PROBLEM SET 1

Due Date: January 25th, 2005

Problem 1. NACA 4-series Airfoils. The camberline of the NACA four-digit series of airfoils is given by the following two parabolas

\[
\bar{Y}(x) = \begin{cases} 
\frac{e x^2 p}{c (2p - \frac{x}{c})} & \text{for } 0 < \frac{x}{c} < p \\
\frac{e (c-x)}{(1-p)^2} (1 + \frac{x}{c} - 2p) & \text{for } p < \frac{x}{c} < 1
\end{cases}
\]

where the first digit of the airfoil designation is the maximum camber ratio, \( \epsilon \) times 100, the second digit is the chordwise location of the maximum camber, \( p \) times 10, and the last two digits are 100 times the thickness ratio \( \tau \). The thickness distribution is given by

\[
T(x) = 10\tau c \left[ 0.2969 \sqrt{x/c - 0.126(x/c) - 0.3537(x/c)^2 + 0.2843(x/c)^3 - 0.1015(x/c)^4} \right]
\]

Find and plot \( c_l \) and \( c_{mac} \) as a function of the angle of attack for the NACA 3412 member of the series (you can work out a general formula based on \( \tau \), \( \epsilon \), and \( p \)) showing all derivations.

Problem 2. Airfoil with Parabolic Camber. Consider an airfoil with a parabolic camber line given by

\[
\bar{Y} = 4\epsilon \frac{x}{c} (c - x).
\]

Find out expressions for \( L' \), \( M' \) and the location of the center of pressure, \( x_{c.p.} \), as a function of the angle of attack and the amount of maximum camber. Is the location of the center of pressure independent of the angle of attack of the airfoil? Where is the aerodynamic center located? Provide expressions for the \( C_l \) and \( C_m \) (about the quarter chord) for this airfoil.

Problem 3. Hess and Smith Panel Method. Write a computer program that implements the Hess-Smith panel method and use it to plot the \( c_l \) vs. \( \alpha \) curve and calculate the \( c_{mac} \) for the NACA 3412 airfoil of the first problem. Compare your results with those of thin airfoil theory.

The implementation suggested by the accompanying MATLAB files consists of following modules:

1. Geometry generation module: generates a surface panelization for an arbitrary NACA 4-Series airfoil with the appropriate bunching. **PROVIDED**.

2. Panel geometry module: sets up the necessary panels, their orientations, lengths, \( \sin \theta_j \), and \( \cos \theta_j \), and the midpoints of each panel \( \bar{x}_j, \bar{y}_j \). **PROVIDED**.

3. Influence coefficient matrix computation for both the flow tangency boundary condition and the vortex strength equation.
4. Solution of the resulting linear system of equations: at the moment, this is achieved by matrix inversion, but you can use more sophisticated routines in MATLAB.

5. Computation of surface velocities and coefficients of pressure from resulting source and vortex distributions. PROVIDED.

6. Force coefficient computation.

In your writeup, answer the following questions:

1. What is a reasonable number of panels to use in these computations? Can you plot the error in $c_l$, $c_d$, and $c_{mac}$ as the number of panels increases?

2. Where do the main sources of error come from?

3. Using a reasonable number of panels, does the output of your code follow the expected trends with varying angle of attack? (Discuss trends in $c_l$, $c_d$, and $c_{mac}$).

**Problem 4.** EXTRA CREDIT / OPTIONAL. 15 points. Analysis of 30P30N multi-element airfoil. From the course web page you can download the coordinates of a three-element airfoil (main element, slat and flap). This optional assignment asks you to modify your Hess-Smith panel code to treat multi-element airfoils (you will have two add two more equations for vortex strength and two more Kutta conditions) and to analyze the flow (at the provided flap and slat settings) at an angle of attack of $15^\circ$. You will then compare your results to the Reynolds-Averaged Navier-Stokes solutions provided.