# Design Optimization of Multi–Element Aerodynamic Configurations Using a Viscous Adjoint Method

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**An adjoint-based Navier-Stokes design and optimization method for multi-element aerodynamic configurations is derived and presented. The compressible Reynolds-Averaged Navier-Stokes (RANS) equations are used as a flow model together with the Spalart-Allmaras turbulence model to account for high Reynolds number effects. Using a viscous continuous adjoint formulation, the necessary aerodynamic gradient information is obtained with large computational savings over traditional finite-difference methods. A study of the accuracy of the gradient information provided by the adjoint method in comparison with finite differences and an inverse design of a single-element airfoil are also presented for validation of the present viscous adjoint method. The multi-element aerodynamic configuration design method uses a compressible RANS flow solver, a point-to-point matched multi-block grid system and the Message Passing Interface (MPI) parallel solution methodology for both the flow and adjoint calculations. As a sample design example for this abstract paper, $C_{l_{max}}$ maximization problem for three-element high-lift airfoil configuration, denoted as 30P30N, is presented. The final paper will present the works that focus on expanding the result of this abstract paper and on utilizing the method to perform realistic multi-element configuration designs.**

## Introduction

AERODYNAMIC shape design has long been a challenging objective in the study of fluid dynamics. Computational Fluid Dynamics (CFD) has played an important role in the aerodynamic design process since its introduction for the study of fluid flow. However, CFD has mostly been used in the analysis of aerodynamic configurations in order to aid in the design process rather than to serve as a direct design tool in aerodynamic shape optimization. Although several attempts have been made in the past to use CFD as a direct design tool,[1–5] it has not been until recently that the focus of CFD applications has shifted to aerodynamic design.[6–11] This shift has been mainly motivated by the availability of high performance computing platforms and by the development of new and efficient analysis and design algorithms. In particular, automatic design procedures which use CFD combined with gradient-based optimization techniques, have made it possible to remove difficulties in the decision making process faced by the aerodynamicist.

Typically, in gradient-based optimization techniques, a control function to be optimized (an airfoil shape, for example) is parameterized with a set of design variables, and a suitable cost function to be minimized or maximized is defined (drag coefficient, lift/drag ratio, difference from a specified pressure distribution, etc.) Then, a constraint, the governing equations in the present study, can be introduced in order to express the dependence between the cost function and the control function. The sensitivity derivatives of the cost function with respect to the design variables are calculated in order to get a direction of improvement. Finally, a step is taken in this direction and the procedure is repeated until convergence to a minimum or maximum is achieved. Finding a fast and accurate way of calculating the necessary gradient information is essential to developing an effective design method since this can be the most time consuming portion of the design algorithm. Gradient information can be computed using a variety of approaches, such as the finite-difference method, the complex step method[12] and automatic differentiation.[13] Unfortunately, their computational cost is proportional to the number of design variables in the problem.

As an alternative choice, the control theory approach has dramatic computational cost advantages when compared to any of these methods. The foundation of control theory for systems governed by partial differential equations was laid by J.L. Lions.[14] The control theory approach is often called the adjoint method, since the necessary gradients are obtained via the solution of the adjoint equations of the governing equations of interest. The adjoint method is extremely efficient since the computational expense incurred in

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the calculation of the complete gradient is effectively *independent* of the number of design variables. The only cost involved is the calculation of *one* flow solution and *one* adjoint solution whose complexity is similar to that of the flow solution. Control theory was applied in this way to shape design for elliptic equations by Pironneau[15] and it was first used in transonic flow by Jameson.[6,7,16] Since then this method has become a popular choice for design problems involving fluid flow .[9,17–19] In fact, the method has even been successfully used for the aerodynamic design of complete aircraft configurations.[8,20]

Most of the early work in the formulation of the adjoint-based design framework used the potential and Euler equations as models of the fluid flow. Aerodynamic design calculations using the Reynolds-Averaged Navier-Stokes equations as the flow model have only recently been tackled. The extension of adjoint methods for optimal aerodynamic design of viscous problems is necessary to provide the increased level of modeling which is crucial for certain types of flows. This cannot only be considered an academic exercise. It is also a very important issue for the design of viscous dominated applications such as the flow in high-lift systems. In 1997, a continuous adjoint method for Aerodynamic Shape Optimization (ASO) using the compressible Navier-Stokes equations was formulated and it has been implemented directly in a three dimensional wing problem .[16,21] In 1998, an implementation of three-dimensional viscous adjoint method was used with some success in the optimization of the Blended-Wing-Body configuration.[22] Since these design calculations were carried out without the benefit of a careful check on the accuracy of the resulting gradient information, a series of numerical experiments in two dimensions that assessed the accuracy of the viscous adjoint gradient information were conducted by the authors.[23]

In this work, our viscous adjoint method is applied to multi-element aerodynamic configuration designs, removing the limitations on the dimensionality of the design space by making use of the viscous adjoint design methodology. In addition to difficulties involved in the prediction of complex flow physics, multi-element aerodynamic configuration provide an additional challenge to the adjoint method: the effect of the changes in the shape of one element must be felt by the other elements in the system. While preliminary studies of the adjoint method in such a situation have already been carried out,[19,24] this research is designed to validate the adjoint method for complex applications of this type.

## Procedure

In this section we outline the overall design procedure used for a design calculation that will be presented later.

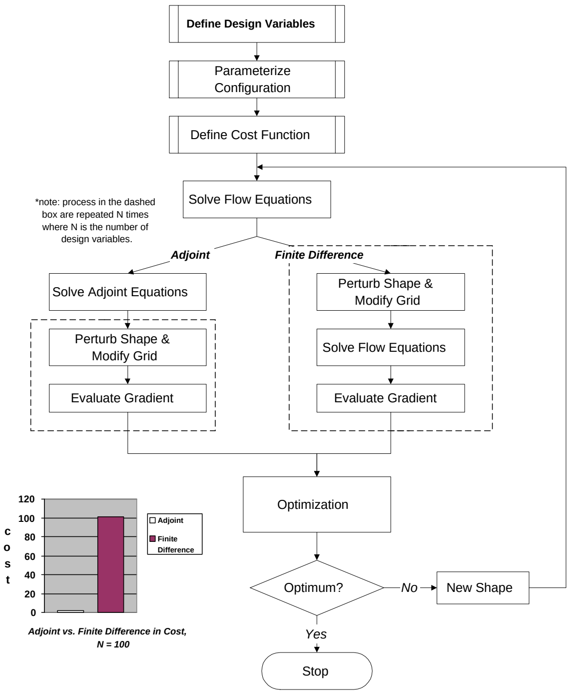

**Fig. 1 Flowchart of the Design Process**

In practical implementations of the adjoint method, a design code can be modularized into several components such as the flow solver, adjoint solver, geometry and mesh modification algorithms, and the optimization algorithm. After parameterizing the configuration of interest using a set of design variables and defining a suitable cost function, which is typically based on aerodynamic performance, the design procedure can be described as follows. First solve the flow equations for the flow variables, then solve the adjoint equations for the costate variables subject to appropriate boundary conditions which will depend on the form of the cost function. Next evaluate the gradients and update the aerodynamic shape based on the direction of steepest descent. Finally repeat the process to attain an optimum configuration. A summary of the design process and a comparison with the finite-difference method are illustrated in Figure 1. Each of the items of the procedure will be fully explained in the final paper. The next section shows a high-lift system design as an example of the multi-element configuration design.

## Results

Figure 2 shows a typical multi-element airfoil configurations. The rigging quantities that describe the relative element positioning of the slat, main element and flap can also be easily seen in Figure 2.

### *Multi-Element Airfoil Results*

Using the newly developed viscous adjoint procedure, the multi-block flow and adjoint solvers, and the lessons learned in the previous design examples, we can now attempt to redesign the $30P30N$ multi-element airfoil to optimize its value of $C_{l_{max}}$. In this

**Fig. 2 Definitions of Gap, Overlap, and Deflection Angles**

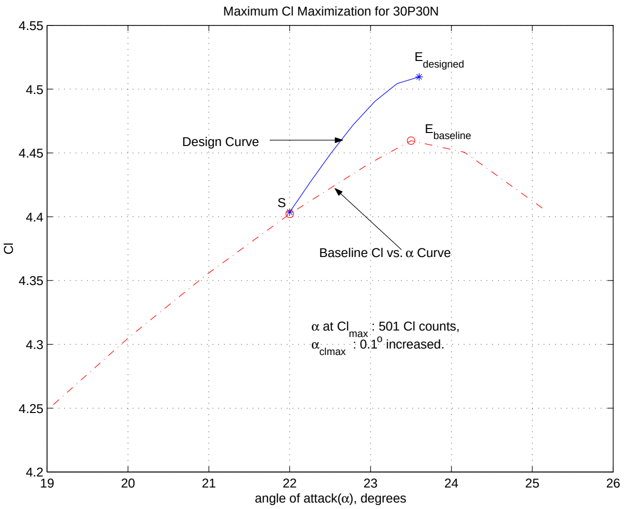

**Fig. 3 Design Curve of the 30P30N $C_{l_{max}}$ Maximization. $M_\infty = 0.20$, $Re = 9 \times 10^6$.**

case, a total of 157 design variables are used, including 50 Hicks-Henne bump functions on each of the three elements, 3 rigging variables for each the slat and flap components, and the angle of attack ($\alpha$) of the complete configuration.

As shown in Figure 3, the design started at $\alpha = 22°$ which is near the $\alpha_{clmax}$ of the baseline $30P30N$ configuration and the baseline was modified in the direction of $C_l$ improvement using all of the design variables. As shown in Figure 4 and Figure 4, $C_{l_{max}}$ improved by 1.12%, 501 counts, increasing from 4.4596 to 4.5097, with a slight change (0.43%) in $\alpha_{clmax}$.

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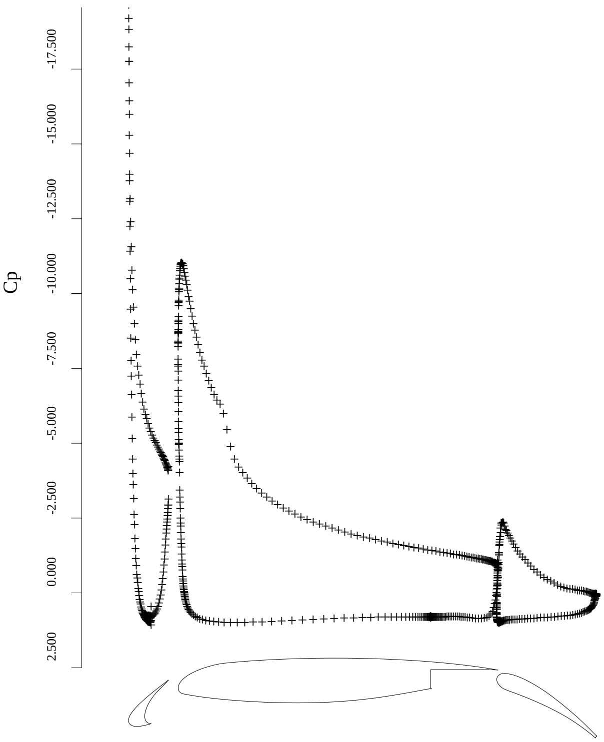
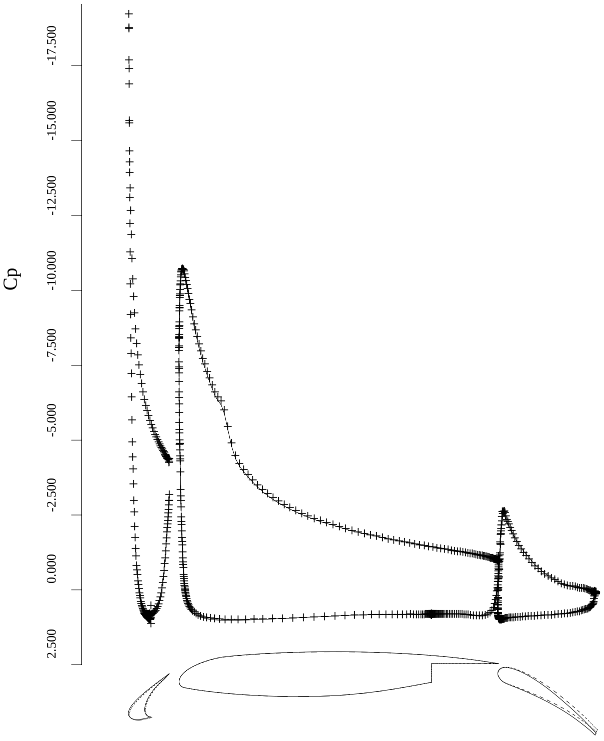


**a)** Baseline, $C_l = 4.4596$, $C_d = 0.1062$, $M_\infty = 0.2$, $\alpha = 23.5°$, $Re = 9 \times 10^6$.



**b)** 6 Design Iterations, $C_l = 4.5097$, $C_d = 0.1236$, $M_\infty = 0.2$, $\alpha = 23.601°$, $Re = 9 \times 10^6$.

**Fig. 4 Multi-Element Airfoil $C_{l_{max}}$ Maximization Calculation for the 30P30N. + Current $C_p$, —— Initial $C_p$. ——— Current Airfoil, – – – Initial Airfoil.**

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