{1}{log\frac{W_1}{W_2}}\frac{\delta W_2}{W_2}\right)$$
$$= -\left(\frac{\delta C_D}{C_D} + \frac{1}{log\frac{C_{W_1}}{C_{W_2}}}\frac{\delta C_{W_2}}{C_{W_2}}\right).$$

Therefore minimizing

$$I = C_D + \alpha C_W,$$

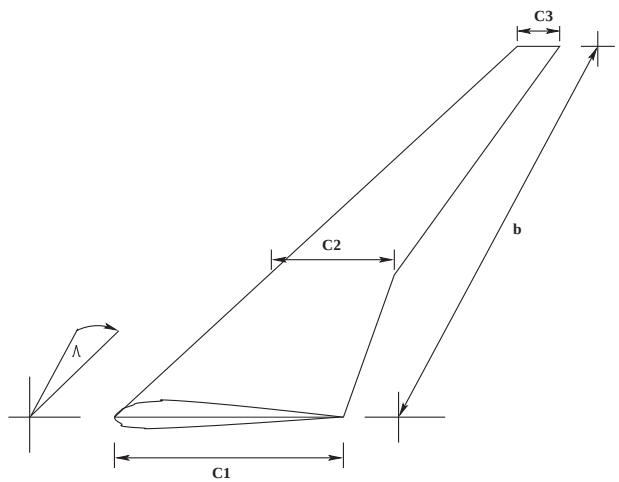
by choosing

$$\alpha = \frac{C_D}{C_{W_2} log\frac{C_{W_1}}{C_{W_2}}}, \tag{4}$$

corresponds to maximizing the range of the aircraft.

☞ Simplified Model for Planform Design

➢ Model the planform by five variables: c_1, c_2, c_3, b, and Λ.



➢ Design variables are now Mesh points on the wing surface plus five planform variables.

➢ Use Adjoint Method to calculate gradientsof both mesh points and planform variables.

☞ Symbolic Development of the Adjoint Method

Let I be the **cost** (or **objective**) function

$$I = I(w, \mathcal{F})$$

where

$$\begin{aligned} w &= \text{flow field variables} \\ \mathcal{F} &= \text{grid variables} \end{aligned}$$

The **first variation** of the cost function is

$$\delta I = \frac{\partial I}{\partial w}^T \delta w + \frac{\partial I}{\partial \mathcal{F}}^T \delta \mathcal{F}$$

The **flow field equation** and its **first variation** are

$$R(w, \mathcal{F}) = 0$$

$$\delta R = 0 = \left[\frac{\partial R}{\partial w}\right] \delta w + \left[\frac{\partial R}{\partial \mathcal{F}}\right] \delta \mathcal{F}$$

☞ Symbolic Development of the Adjoint Method (cont.)

Introducing a **Lagrange Multiplier**, ψ, and using the **flow field equation** as a **constraint**

$$\begin{aligned}\delta I &= \frac{\partial I}{\partial w}^T \delta w + \frac{\partial I}{\partial \mathcal{F}}^T \delta \mathcal{F} - \psi^T \left\{ \left[\frac{\partial R}{\partial w}\right] \delta w + \left[\frac{\partial R}{\partial \mathcal{F}}\right] \delta \mathcal{F} \right\} \\ &= \left\{ \frac{\partial I}{\partial w}^T - \psi^T \left[\frac{\partial R}{\partial w}\right] \right\} \delta w + \left\{ \frac{\partial I}{\partial \mathcal{F}}^T - \psi^T \left[\frac{\partial R}{\partial \mathcal{F}}\right] \right\} \delta \mathcal{F}\end{aligned}$$

By choosing ψ such that it satisfies the **adjoint equation**

$$\left[\frac{\partial R}{\partial w}\right]^T \psi = \frac{\partial I}{\partial w},$$

we have

$$\delta I = \left\{ \frac{\partial I}{\partial \mathcal{F}}^T - \psi^T \left[\frac{\partial R}{\partial \mathcal{F}}\right] \right\} \delta \mathcal{F}$$

This reduces the **gradient** calculation for an arbitrarily large number of design variables at a **single design point** to

☞ **One Flow Solution + One Adjoint Solution**

☞ Mathematical Formulation for Euler Equations (cont.)

Consider the Euler equations of steady flow

$$\frac{\partial}{\partial x_i} f_i(w) = 0.$$

In a fixed computational domain, with coordinates ξ_i, the Euler equations become,

$$R(w, S) = \frac{\partial}{\partial \xi_i} F_i(w) = \frac{\partial}{\partial \xi_i} S_{ij} f_j(w) = 0.$$

Suppose one wishes to minimize the cost function of a boundary integral

$$I = \int_{\mathcal{B}} \mathcal{M}(w, S)\, d\mathcal{B}_\xi + \int_{\mathcal{B}} \mathcal{N}(S)\, d\mathcal{B}_\xi$$

where

$\int_{\mathcal{B}} \mathcal{M}(w, S)\, d\mathcal{B}_\xi$ could be an **aerodynamic cost function**, e.g. C_D,

$\int_{\mathcal{B}} \mathcal{N}(S)\, d\mathcal{B}_\xi$ could be a **structural cost function**, e.g. **wing weight**.

☞ Mathematical Formulation for Euler Equations (cont.)

Then one can augment the cost function through Lagrange multiplier ψ as

$$I = \int_{\mathcal{B}} \mathcal{M}(w, S)\, d\mathcal{B}_\xi + \int_{\mathcal{D}} \psi^T \mathcal{R}(w, S)\, d\mathcal{D}_\xi + \int_{\mathcal{B}} \mathcal{N}(S)\, d\mathcal{B}_\xi.$$

A shape variation δS causes a variation

$$\delta I = \int_{\mathcal{B}} \delta\mathcal{M}\, d\mathcal{B}_\xi + \int_{\mathcal{D}} \psi^T \frac{\partial}{\partial \xi_i} \delta F_i\, d\mathcal{D}_\xi + \int_{\mathcal{B}} \delta\mathcal{N}\, d\mathcal{B}_\xi$$

The second term can be integrated by parts to give

$$\int_{\mathcal{B}} n_i \psi^T \delta F_i d\mathcal{B}_\xi - \int_{\mathcal{D}} \frac{\partial \psi^T}{\partial \xi_i} \delta F_i\, d\mathcal{D}_\xi.$$

Now, choosing ψ to satisfy the adjoint equation

$$(S_{ij} \frac{\partial f_j}{\partial w})^T \frac{\partial \psi}{\partial \xi_i} = 0$$

with appropriate boundary conditions, explicit dependence on δw is eliminated, allowing the cost variations to be expressed in terms of δS and the adjoint solution.

☞ Mathematical Formulation for Euler Equations (cont.)

Thus one obtains

$$\delta I = \int \mathcal{G} \delta \mathcal{F} \, d\xi = \langle \mathcal{G}, \delta \mathcal{F} \rangle$$

where $\mathcal{G}$ is the infinite dimensional gradient (Frechet derivative) and $\delta \mathcal{F}$ is the shape variations. Then one can make an improvement by setting

$$\delta \mathcal{F} = -\lambda \mathcal{G}$$

In fact the gradient $\mathcal{G}$ is generally of a lower smoothness class than the shape $\mathcal{F}$. Hence it is important to restore the smoothness. This may be effected by passing to a weighted Sobolev inner product of the form

$$\langle u, v \rangle = \int (uv + \epsilon \frac{\partial u}{\partial \xi} \frac{\partial v}{\partial \xi}) \, d\xi$$

This is equivalent to replacing $\mathcal{G}$ by $\bar{\mathcal{G}}$, where in one dimension

$$\bar{\mathcal{G}} - \frac{\partial}{\partial \xi} \bar{\mathcal{G}} \frac{\partial \bar{\mathcal{G}}}{\partial \xi} = \mathcal{G}, \quad \bar{\mathcal{G}} = \text{zero at end points}$$

and making a shape change $\delta \mathcal{F} = -\lambda \bar{\mathcal{G}}$.

☞ Aerodynamic Gradient for Planform Variables

Consider the aerodynamic contribution of the cost function (1)

$$\delta I = \int_{\mathcal{B}} \delta\mathcal{M}\, d\mathcal{B}_\xi + \int_{\mathcal{D}} \psi^T \delta R\, d\mathcal{D}_\xi$$

This can be split as

$$\delta I = [I_w]_I\, \delta w + \delta I_{II}$$

with

$$\delta\mathcal{M} = [\mathcal{M}_w]_I \delta w + \delta\mathcal{M}_{II}$$

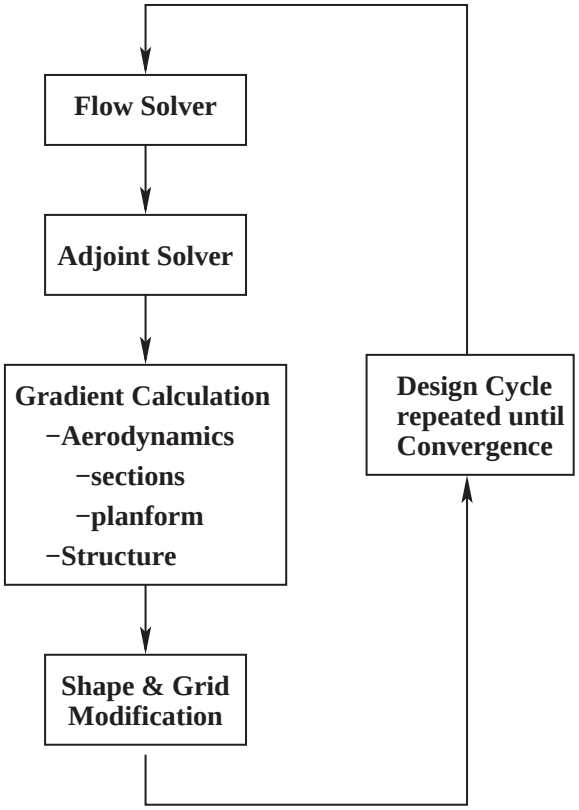
where the subscripts I and II are used to distinguish between the contributions associated with variation of the flow solution δw and those associated with the metric variations δS. Choose ψ to satisfy

$$(S_{ij}\frac{\partial f_j}{\partial w})^T \frac{\partial \psi}{\partial \xi_i} = 0,$$

Then,

$$
\begin{aligned}
\delta I(w,S) &= \delta I(S) \\
&= \int_{\mathcal{B}} \delta \mathcal{M}_{II}\, d\mathcal{B}_\xi + \int_{\mathcal{D}} \psi^T \delta R\, d\mathcal{D}_\xi \\
&\approx \sum_{\mathcal{B}} \delta \mathcal{M}_{II} \Delta \mathcal{B} + \sum_{\mathcal{D}} \psi^T \Delta \bar{R} \\
&\approx \sum_{\mathcal{B}} \delta \mathcal{M}_{II} \Delta \mathcal{B} + \sum_{\mathcal{D}} \psi^T \left(\bar{R}|_{S+\delta S} - \bar{R}|_S \right),
\end{aligned}
$$

Design Cycle



☞ Validation of Aerodynamic Gradient with respect to Planform Variables

- ➢ Compare aerodynamic Planform Variables gradient using Adjoint and Finite-Difference methods.
- ➢ Test case: Boeing 747-200 wing-fuselage and modified geometries at the following flow conditions

  $M_\infty = 0.87$, $\alpha = 2.3$ degrees (fixed)

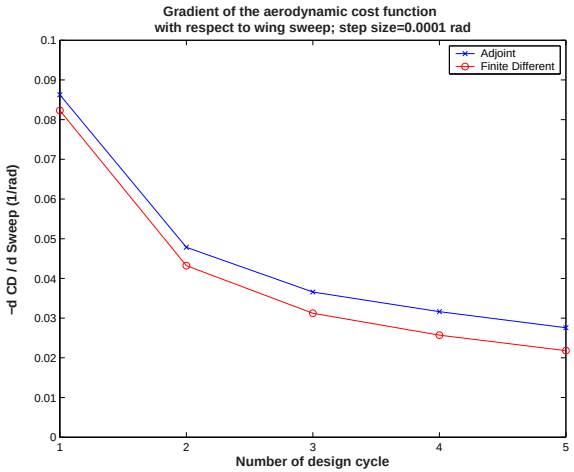


Figure 4: Sweep

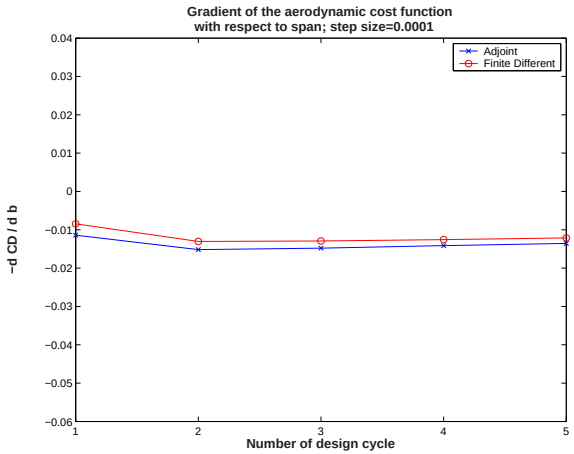


Figure 5: Span

☞ Validation of Aerodynamic Gradient with respect to Planform Variables(cont.)

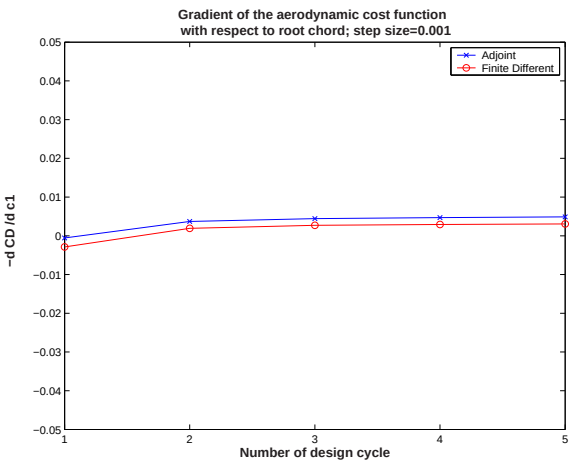


Figure 6: c_1

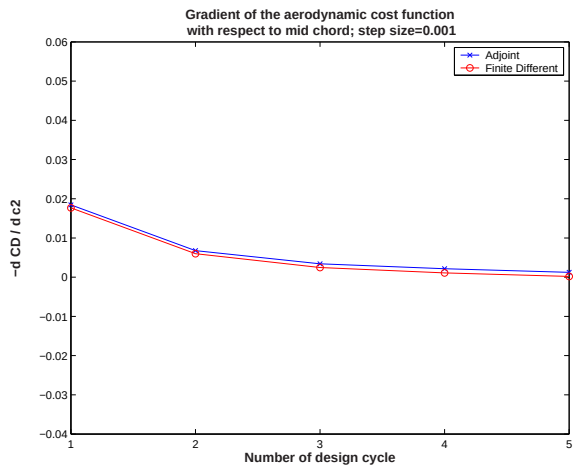


Figure 7: c_2

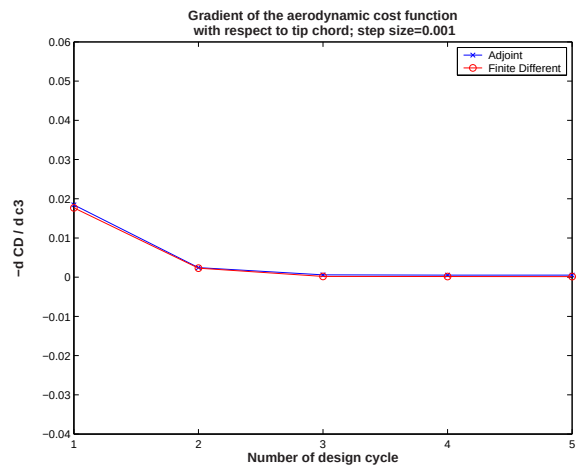


Figure 8: c_3

Adjoint gradients match Finite-Difference gradients very well but at lower cost.

☞ Sample of Planform Optimization

➢ Test case: Boeing 747-200 wing-fuselage and modified geometries at the following flow conditions

$M_\infty = 0.87$

$C_L = 0.42$ (fixed)

Choose α_3 according to (4) (maximize range).

☞ Sample of Planform Optimization : Fixed Planform

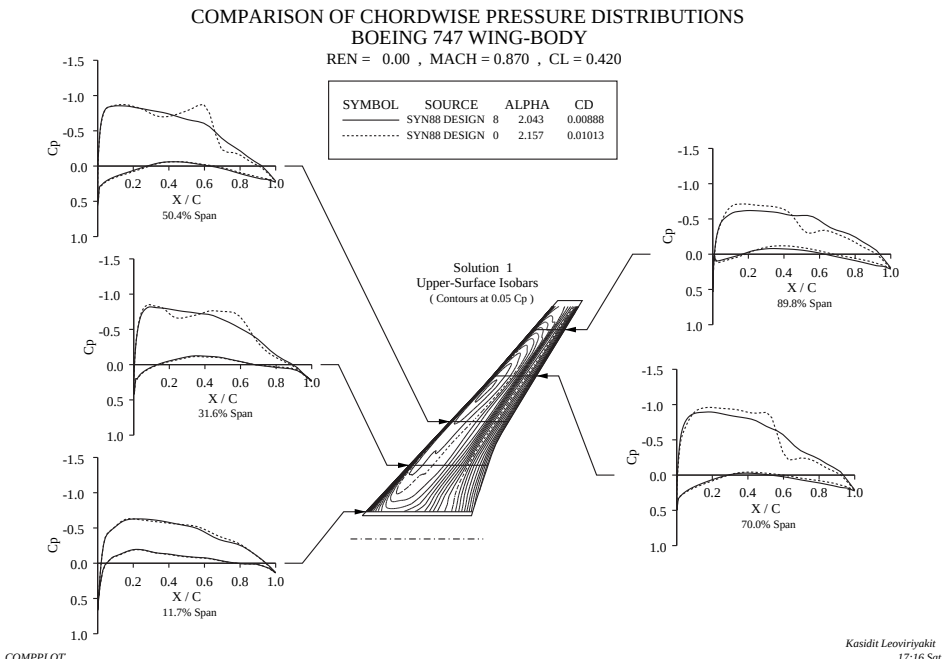


Figure 9: Redesign of B747-200

➢ a C_D is reduced from 101.3 drag counts to 88.8 drag counts (12.3 % reduction).

☞ Sample of the Planform Optimization : Sweepback, Span, and Chord Variations & Maximized Range

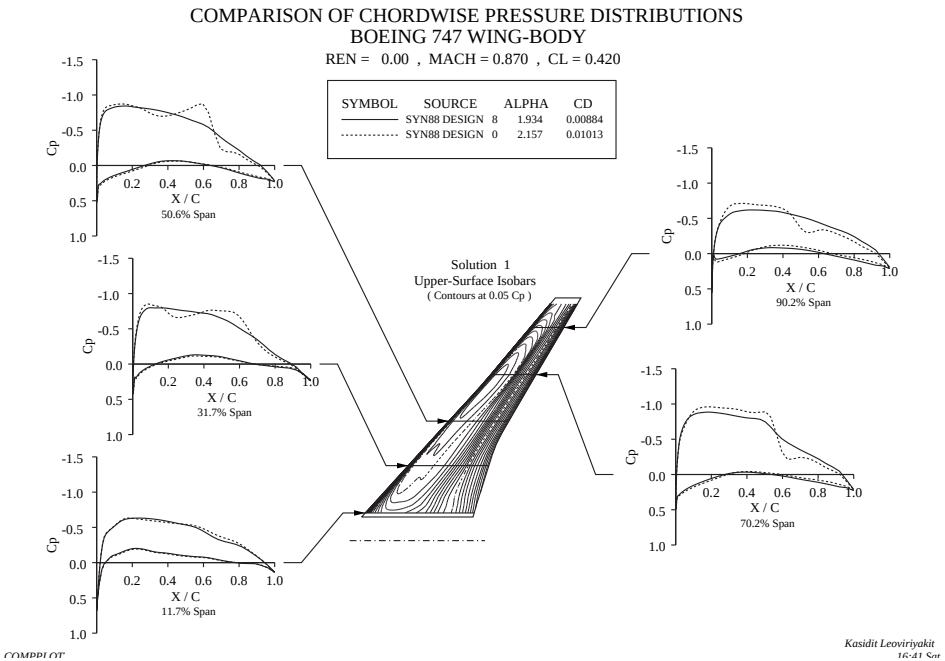


Figure 10: Redesign of B747-200

- ➢ C_D is reduced from 101.3 drag counts to 88.4 drag counts (12.7 % reduction).
- ➢ C_w is reduced from 0.03215 to 0.03211 (0.1 % reduction).

- sweepback decreases from 42.11 degrees to 41.34degrees.
- span increases from 191.47 ft to 192.29 ft.
- c_1 increases from 48.16 ft to 48.40 ft.
- c_2 increases from 30.59 ft to 30.87 ft.
- c_3 increases from 10.79 ft to 11.07 ft.

  Ideal!!

☞ Sample of Planform Optimization : Extreme case

- To demonstrate the trade between aerodynamic and structural cost functions.
- Test case: Boeing 747-200 wing-fuselage and modified geometries at the following flow conditions

  $M_\infty = 0.87$

  $C_L = 0.42$ (fixed)

  $\alpha_3 =$ large!.

☞ Sample of Planform Optimization : Sweepback, Span, and Chord Variations & Extreme case

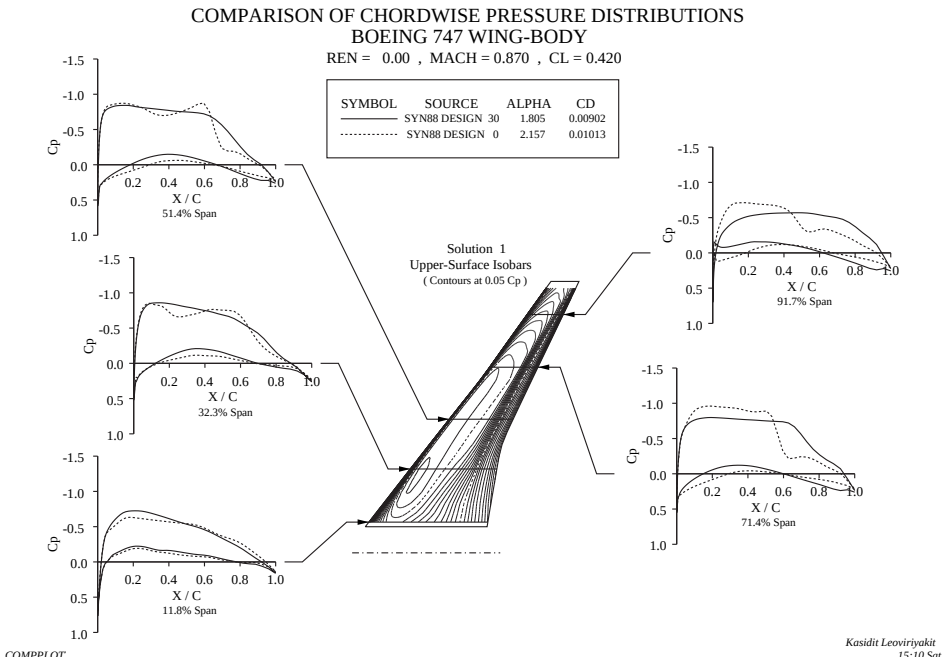


Figure 11: Redesign of B747-200

- ➢ C_D is reduced from 101.3 drag counts to 90.2 drag counts (11.0 % reduction).
- ➢ C_w is reduced from 0.03215 to 0.03042 (5.4 % reduction).

- sweepback decreases from 42.11 degrees to 36.99degrees.
- span increases from 191.47 ft to 195.83 ft.
- c_1 decreases from 48.16 ft to 48.08 ft.
- c_2 decreases from 30.59 ft to 29.88 ft.
- c_3 increases from 10.79 ft to 11.06 ft.

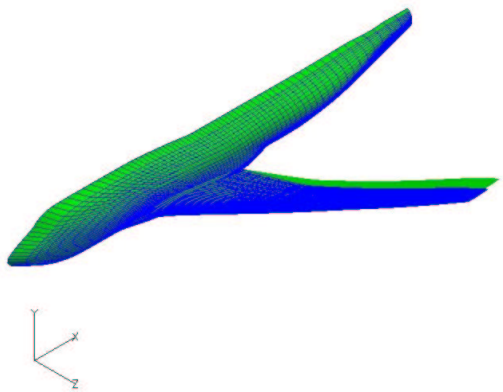


Figure 12: Superposition of the baseline and the optimized geometries of B747-200

- Caution!: Watch out the span load

☞ Sample of Planform Optimization : Sweepback, Span, and Chord Variations & Extreme case

- ➢ The statistical weight model does NOT capture aerodynamic load.
- ➢ To achieve a practical design, force the span load.

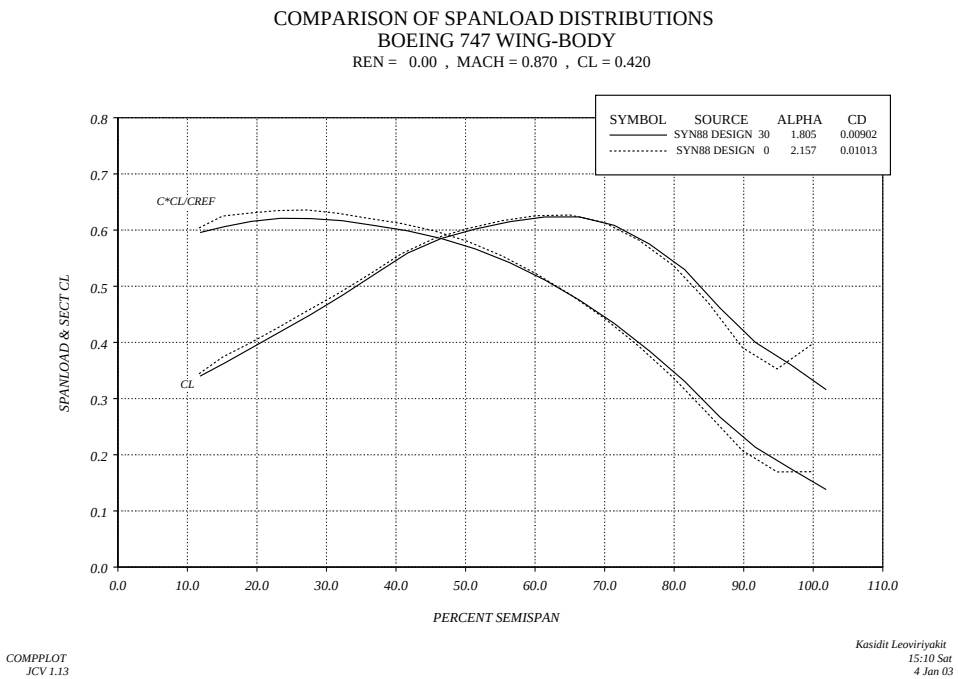


Figure 13: Redesign of B747-200

☞ Conclusions: (1)

- An aerodynamic design methodology for planform optimization has been validated.
- In order to realize meaningful designs, a model of structure weight needs to be included in the cost function.
- The trade between the structure cost function and the aerodynamic cost function prevents an unrealistic results, leading to a useful design.
- Currently, the simplified structure model is a simplified model;
  - only a function of planform variables
  - Independent of wing loading
- A more realistic model to estimate structure weight should be used.

☞ Conclusions: (2)

- ➢ To realize a practical optimization tool, the Navier-Stokes equations with an appropriate turbulence model should be used.
- ➢ The necessary extensions to the software are currently in process:.

  Viscous Aerodynamic Shape Optimization of Wings including Planform Variables, K. Leoviriyakit, S. Kim, and A. Jameson, **21st Applied Aerodynamics Conference,Orlando, Florida/June 23–26, 2003.**

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