ADAPTIVE WALL FUNCTIONS WITH APPLICATIONS

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ABSTRACT

A novel wall function formulation for Spalart-Allmaras and $k$-$\omega$ model derived for the flow over a flat plate at zero-pressure gradient is applied to complex flows: the flow over a ramp, flow over the RAE2822 airfoil and ONERA M6 wing. The performance of the proposed adaptive wall functions is investigated in the presence of regions of recirculating flow, strong pressure gradients, stagnation regions and shocks. It is demonstrated that the proposed wall functions are indeed adaptive and that, despite the simplicity of the approach, the flow features are captured on coarse grids with sufficient accuracy.

KEYWORDS

Adaptive wall functions, RANS, Spalart-Allmaras, $k$-$\omega$, recirculating flow, pressure gradient

NOMENCLATURE

$U$  
tagential velocity component
$y$  
wall normal coordinate
$u_\tau$  
friction velocity
$\mu, \mu_t$  
dynamic molecular and turbulent viscosities
$\nu, \nu_t$  
kinematic molecular and turbulent viscosities
$\rho$  
density
$\tau_w$  
wall shear stress
$\theta$  
momentum thickness
$Re_\theta = U_\infty \theta / \nu$  
momentum thickness based Reynolds number
INTRODUCTION

Recently, we have proposed an efficient, accurate and robust approach for adaptive wall functions that is applicable to different RANS turbulence models [1]. The proposed wall functions are formulated for zero pressure gradient flows. They are based on look-up tables for the turbulence quantities and the friction velocity $u_r$. These tables are built upon well-resolved, grid-converged numerical solutions obtained with the particular turbulence model. The boundary conditions derived from these tables are valid for the viscous, logarithmic as well as the intermediate (buffer) region. The friction velocity $u_r$ is computed explicitly avoiding a costly iterative method [1].

The validation of the proposed adaptive wall functions was performed for the flow over a flat plate. In [1], a detailed analysis of the discrepancy between the coarse grid wall function solutions and the corresponding wall integration was performed. A method that allows to isolate the errors resulting from the modeling of the physics was proposed. It is based on a computational grid that is identical to the wall integration grid but shifted for a distance $\delta$ from the wall. This eliminates the numerical errors caused by the coarseness of the wall functions grid in the near-wall region. Indeed, this method confirms the validity of the proposed boundary conditions based on look-up tables. The discrepancies on the coarse grid are a consequence of numerical errors.

In the present paper we further investigate this adaptive wall function concept for the Spalart-Allmaras [2] and $k$-$\omega$ model [3]. The wall functions are applied to complex flows about curved geometries that include pressure gradient and convection effects: incompressible flow over a ramp that includes a recirculation region and transonic flows over the RAE2822 airfoil and the ONERA M6 wing that include stagnation regions and shock-boundary layer interaction. A grid-dependence study demonstrates the adaptivity of the wall functions. Comparison with wall integration results and experimental data as well as computational cost comparisons provide a qualitative assessment of the proposed wall functions for industrial application.

ADAPTIVE WALL FUNCTIONS

In the case of turbulent flow of incompressible fluid with constant molecular viscosity, the velocity profile in the boundary layer can be split in three distinguished regions: the viscous sublayer, the logarithmic layer and
the defect layer. The location of the outer edge of the logarithmic layer depends on the Reynolds number, i.e. the extent of the logarithmic layer grows with increasing Reynolds number. In a quasi-equilibrium boundary layer (e.g. flow over a flat plate at zero pressure gradient), the region between the wall and the outer edge of the logarithmic layer is universal; i.e. the profiles of the flow variables collapse when scaled with the friction velocity \( u_\tau \) and molecular viscosity \( \nu \). This universality allows the derivation of wall functions.

Near the wall, derivatives in streamwise direction can be neglected and the flow and turbulence variables depend only on the coordinate \( y \) which is directed normal to the wall. Essentially, this is a turbulent Couette flow approximation. To derive solutions for the viscous sublayer and logarithmic layer, the equations are recast in non-dimensional form. For flow over a flat plate at zero pressure gradient the RANS equations simplify in the region between the wall and the outer edge of the logarithmic layer to:

\[
\frac{d}{dy} \left( (\mu + \mu_t) \frac{dU}{dy} \right) = 0 \quad (1)
\]

Integration along the wall normal coordinate \( y \) yields:

\[
(\mu + \mu_t) \frac{dU}{dy} = \rho u_\tau^2 \quad (2)
\]

This equation states that the sum of the viscous and turbulent shear stress is constant and equal to the wall shear stress \( \tau_w = \mu \left( \frac{\partial u}{\partial y} \right)_w = \rho u_\tau^2 \). Introducing the non-dimensionalization, \( U^+ = U/u_\tau \), \( y^+ = yu_\tau/\nu \) and \( \nu_t^+ = \nu_t/\nu \), equation (2) becomes

\[
(1 + \nu_t^+) \frac{dU^+}{dy^+} = 1 \quad (3)
\]

The linear law, \( U^+ = y^+ \), follows for the viscous sublayer where \( \nu_t^+ << 1 \). In the logarithmic layer, \( \nu_t^+ \) is large and the equation (3) is usually approximated with \( \nu_t^+ \frac{dU^+}{dy^+} = 1 \). Using Prandtl’s assumption for the turbulent viscosity [4], \( \nu_t^+ = \kappa y^+ \), the logarithmic law \( U^+ = \frac{1}{\kappa} \log(y^+) + B_{log} \) follows (the experimentally fitted constants are \( \kappa = 0.41 \) and \( B_{log} = 5.0 \)).

Suppose \( \nu_t^+ \) were known. Then (3) could be integrated to find the universal function \( U^+(y^+) \). An early approach was to assume a form for that function, such as Spalding’s profile [5]. However, it is more consistent to develop the function that is appropriate to each model by solving the wall layer equations numerically. Now, knowing the universal function \( U^+(y^+) \), the friction velocity can be solved from the computed velocity
at the first grid point. Say that is \( U_1 \), evaluated at \( y_1 \). If this lies in the wall layer then

\[
Re_1 \equiv y_1 U_1 / \nu = y_1^+ U^+(y_1^+) = F(y_1^+) \tag{4}
\]

\( Re_1 \) is the Reynolds number obtained from the first cell. The right is a universal function. Inverting this function gives

\[
y^+ = y_1 u_r / \nu = F^{-1}(Re_1)
\]

Hence, \( u_r \) can be found. Given \( F(y_1^+) \), the inversion can be done iteratively by Newton’s method. However, this can be done once and for all, and the inverse function stored in a table, as discussed in [1]. That is the method used in our computations. In a finite volume formulation, the friction velocity is used in the momentum balance. Hence, it serves as a boundary condition for computing \( U_1 \).

The eddy-viscosity is explicitly related to the velocity profile. In the entire region between the wall and the outer edge of the logarithmic boundary layer the eddy-viscosity is related to the velocity gradient according to the equation (3). In some wall function formulations the velocity profile and eddy-viscosity do not obey this relation. The result is grid dependence. One might use (3) to obtain a boundary condition on one turbulence variable, thereby ensuring consistency between the eddy-viscosity and the velocity gradient: for instance, in the \( k-\omega \) model the \( k \) condition can be obtained from (3), given a condition on \( \omega \). In the present paper, consistency comes from solving the model equation in the wall layer. Formally, the wall function should match smoothly to the outer, computational region. Consistent matching would produce a grid independent formulation — although the issue of numerical accuracy must be addressed, and the assumption of quasi-equilibrium is a caveat.

For the turbulence equations, analytical solutions can often be derived for the viscous sublayer and for the logarithmic layer. However, the intermediate region is problematic. This can be addressed by generating a look-up table for each non-dimensionalized turbulence variable from a well-resolved numerical solution. This table can take the form of spline coefficients.
Numerical implementation

The adaptive wall functions described above are implemented in a three-dimensional compressible RANS flow solver TFLO2000 [6]. TFLO2000 has been recently developed at Stanford under the scope of the Department of Energy’s (DoE) ASC (Advanced Simulation and Computing) program. It is a state-of-the-art multiblock flow solver, based on a finite-volume formulation, a modified Runge-Kutta scheme, a central-difference discretization augmented with artificial dissipation terms and the multigrid solution procedure to enhance convergence to a steady state. The wall functions are implemented in the following way. The $u$-velocity component requires a condition that ensures the correct shear stress at the wall. To compute the correction, $u_r$ is needed. As explained in [1], using the values $y_1$ and $U_1$ from the first cell center a local Reynolds number $Re_1$ is computed. A value of $u_r$ is obtained from the corresponding look-up table. Look-up tables are also employed for the turbulence variables. An off-wall boundary condition is adopted for the turbulence variables by enforcing the first cell center values.

The look-up tables were created from an accurate numerical wall integration solution of a flow over a flat plate at zero pressure gradient. At a sufficiently high Reynolds number ($Re_\theta > 5000$) profiles of non-dimensional variables were approximated using cubic splines. For variables with large variations in the boundary layer the curve fit is done for their logarithm (e.g. $log \omega^+$ is fitted in place of $\omega^+$). This reduces the interpolation error.

Flow over a flat plate

Wall functions are designed to be used with coarse near-wall grids. The solution of the discrete RANS equations is associated with a numerical error that increases with decreasing grid resolution. The difficulty in testing wall function implementations is to distinguish between this numerical error and inaccuracies that may result from the physical model (boundary condition). What we will call a $\delta$-grid eliminates the numerical error and provides a test for the correctness of the applied boundary condition. In the $\delta$-grid, a wall integration grid is shifted by a distance $\delta$ into the flow to provide the desired $y^+$ location of the first cell (see [1]). The wall functions provide the desired boundary conditions. At the same time, the grid resolution and the associated numerical error is of the same order as for the wall integration. Note that this grid is only used to test the physical boundary conditions. Standard grids are subsequently used to test the adaptive wall functions.
Flow over a flat plate was solved to the downstream location defined by momentum thickness based Reynolds number of $Re_\theta = 7700$. All turbulence models have been solved on $\delta$-grids with first cell center at $y^+ = 0.11, 1.1, 2.5, 5, 11, 25, 111$. After considering the $\delta$-grid, solutions are given on standard coarsened grids with the same first cell center $y^+$ values.

When $\delta$-grids are used with the correct boundary conditions, the computations collapse onto the wall integration profile. This is true for both models, as shown in Figure 1. Since the results for all the grids are practically coinciding, the colors of the curves are chosen to alternate between grey and black to visualize the first grid point for each grid. The results obtained on “classic” coarse grids are presented in Figure 2. The spread in the results is the consequence of numerical errors. This is inferred from the fact that solutions on the $\delta$-grids collapse on the wall integration result.

The spread of the $U^+$ profiles is very important. In the defect region, this spread will affect the skin friction prediction. The numerical error is dependent on the turbulence model. The Spalart-Allmaras model seems more sensitive to the location of the first cell center. The results also show that the error is largest when the first cell center is located in the region $5 < y^+ < 11$, despite the use of correct boundary conditions. The deviation of the eddy-viscosity $\nu_i^+$ for grids with the first cell center at $y^+ = 25$ or $y^+ = 111$ is present for all models. That can be explained by the relative coarseness of the grid throughout the boundary layer.

The investigation presented in [1] has shown that the numerical error discussed in the previous section is mainly of local character. An increase in the accuracy of the momentum flux at face 12 (0 is the halo cell, 1 is the wall adjacent cell and 2 is its first neighbor) has a significant impact on the results. Although the eddy-viscosity is a highly non-linear function of wall distance in the viscous sublayer and the buffer region, the numerical schemes usually use linear interpolation to compute the eddy-viscosity at face 12 when computing the diffusive flux. A simple remedy is to enforce the correct value of $\nu_{i12}^+$ using the corresponding $y_{i12}^+$ from the look-up table. This can be seen as having a more accurate interpolation at our disposal. Alternatively, the interpolation at 12 can be done directly for turbulence variables (e.g., for Spalart-Allmaras model one would first interpolate $\tilde{\nu}_{12}$ and then compute $\nu_{i12}^+$ from $\tilde{\nu}_{12}^+$). The error in the turbulence equations is also of local character, as shown in [1]: by fixing the correct turbulence variables values at 2, the error has been almost completely eliminated.
Velocity and eddy-viscosity profiles (a) and $k$ and $\omega$ profiles (b), $k$-$\omega$ model

Velocity and eddy-viscosity profiles, Spalart-Allmaras (c)

Figure 1: Application of adaptive wall functions using $\delta$-grids

Velocity and eddy-viscosity profiles (a) and $k$ and $\omega$ profiles (b), $k$-$\omega$ model

Velocity and eddy-viscosity profiles, Spalart-Allmaras (c)

Figure 2: Standard application of adaptive wall functions using coarse grids
APPLICATIONS

Flow over a ramp

Incompressible flow over a two-dimensional ramp was studied experimentally at Stanford [7], where detailed experimental database was assembled for a range of Reynolds numbers (the intent was to study the effect of Reynolds number on the recirculation). The ramp is a circular arc with the radius \( R = 1.814L \), where \( L \) is the length of the ramp. The height of the ramp is \( h = 0.3L \) and the height of the channel upstream of the ramp is \( H = 1.871L \) (see Figure 3). We considered flows at two Reynolds numbers: \( Re_L = U_\infty L/\nu = 7 \cdot 10^5 \) and \( Re_L = 10^5 \), with the momentum thickness based Reynolds number for the velocity profile at \( x/L = -0.2 \) of \( Re_\theta = 20100 \) and \( Re_\theta = 3400 \), respectively.

Several grids with different values of the first cell center \( y^+ \) were used in this analysis. For both Reynolds numbers, four grids were generated with \( y^+_1 \) of 1, 5, 17.5 and 85 in the channel upstream of the ramp. The grid size varies from \( 161 \times 121 \) cells for the finest grid to \( 161 \times 81 \) for the coarsest grid. Despite the constant size of the wall adjacent cells, the first cell center \( y^+ \) varies significantly in the recirculation region, as shown in Figure 4a. Convergence curves for Spalart-Allmaras model using those four grids are shown in Figure 4b. Note that the convergence is improved as the grid is coarsened.

First, we present the results for \( Re_\theta = 20100 \). Skin friction coefficient distribution for the Spalart-Allmaras and \( k-\omega \) models is shown in Figures 5a and 5b. As already discussed in [1], even for recirculating flows the numerical solution in the near-wall region scales similarly to the flow over flat plate. In addition, \( y^+ \) depends on \( u_\tau \) and therefore the actual \( y^+ \) in the recirculation is much smaller than \( y^+ \) at the inflow on a given grid.
(e.g., it drops from 85 to 15). The results for Spalart-Allmaras and $k$-$\omega$ model practically coincide on all four grids. In the recirculation region (as well as in the recovery region) the results for $k$-$\omega$ model with the coarsest grid ($y^+ = 85$) start to deviate from the other grids. Similar behavior was also observed in the recirculation flow test case discussed in [1] and it was attributed to the fact that kinetic energy of turbulence in the recirculation region differs from the universal solution at lower $y^+$ values than the velocity, as shown in Figure 7.

Figure 4: First cell center $y^+$ distribution for $Re_\theta = 20100$ (a); convergence for Spalart-Allmaras model (b).

Figure 5: Skin friction coefficient for $Re_\theta = 20100$; Spalart-Allmaras (a) and $k$-$\omega$ (b).

The comparison of tangential velocity profiles (in non-dimensional form) computed with the Spalart-Allmaras model in the recirculation region, at $x/L = 1.0$ and $x/L = 1.2$, with the universal solution for the flat plate (the look-up table) is presented in Figure 6a. At $x/L = 1.0$ the universal solution is a relatively reasonable
boundary condition even for the coarsest grid \((y^+ = 85)\). However, at \(x/L = 1.2\) (closer to the reattachment point) the discrepancy is more significant. For the finer grids, the look-up table provides a good boundary condition at both locations. In Figure 6b, the streamlines for the Spalart-Allmaras solution computed on the finest grid \((y^+ = 1)\) are overlayed over the coarsest grid \((y^+ = 85)\). The resolution is still sufficient for the recirculation region to be captured accurately.

As discussed in [7], the size of the recirculation region for this flow problem depends only weakly on the Reynolds number. Therefore, same grid resolution is needed for the high and the low Reynolds number case. However, the same grid resolution does not mean the same \(y^+\) values (e.g. the grid with \(y^+\) of 85 for the high Re number will have a \(y^+\) of 15 for the low Re number). The coarsest grid with \(y^+ = 85\) for the low Reynolds number case is presented in Figure 8b, overlayed with the streamlines computed on the finest grid. Obviously, this grid is simply too coarse to properly capture the recirculation region. The skin friction coefficient distribution presented in Figure 8a shows exactly that - the flow does not separate on the coarsest grid. This shows that the criterion describing the wall function grid quality necessary for recirculating flows cannot be based solely on \(y^+\). It has to take into account the adequate resolution of the recirculating flow structure.
Figure 6: Profiles of tangential velocity $U_t^+$ in the recirculation region for $Re_\theta = 20100$ computed with Spalart-Allmaras model (a). Streamlines for $Re_\theta = 20100$ computed with Spalart-Allmaras model on the finest grid ($y^+ = 1$) overlayed over the coarsest grid ($y^+ = 85$) (b).

Figure 7: Profiles of tangential velocity $U_t^+$ and turbulent kinetic energy $k^+$ in the recirculation region for $Re_\theta = 20100$ computed with $k$-$\omega$ model.

Figure 8: Skin friction coefficient for $Re_\theta = 3400$ computed with Spalart-Allmaras model (a). Streamlines for $Re_\theta = 3400$ computed with Spalart-Allmaras model on the finest grid ($y^+ = 1$) overlayed over the coarsest grid ($y^+ = 85$) (b).
RAE2822 Airfoil

Transonic flow around the supercritical RAE 2822-airfoil for the flow conditions described as Case 9 in [8] involves shock/boundary layer interaction. The experimental data was obtained in the wind tunnel for the flow conditions: \( M_\infty = 0.73 \), \( Re = 6.50 \times 10^6 \), \( \alpha = 3.19^\circ \). In the experiments, transition has been tripped near the leading edge of the airfoil at \( x/c = 0.03 \) on both upper and lower surfaces of the airfoil. Transition was not modeled in the computations. To compare the experimental data with a flow around an airfoil in free-flight conditions, corrections to the tunnel data are required. Different wind tunnel corrections have been used in various studies published in the literature. The flow conditions used in the EUROVAL project [8] are adopted here. They are equal to \( M_\infty = 0.734 \), \( Re = 6.50 \times 10^6 \), \( \alpha = 2.54^\circ \).

The present computations were carried out with two grids. The finer grid consists of \( 256 \times 64 \) nodes and has 192 nodes on the airfoil surface. For this grid the \( y^+ \) value of the first cell above the wall is below 1. The far field is about 15 cord lengths from the airfoil. The coarser grid that consists of \( 256 \times 48 \) nodes was used with wall functions. It has been generated by coarsening the fine grid in the near-wall region and has the first cell center \( y^+ \) values in the range \( 1 \leq y^+ \leq 30 \), as shown in Figure 10.

Pressure and skin friction distributions for Spalart-Allmaras and \( k-\omega \) models are shown in Figures 11 and 12. The pressure distribution, particularly on the suction side, is influenced by the outer extent of the computational domain and is usually corrected by using a vortex correction to the far-field boundary condition, which was not used in the present computations. The results on the coarser grid differ from the wall integration solution only in the stagnation and transition regions. The pressure coefficient distributions agree quite well with the experimental data over the entire airfoil. While the difference in pressure distributions and shock locations between wall function and wall integration solutions is quite small, it becomes more apparent in the skin friction data. The assumption of fully developed turbulent boundary layer used in the wall functions is obviously not valid in the stagnation and transition regions. As for the flow over a ramp, the wall functions for Spalart-Allmaras are less dependent on the first cell center \( y^+ \) than for \( k-\omega \) model.
ONERA M6 Wing

Transonic flow over the ONERA M6 wing [9] has been computed for Mach number $M = 0.8395$, Reynolds number $Re_c = 11.72 \times 10^6$ based on the mean chord of $c = 0.646m$ and incidence $\alpha = 3.06^\circ$. This is a swept wing with a root chord of about $c_{root} = 0.8m$ and a span of about $b = 1.2m$. The airfoil profile is symmetric. The computational grid is a multiblock grid that consists of approximately 320000 cells. The inflow is located about 15 chord lengths into the far field. The surface mesh on the wing is $72 \times 32$ cells on both the upper and lower surfaces with the first cell center $y^+$ in the range of $1 \leq y^+ \leq 20$, as shown in Figure 14.

A comparison of pressure coefficient distribution computed with the Spalart-Allmaras and $k-\omega$ models with the experimental data at selected spanwise locations is shown in Figure 15. The corresponding pressure coefficient distribution on the upper wing surface and the location of the spanwise cross-sections are shown in Figure 13. In general, the results using the proposed wall functions show a good agreement with the experimental data despite the omission of pressure gradient in the formulation of wall functions. The largest discrepancy can be seen with the pressure distribution at station $y/b = 0.80$ which shows both branches of the shock merging prematurely. However, this is not due to the wall functions since this has also been observed on wall integration grids.
Figure 10: First cell center $y^+$ distribution for Spalart-Allmaras model.

Figure 11: Pressure coefficient and skin friction coefficient for Spalart-Allmaras model.

Figure 12: Pressure coefficient and skin friction coefficient for $k$-$\omega$ model.
Figure 13: Surface distribution of pressure coefficient for ONERA M6 computed using Spalart-Allmaras with the location of spanwise cross-sections with experimental data.

CONCLUSIONS

This paper presents efficient, accurate and robust adaptive wall functions for Spalart-Allmaras and $k$-$\omega$ models. These wall functions are based on look-up tables created for the flow over a flat plate with zero pressure gradient. It is shown that, in contrast to existing wall functions, the proposed wall functions impose the correct physical boundary condition even in the intermediate region. The consistency relation (3) between the velocity and eddy-viscosity is obeyed, resulting in grid independence.

The look-up table concept seems to be an accurate and practical approach. In addition, the individual look-up table for each turbulence models assures consistency of the boundary conditions with the actual numerical solution of the model. The method allows an explicit evaluation of the friction velocity $u_\tau$ which is usually obtained with an iterative method at much larger computational cost.

The adaptive wall functions were applied to complex flows with pressure gradient and recirculation on curvilinear grids. For recirculating flow the numerical solution in the near-wall region scales similarly to the flow
over a flat plate - that is the reason why wall functions function for these flows. The computation of flow over a ramp at two Reynolds numbers shows that the criterion describing the wall function grid quality for recirculating flow cannot be based only on the first cell center $y^+$. The recirculating flow structure also has to be sufficiently resolved.

For the transonic flow over the RAE2822 airfoil the wall functions work well over the entire airfoil except in the stagnation and transition regions. The assumption of fully developed turbulent boundary layer used in the wall functions is obviously not valid in those regions. An example of successful application for three-dimensional flows is shown for the ONERA M6 wing.

References


Figure 14: $y_1^+$ distribution for ONERA M6 wing computed with Spalart-Allmaras model.
Figure 15: Pressure coefficient distribution for ONERA M6 wing computed with Spalart-Allmaras and $k$-$\omega$ model.