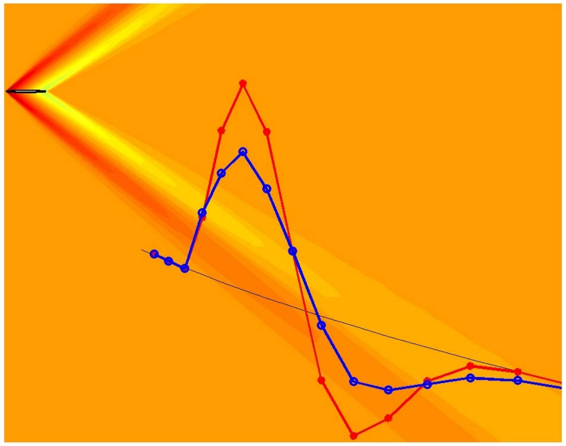
Studies of the Continuous and Discrete Adjoint Approaches to Viscous Automatic Aerodynamic Shape Optimization



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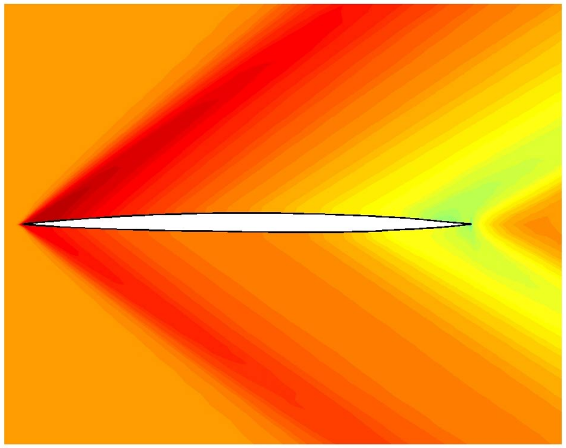

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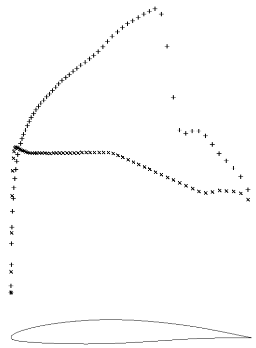
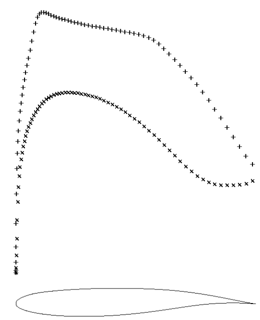
Introduction: CFD as a Design Tool

- **Surface Pressure Inverse Design**

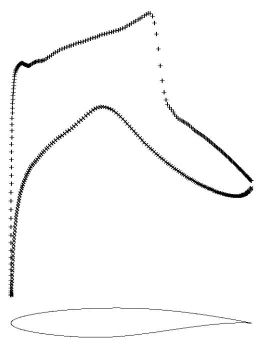


Initial Surface Pressure
Airfoil: NACA 64A410

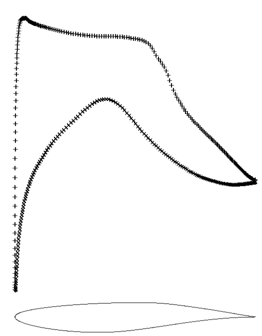


Target Surface Pressure
Airfoil: Korn

- **Drag Minimization**



Initial Surface Pressure
Airfoil: RAE 2822



Final Surface Pressure

Introduction: CFD as a Design Tool

Optimization

Gradient Based

Non Gradient Based

Finite Difference Method

- $\mathcal{G} = \dfrac{(I + \Delta I) - I}{\Delta \mathcal{F}}$

where, $I = (w, \mathcal{F})$ is the Cost Function

w = Flow Variable

$\mathcal{F}$ = Geometry Function

- N design variables $\longrightarrow$ N Flow Calculation

Control Theory Approach
Adjoint Method

- N design variables

$\downarrow$

1 Flow and
1 Adjoint Calculation

Objectives

- Review the formulation and development of the viscous adjoint equations for both the continuous and discrete approach.
- Investigate the differences in the implementation of boundary conditions for each method for various cost functions.
- Compare the gradients of the two methods to complex step gradients for inverse pressure design and drag minimization.
- Study the differences in calculating the exact gradient of the inexact cost function (discrete adjoint) or the inexact gradient of the exact cost function (continuous).

Overview of Adjoint Method

Let I be the **cost** (or **objective**) function and R the flow field equation

$$I = I(w, \mathcal{F})$$

$$R(w, \mathcal{F}) = 0$$

where

$$\begin{aligned} w &= \text{flow field variables} \\ \mathcal{F} &= \text{design variables} \end{aligned}$$

The **first variation** of the cost function and flow field equation are

$$\delta I = \frac{\partial I}{\partial w}^T \delta w + \frac{\partial I}{\partial \mathcal{F}}^T \delta \mathcal{F}$$

Here the variation δw of the flow variables will depend on the variation $\delta \mathcal{F}$ of the design variables through the variation of the flow equation.

$$\delta R = 0 = \left[\frac{\partial R}{\partial w}\right] \delta w + \left[\frac{\partial R}{\partial \mathcal{F}}\right] \delta \mathcal{F}$$

Overview of Adjoint Method

Introducing a **Lagrange Multiplier**, ψ, and using the **flow field equation** as a **constraint**

$$\begin{aligned}
\delta I &= \frac{\partial I}{\partial w}^T \delta w + \frac{\partial I}{\partial \mathcal{F}}^T \delta \mathcal{F} - \psi^T \left\{ \left[\frac{\partial R}{\partial w}\right] \delta w + \left[\frac{\partial R}{\partial \mathcal{F}}\right] \delta \mathcal{F} \right\} \\
&= \left\{ \frac{\partial I}{\partial w}^T - \psi^T \left[\frac{\partial R}{\partial w}\right] \right\} \delta w + \left\{ \frac{\partial I}{\partial \mathcal{F}}^T - \psi^T \left[\frac{\partial R}{\partial \mathcal{F}}\right] \right\} \delta \mathcal{F}
\end{aligned}$$

By choosing ψ such that it satisfies the **adjoint equation**

$$\left[\frac{\partial R}{\partial w}\right]^T \psi = \frac{\partial I}{\partial w},$$

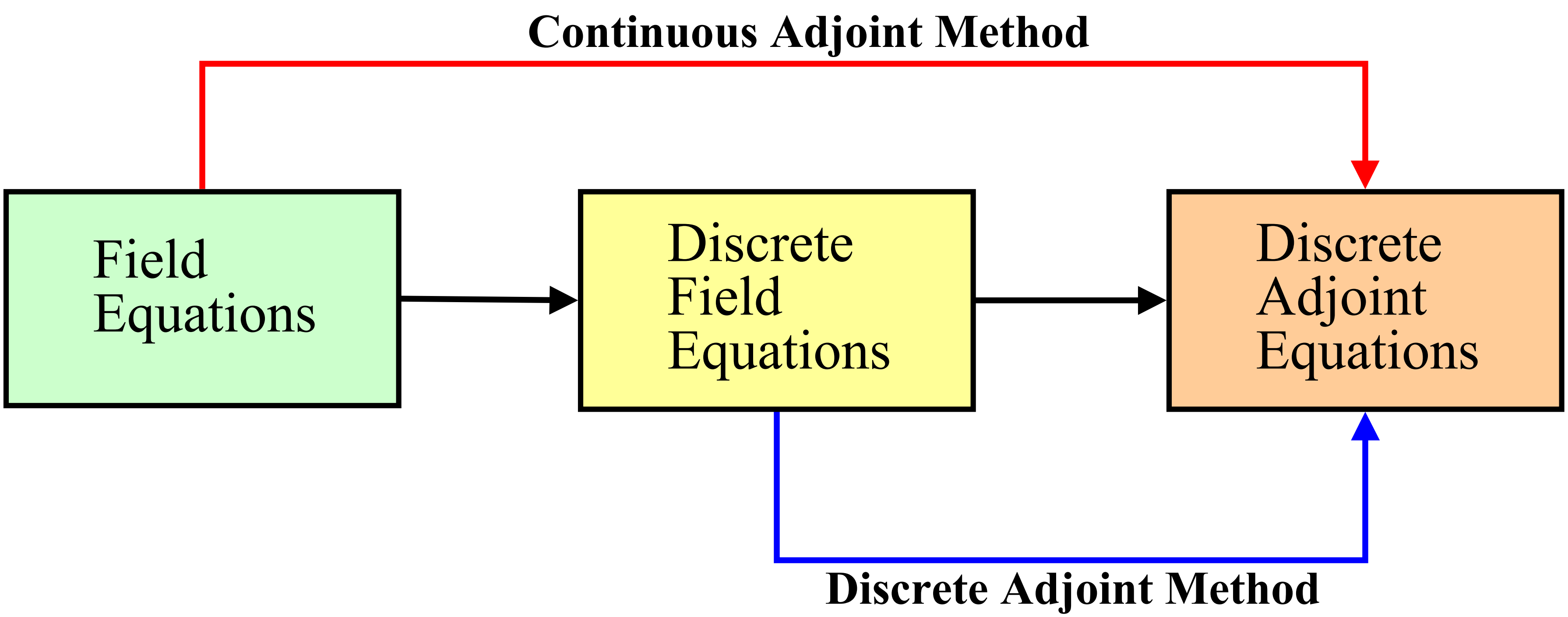
we have

$$\begin{aligned}
\delta I &= \left\{ \frac{\partial I}{\partial \mathcal{F}}^T - \psi^T \left[\frac{\partial R}{\partial \mathcal{F}}\right] \right\} \delta \mathcal{F} \\
&= \mathcal{G}^T \delta \mathcal{F}
\end{aligned}$$

Hence setting $\delta \mathcal{F} = -\lambda \mathcal{G}$ we get an improvement

$$\delta I = -\lambda \mathcal{G}^T \mathcal{G} < 0, \text{ unless } \mathcal{G} = 0$$

Adjoint Method



Continuous Adjoint Method

The Navier-Stokes equations in steady state,

$$R(w,\mathcal{F}) = \frac{\partial}{\partial \xi_k}\left(F_k - F_{v_k}\right) = 0$$

The first variation of the flow field equation is

$$\delta R(w,\mathcal{F}) = \frac{\partial}{\partial \xi_k}\delta\left(F_k - F_{v_k}\right) = 0$$

Then,

$$\int_{\mathcal{D}} \psi^T \frac{\partial}{\partial \xi_k}\delta\left(F_k - F_{v_k}\right) d\mathcal{D} = 0.$$

Integration by parts,

$$-\int_{\mathcal{D}} \frac{\partial \psi^T}{\partial \xi_k}\delta\left(F_k - F_{v_k}\right) d\mathcal{D} + \int_{\mathcal{B}} n_k \psi^T \delta\left(F_k - F_{v_k}\right) d\mathcal{B} = 0,$$

Continuous Adjoint Method

The first variation of the cost function,

$$\delta I = \int_{B_W} (p - p_d)\, \delta p \;\; ds + \frac{1}{2} \int_{B_W} (p - p_d)^2\, \delta ds.$$

The variation of the cost function is added to the variation of the flow field equation,

$$\begin{aligned} \delta I \;=\; & \int_{B_W} (p - p_d)\, \delta p \;\; ds + \frac{1}{2} \int_{B_W} (p - p_d)^2\, \delta ds \\ & - \int_{\mathcal{D}} \frac{\partial \psi^T}{\partial \xi_k} \delta \left(F_k - F_{v_k}\right) d\mathcal{D} + \int_{\mathcal{B}} n_k \psi^T \delta \left(F_k - F_{v_k}\right) d\mathcal{B}. \end{aligned}$$

Collect δw terms,

$$Continuous\ Adjoint\ equation: \frac{\partial \left(F_k - F_{v_k}\right)^T}{\partial w} \frac{\partial \psi}{\partial \xi_k} = 0$$

$$Continuous\ Adjoint\ Boundary\ Condition: n_j \psi_j = p - p_d$$

Discrete Adjoint Method

The discrete Navier-Stokes equations in steady state,

$$R_{i,j} = h_{i+\frac{1}{2},j} - h_{i-\frac{1}{2},j} + h_{i,j+\frac{1}{2}} - h_{i,j-\frac{1}{2}}$$

The first variation of the flow solver is,

$$\delta R_{i,j} = \delta h_{i+\frac{1}{2},j} - \delta h_{i-\frac{1}{2},j} + \delta h_{i,j+\frac{1}{2}} - \delta h_{i,j-\frac{1}{2}}$$

Then,

$$\sum_i \sum_j \psi_{i,j}^T \delta R_{i,j} = \sum_i \sum_j \psi_{i,j}^T \delta \left(Q_{i,j} + D_{i,j} + V_{i,j} \right) = 0$$

The discrete cost function,

$$I = \frac{1}{2} \sum_i (p - p_d)^2 ds_i,$$

The variation of the cost function is added to the variation of the flow solver,

$$\delta I = \sum_i (p - p_d) ds_i + \sum_i \sum_j \psi_{i,j}^T \delta \left(Q_{i,j} + D_{i,j} + V_{i,j} \right)$$

Adjoint Method for the Calculation of Remote Sensitivities

Continuous Adjoint Boundary Condition

$$
\begin{aligned}
\psi_{1_{i,1}} &= \psi_{1_{i,2}} \\
\psi_{2_{i,1}} &= \psi_{2_{i,2}} + 2n_2\left((p-p_d) - n_2\psi_{2_{i,2}} + n_1\psi_{3_{i,2}}\right) \\
\psi_{3_{i,1}} &= \psi_{3_{i,2}} - 2n_1\left((p-p_d) - n_2\psi_{2_{i,2}} + n_1\psi_{3_{i,2}}\right) \\
\psi_{4_{i,1}} &= \psi_{4_{i,2}}
\end{aligned}
$$

Discrete Adjoint Boundary Condition

$$
V\frac{\partial\psi_{i,2}}{\partial t} = \frac{1}{2}\left[-A^T_{i-\frac{1}{2},2}\left(\psi_{i,2}-\psi_{i-1,2}\right) - A^T_{i+\frac{1}{2},2}\left(\psi_{i+1,2}-\psi_{i,2}\right)\right] + \frac{1}{2}\left[-B^T_{i,\frac{5}{2}}\left(\psi_{i,3}-\psi_{i,2}\right)\right] + \Phi
$$

where Φ is the source term for inverse design,

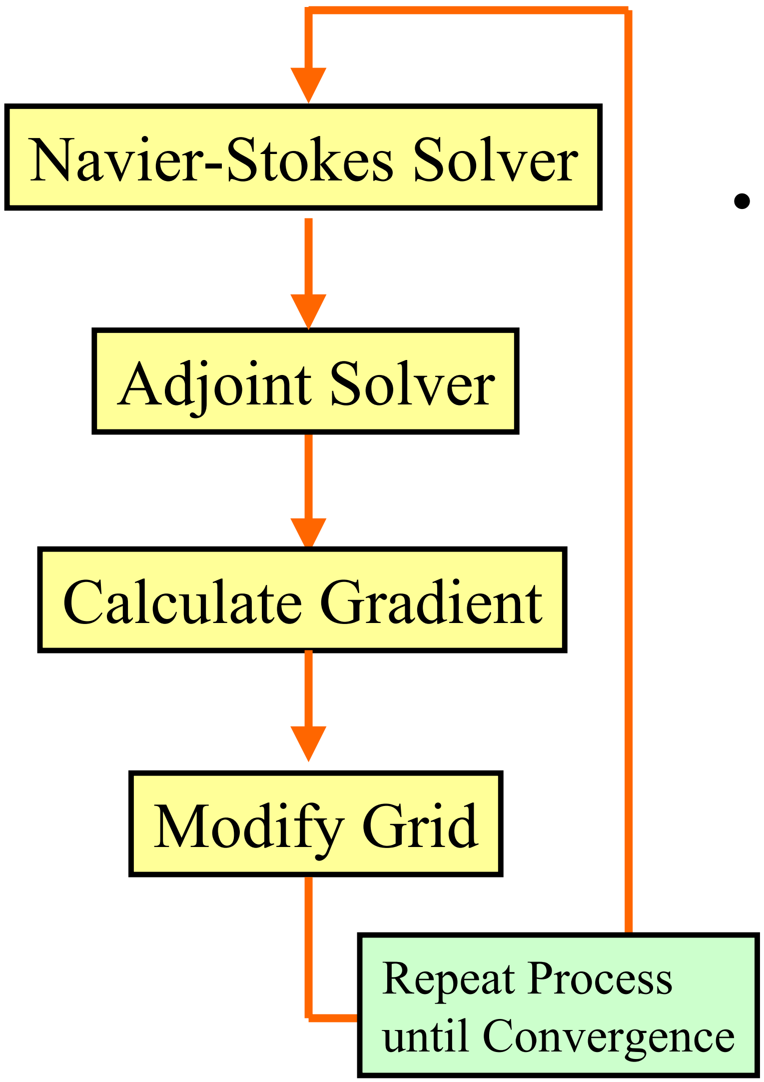
$$
\Phi = \left(-\Delta y_\xi\psi_{2_{i,2}} + \Delta x_\xi\psi_{3_{i,2}} - (p-p_T)\Delta s_i\right)\delta p_{i,2}
$$

$$
\lim_{\Delta x\to 0,\ \Delta t\to 0} \textbf{Discrete Adjoint BC} \Longrightarrow \textbf{Continuous Adjoint BC}
$$

Adjoint Boundary Conditions for Various Cost Functions

Boundary Condition	Continuous Adjoint Boundary Condition	Discrete Adjoint Boundary Condition
Inverse Design	$\phi_k n_k = p - p_d$	$\Phi_{inv} = \left(-\Delta y_\xi \psi_{2_{i,2}} + \Delta x_\xi \psi_{3_{i,2}} - (p - p_T)\Delta s_i\right) \delta p_{i,2}$
Pressure Drag Minimization	$\phi_k n_k = -\frac{1}{\frac{1}{2}\gamma P_\infty M_\infty^2} \begin{bmatrix} \cos\alpha \\ \sin\alpha \end{bmatrix} n_k$	$\Phi_{pressure\ drag} = Refer\ to\ Paper$
Skin Friction Drag Minimization	$\phi_k = -\frac{1}{\frac{1}{2}\gamma P_\infty M_\infty^2} \begin{bmatrix} \cos\alpha \\ \sin\alpha \end{bmatrix}$	$\Phi_{skin\ friction\ drag} = Refer\ to\ Paper$
Total Drag Minimization	$\phi_k = -\frac{1}{\frac{1}{2}\gamma P_\infty M_\infty^2} \begin{bmatrix} \cos\alpha \\ \sin\alpha \end{bmatrix}$	$\Phi_{total} = \Phi_{pressure\ drag} + \Phi_{skin\ friction\ drag}$
Remote Inverse Design	$(\phi_k n_k)_{NF} = (p - p_d)_{NF}$	$\Phi_{NF} = -(p - p_T)\Delta s_i \delta p_{i,NF}$

Design Procedure



- **FLO103 Navier-Stokes Solver**
  - » Modified Runge-Kutta Explicit Time Stepping
  - » Jameson-Schmidt-Turkel (JST) Scheme for Artificial Dissipation
  - » Local Time Stepping, Implicit Resdiual Smoothing, and Multigrid.

Optimization Procedure

Let $\mathcal{F}$ represent the design variable, and $\mathcal{G}$ the gradient. An improvement can then be made with a shape change

$$\delta\mathcal{F} = -\lambda\mathcal{G},$$

The gradient $\mathcal{G}$ can be replaced by a smoothed value $\overline{\mathcal{G}}$ in the descent process. This ensures that each new shape in the optimization sequence remains smooth and acts as a preconditioner which allows the use of much larger steps.

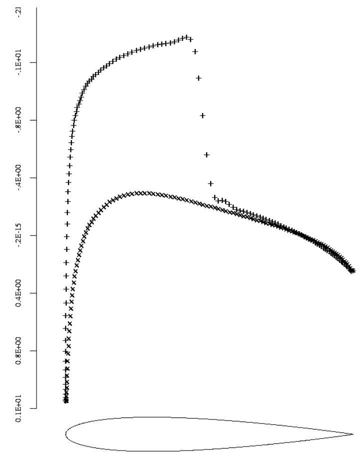
$$\overline{\mathcal{G}} - \frac{\partial}{\partial\xi_1}\epsilon\frac{\partial}{\partial\xi_1}\overline{\mathcal{G}} = \mathcal{G}$$

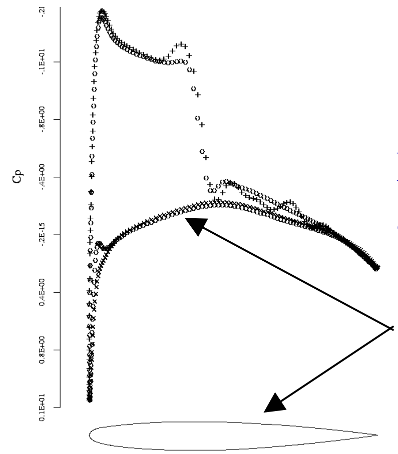
where ϵ is the smoothing parameter.

Inverse Design of NACA 0012 to Onera M6 at Fixed Cl

(Medium Grid - 512x64, M = 0.75, Cl = 0.65)

Initial Pressure Distribution of NACA 0012



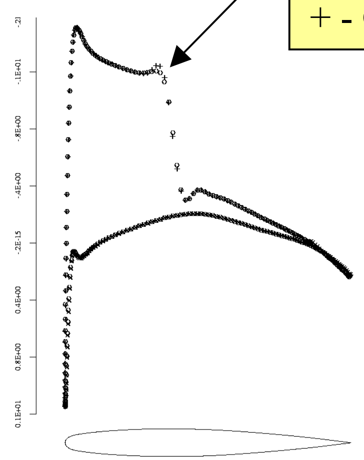


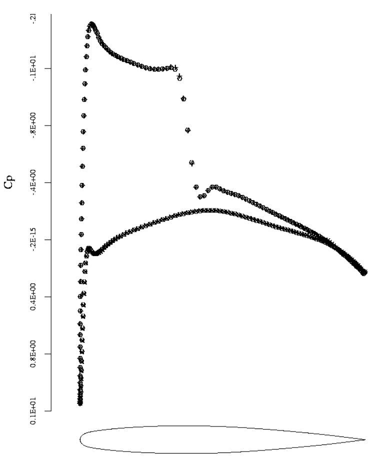
Pressure Distribution after 4 Design Cycles

General Shape of Target Airfoil is Achieved

o - Target Pressure
+ - Current Pressure

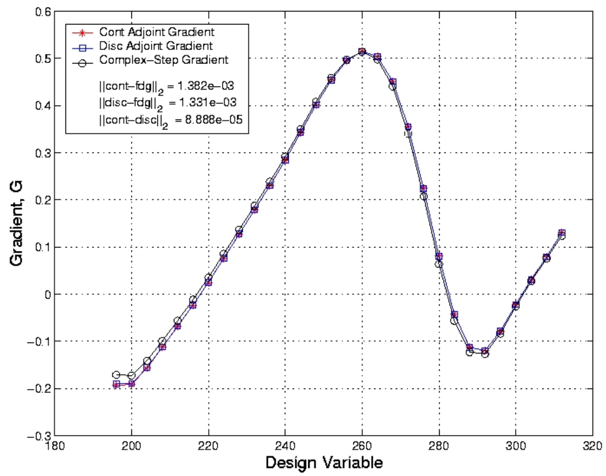
Pressure Distribution after 50 Design Cycles



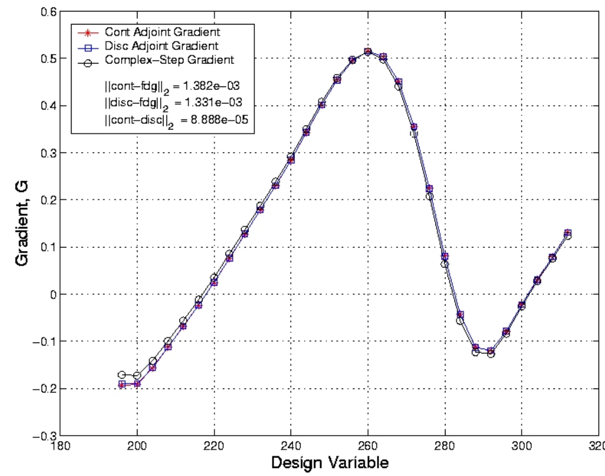


Pressure Distribution after 100 Design Cycles

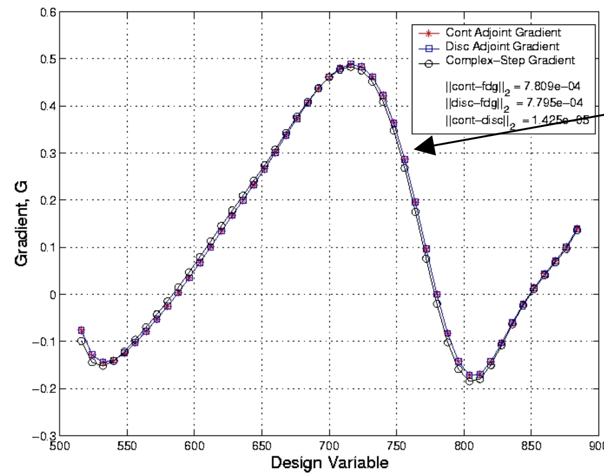
Adjoint Versus Complex-Step Gradients for Inverse Design
(RAE to NACA 64A410, M = 0.75, Fixed Cl = 0.65)



Coarse Grid - 384 x 64



Medium Grid - 512 x 64



Fine Grid - 1024 x 64

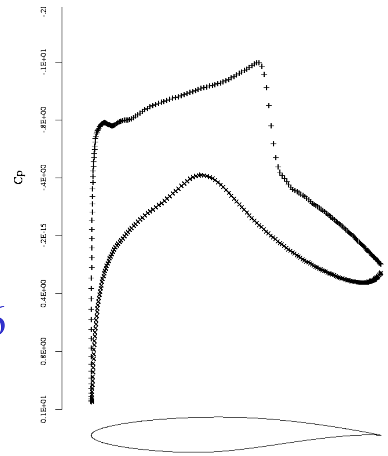
* Continuous Adjoint
□ Discrete Adjoint
o Complex-Step

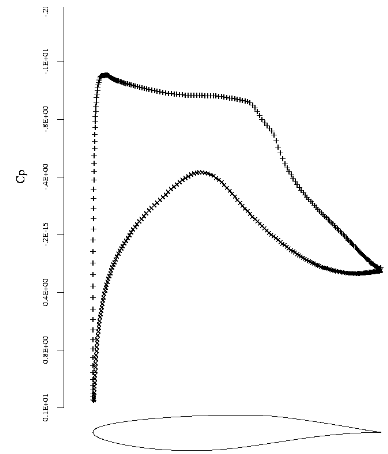
Continuous Adjoint: Drag Minimization of RAE 2822 at Fixed Cl

(Medium Grid - 512x64, M = 0.75, Cl = 0.65, AOA = 1 degree)

Initial Pressure
Distribution of
RAE 2822

Skin Friction = 0.0056
Total Drag = 0.0148



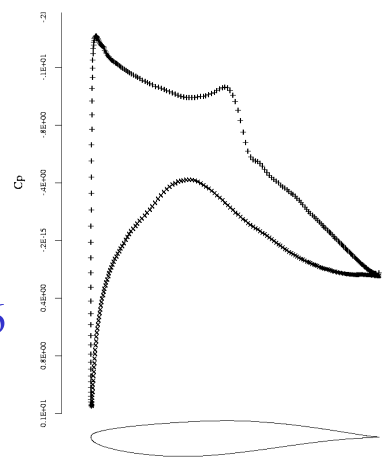


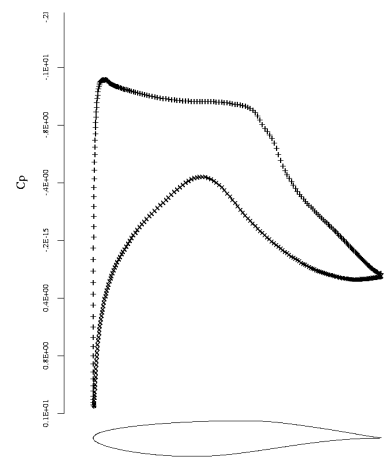
Final Design
Pressure Drag Min

Skin Friction = 0.0057
Total Drag = 0.0098

Final Design
Skin Friction
Drag Min

Skin Friction = 0.0056
Total Drag = 0.0104





Final Design
Total Drag Min

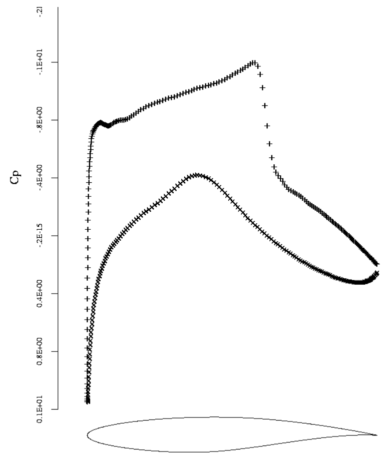
Skin Friction = 0.0057
Total Drag = 0.0098

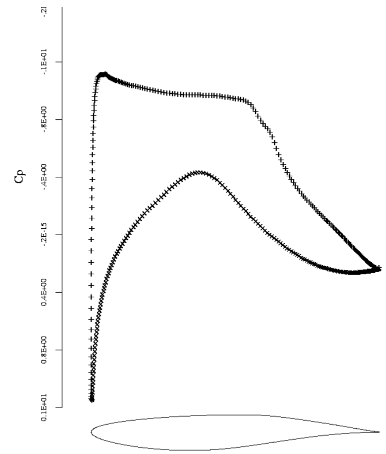
Discrete Adjoint: Drag Minimization of RAE 2822 at Fixed Cl

(Medium Grid - 512x64, M = 0.75, Cl = 0.65, AOA = 1 degree)

Initial Pressure Distribution of RAE 2822

Skin Friction = 0.0056
Total Drag = 0.0148



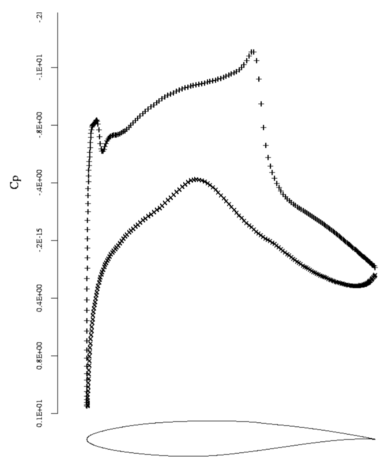


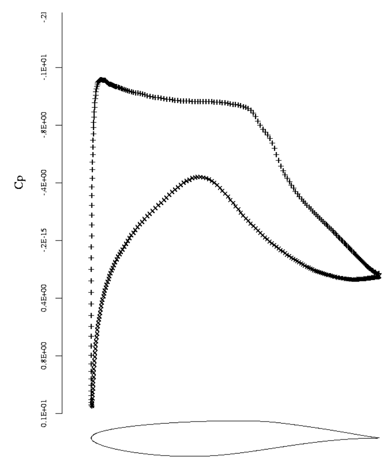
Final Design Pressure Drag Min

Skin Friction = 0.0058
Total Drag = 0.0099

Final Design Skin Friction Drag Min

Skin Friction = 0.0054
Total Drag = 0.0188

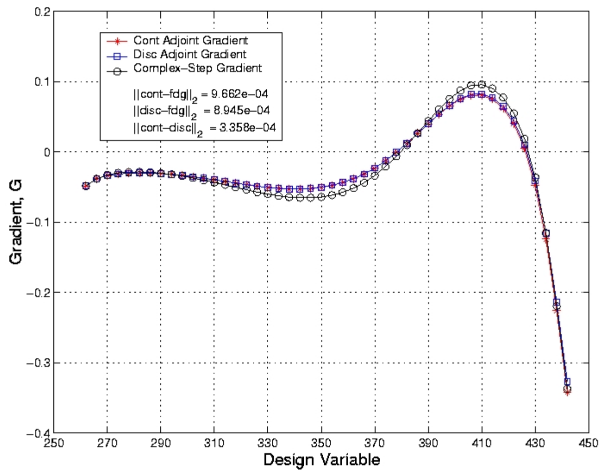




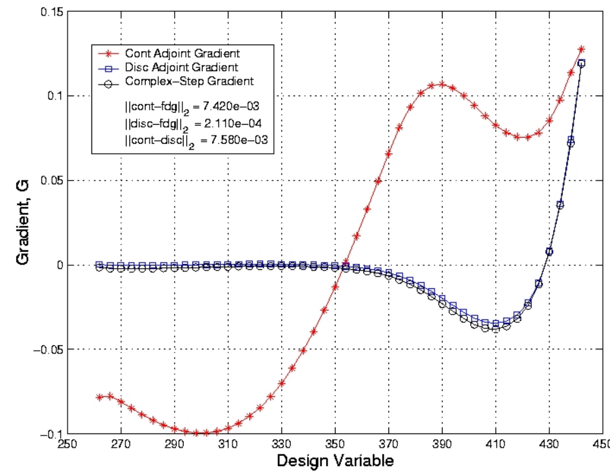
Final Design Total Drag Min

Skin Friction = 0.0057
Total Drag = 0.0098

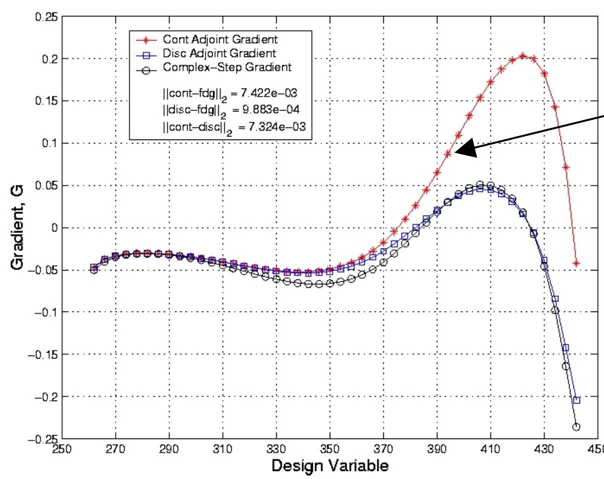
Adjoint Versus Complex-Step Gradients for Drag Minimization
(RAE Airfoil, M = 0.75, Fixed Cl = 0.65)



Pressure Drag Minimization



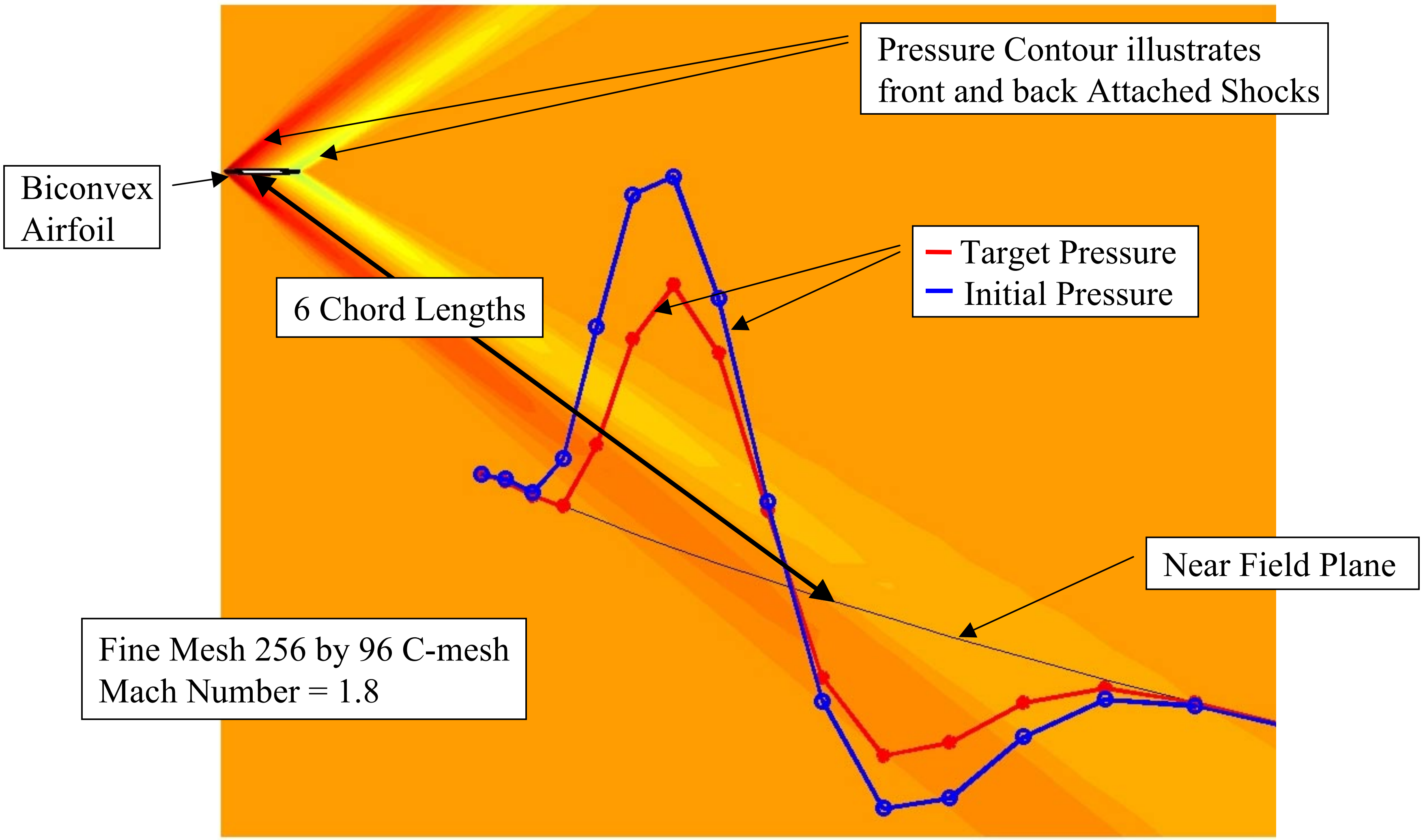
Skin Friction Drag Minimization



Total Drag Minimization

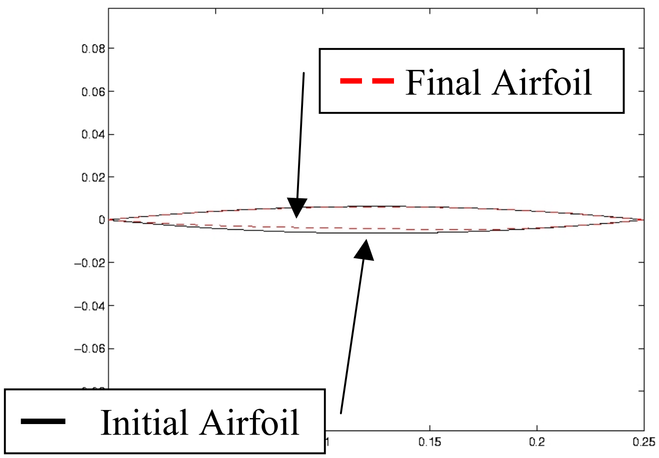
* Continuous Adjoint
□ Discrete Adjoint
o Complex-Step

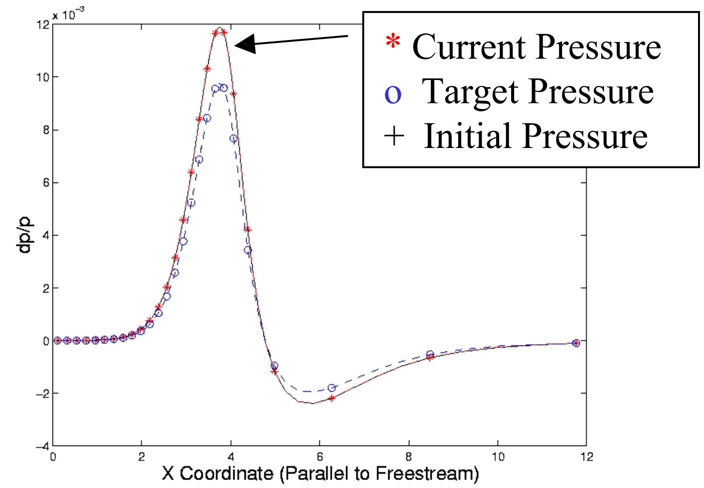
Geometry and Near Field Plane Description

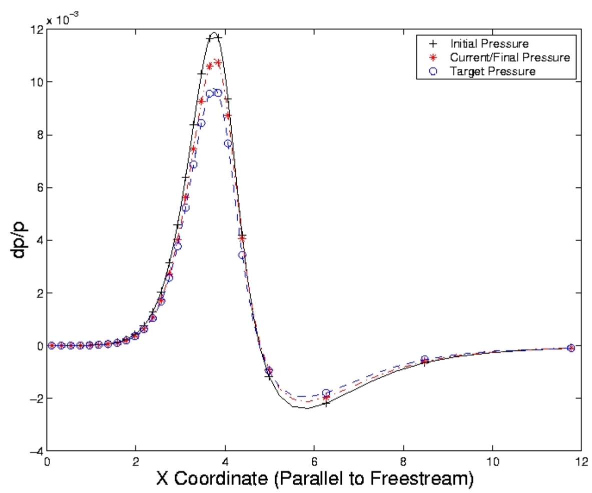


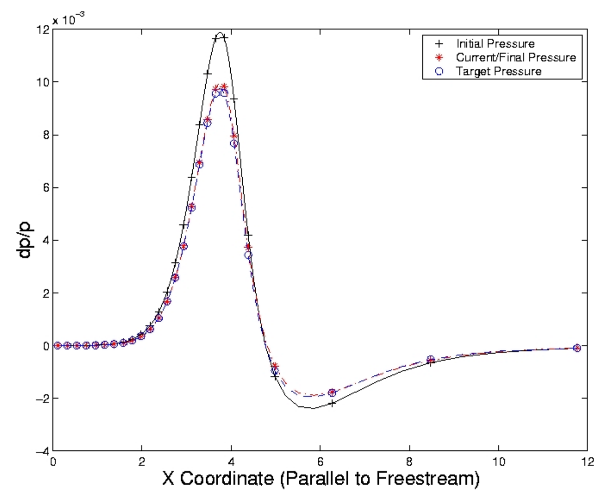
Adjoint Versus Complex-Step Gradients for Drag Minimization

(RAE Airfoil, M = 0.75, Fixed Cl = 0.65)









Conclusions and Future Work

- The continuous adjoint boundary condition appears as an update in contrast to the discrete adjoint which appears as a source term in the adjoint fluxes. As the mesh is reduced, the continuous adjoint boundary condition is recovered from the discrete adjoint boundary condition.
- The viscous continuous adjoint skin friction minimization boundary condition does not provide the right gradients. It appears that the extrapolation of the first and fourth multipliers, as used in this work, is not adequate. The discrete version does.
- Discrete adjoint gradients have better agreements with complex-step gradients
- The difference between the continuous and discrete adjoint gradients reduce as the mesh size increases.
- The discrete adjoint may provide a route to improving the boundary conditions for the continuous adjoint for viscous flows.
- The best compromise may be to use the continuous adjoint formulations in the interior of the domain and the discrete adjoint boundary condition.

Conclusions and Future Work

- The discrete adjoint may provide a route to improving the boundary conditions for the continuous adjoint for viscous flows.

- The best compromise may be to use the continuous adjoint formulations in the interior of the domain and the discrete adjoint boundary condition.